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AN IMPROVED MODEL FOR THE SUCKER ROD PUMPING SYSTEM

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Abstract

In this paper, three-dimentional vibration of the sucker rod string, tubing and liquid column in the sucker rod pumping system has been comprehensively studied. A new set of partial differential equations describing the movement regularity of sucker rod pumping systems was obtained and numerical solutions of the partial differential equations were found using finite difference technique in a manner of linear analysis. This mathematical model was used to evaluate performance of the sucker rod pumps (pumping rate as a function, number of strokes, stroke length, rod length, plunger to cylinder diameter ratio etc.). The results have shown that the developed model is much superior over the existing one-dimentional or two-dimentional models. This model can be used as a powerful tool for predicting the dynamic parameters of a sucker rod pumping system and well site diagnosis.

Introducton

Sucker rod pumping systems are used in approximately 70% of artificially lifted wells. In view of this wide application, it behooves the industry to have an accurate understanding of the sucker rod pumping process. Recently there has been a substantial effort to develop sucker rod pumping models that can be solved with the aid of a computer[1]-[3]. These efforts, however, have been restricted solely to the analysis of the dynamics of the sucker rod string, i.e. one-dimensional vibration. As such, these model ignore the dynamics of the fluid and tubing columns as well as the physical properties of the fluid.

The study of Doty and Schmidt[4] overcame one of these shortcomings by including the dynamics of the fluid as well as the rod. So it is possible to analyze the effects of fluid physical properties on a sucker rod pumping system. This model has been called two-dimensional vibration which is more accurate than one-dimensional vibration model. In the above study, the following assumption is made: tubing is anchored. In fact, there are many sucker rod pumping installations in which tubing is not anchored.

In this paper, three-dimensional vibration of the sucker rod string, tubing and liquid column in the sucker rod pumping system has been comprehensively studied. The purpose of current study is to illustrate the effect of fluid properties and vibration of tubing on a sucker rod pumping installation. It is hoped that this study and techniques which may evolve from it, will prove to be the tool needed by industry to obtain the most efficient use of rod pumping equipment.

Nomenclature

A:distance between O_2 and O_5 in Fig.2 (m)
A_c:rod coupling area (m^2)
A_h:tubing external area (m^2)
A_p:plunger area (m^2)
A_r:rod area (m^2)
A_t:tubing internal area (m^2)
B:distance between O_3 and O_4 in Fig.2 (m)
b_r,b_{fr}:coefficients defined by eqn.(13) (NS/m^2)
b_t,b_{ft}:coefficients defined by eqn.(16) (NS/m^2)
C:distance between O_2 and O_3 in Fig.2 (m)
C_1:fluid friction factor
C_2:rod coupling friction factor
C_f:sound speed in the liquid column (m/s)
C_r:sound speed in the rod (m/s)
C_t:sound speed in the tubing (m/s)
D_c:coupling diameter (m)
D_h:tubing external diameter (m)
D_p:plunger diameter (m)
D_r:rod diameter (m)
D_t:tubing internal diameter (m)
E_f:modulus of elasticity for liquid column (n/m^2)
E_r:modulus of elasticity for rod (n/m^2)
E_t:modulus of elasticity for tubing (n/m^2)
F_f:viscous force per unit length of liquid column (n/m)
F_r:viscous force per unit length of rod (n/m)
F_t:viscous force per unit length of tubing (n/m)
g:gravitational constant (m/s^2)
J:distance between O_2 and O_4 in Fig.2 (m)
K:distance between O_1 and O_2 in Fig.2 (m)
L:length of all of sucker rod string (m)
l:length of one sucker rod string (m)
N:crank angular speed (1/min)
PR:ratio of stroke
P:tubing pressure at pumping cavity (n/m^2)
P_f:fluid pressure (n/m^2)
P_0:tubing head pressure (n/m^2)
P_1:casing pressure at plunger level (n/m^2)
Q_r:rod tension (n)
Q_t:tubing tension (n)
R:crank length (m)
R_e:Reynolds number for the fluid
R_e':Reynolds number for the rod and the tubing
S:polished-rod stroke (m)
t:time (s)

V_f:fluid velocity (m/s)
V_r:rod velocity (m/s)
V_t:tubing velocity (m/s)
X:depth below polished rod (m)
Z_i: transformation defined by eqns.(20)
η:liquid dynamic viscosity (ns/m)
λ:stroke lose (m)
μ:damping coefficient between plunger and cylinder (ns/m)
ρ_f:fluid density (kg/m^3)
ρ_r:rod density (kg/m^3)
ρ_t:tubing density (kg/m^3)
ω:crank angular speed (r/s)
θ:crank angle (r)
α,β,ϵ,ξ,ϕ,ψ:angles in Fig.2 (r)

Formulation of the Problem

Fig.1 shows a sucker rod pumping system. To simplify the analysis the following assumptions are made:

(a) The conventional pumping unit has been used. The prime mover has no slip, the crank angular speed ω is a constant.

(b) Fluid column contains no gas, and ρ_f, P_0, P_1, η are constants.

(c) The valve resistance is ignored, and μ is constant.

The equation of the motion of the rod string has been yielded according the theory of vertical vibration of a rod:

$$\rho_r A_r \frac{\partial V_r}{\partial t} = \frac{\partial Q_r}{\partial X} + \rho_r g A_r - F_r \tag{1}$$

The second equation governing the motion of the rod relates the amount of rod deformation to the tention in the rods. Here Hooke's law is taken to obtain:

$$E_r A_r \frac{\partial V_r}{\partial X} = \frac{\partial Q_r}{\partial t} \tag{2}$$

It is also assumed that the rod has a constant modulus of elasticity and that Hooke's law applies.

Similarly, the first-order partial differential equations which describe the motion of the tubing are:

$$\rho_t(A_h - A_t)\frac{\partial V_t}{\partial t} = \frac{\partial Q_t}{\partial X} + \rho_t g(A_h - A_t) - F_t \tag{3}$$

$$E_t(A_h - At)\frac{\partial V_t}{\partial X} = \frac{\partial Q_t}{\partial t} \tag{4}$$

According to the assumption (b), the liquid column could be known as an elastic object which can not be pulled and the equations of the motion of it are:

$$\rho_f(A_t - A_r)\frac{\partial V_f}{\partial t} = -(A_t - A_r)\frac{\partial P_f}{\partial X} + \rho_f g(A_t - A_r) - F_f \tag{5}$$

$$E_f \frac{\partial V_f}{\partial X} = -\frac{\partial P_f}{\partial t} \tag{6}$$

In fact, the liquid density, ρ_f changes very little when the pressure changes, so it is considered as constant. Thus, the motion of liquid is linearizated. Notice that the liquid pressure P_f is always positive.

The function, F_r of eqn.(1) is the force per unit length of the rod arising from the viscous force of the fluid acting on the rod surface. The equation for F_r is [4]:

$$F_r = 0.5\rho_f V_f|V_f|\pi D_r C_1 + 0.5\rho_f V_f|V_f|(A_c - A_r)C_2/l \tag{7}$$

where, C_1 and C_2 are friction factors. Experiments were performed by Valeev and Repin to determine the dimentionless friction factors [2]. For the sucker rod they obtained the equations for friction factors as:

$$C_1 = \frac{24}{R_e}[1 \pm \frac{R_e'}{R_e}(0.2 + 0.39\frac{D_r}{D_t})] \tag{8}$$

$$C_2 = \frac{5.2 \times 10^4}{R_e}(\frac{D_c}{D_t} - 0.381)^{2.57}(2.77 \pm 1.69\frac{R_e'}{R_e}) \tag{9}$$

where, the Reynolds number R_e and R_e' are associated with the liquid and rod velocities. They are defined as:

$$R_e = \frac{V_f(D_t - D_r)\rho_f}{\eta} \tag{10}$$

$$R_e' = \frac{V_r(D_t - D_r)\rho_f}{\eta} \tag{11}$$

So, eqn.(7) can be written as:

$$F_r = b_r V_r - b_{fr} V_f \tag{12}$$

where:

$$b_r = 12\pi\eta[(0.2 + 0.39\frac{D_r}{D_t}) + 915.42(\frac{D_c}{D_t} - 0.381)^{2.57} ((\frac{D_c}{D_r})^2 - 1)D_r](\frac{D_t}{D_r} - 1)^{-1} \tag{13}$$

and

$$b_{fr} = 12\pi\eta[1 + 1500.42(\frac{D_c}{D_t} - 0.381)^{2.57} ((\frac{D_c}{D_r})^2 - 1)D_r](\frac{D_t}{D_r} - 1)^{-1} \tag{14}$$

similarly:

$$F_t = b_t V_t - b_{ft} V_f \tag{15}$$

where:

$$b_t = 12\pi\eta(0.2 + 0.39\frac{D_r}{D_t})(\frac{D_t}{D_r} - 1)^{-1} \tag{16}$$

$$b_{ft} = 12\pi\eta(\frac{D_t}{D_r} - 1)^{-1} \tag{17}$$

and

$$F_f = -(F_r + F_t) \tag{18}$$

Boundary Conditions

1. Surface Boundary Conditions (X=0)

Fig.2 shows a convetional pumping unit. The motion of the polished rod is determined by the geometry of the surface pumping unit and the torque-speed characteristics of its prime mover. By determining the motion of the polished rod, one can formulate a surface boundary condition of the sucker rod string as:

$$V_r(0,t) = -\frac{A}{C}R\omega\frac{\sin\alpha}{\sin\beta} \tag{19}$$

Let the value of ω be positive when the crank rotates clockwise, and

$$\theta = \omega t \tag{20}$$

where, the crank angular speed ω is constant according to the assumption (a).
Eqn.(19) is obtaind from the general solution of the "four-bar" linkage problem and can be used to describe the kinematics of any modern pumping unit. Notice

$$\alpha = \phi - \theta + \beta + \psi \tag{21}$$

where:

$$\psi = \begin{cases} \epsilon - \xi & sin(\theta - \phi) \geq 0 \\ \epsilon + \xi & sin(\theta - \phi) < 0 \end{cases} \tag{22}$$

and

$$\beta = cos^{-1}(\frac{C^2 + B^2 - J^2}{2CB}) \tag{23}$$

$$\epsilon = cos^{-1}(\frac{C^2 + J^2 - B^2}{2CJ}) \tag{24}$$

$$\xi = cos^{-1}(\frac{K^2 + J^2 - R^2}{2CJ}) \tag{25}$$

where:

$$J = \sqrt{K^2 + R^2 - 2KRcos(\theta - \phi)} \tag{26}$$

Notice, when the crank rotates anticlockwise, the negative sign of eqn.(19) should cancel.
The surface boundary condition of the tubing is:

$$V_t(0, t) = 0 \tag{27}$$

Whereas the surface boundary condition of the liquid column is:

$$P_f(0, t) = P_0 \tag{28}$$

2. Downhole Boundary Conditions (X=L)
Fig.3 shows a downhole pump. One can get three equilibrium equations with the theory of mechanics.

(a) The equilibrium equation of the pumping plunger is:

$$\begin{aligned} Q_r(L,t) + PA_p - P_f(L,t)(A_p - A_r) \\ -\mu[V_t(L,t) - V_r(L,t)] = 0 \end{aligned} \tag{29}$$

(b) The equilibrium equation of the cylinder is:

$$\begin{aligned} Q_t(L,t) - PA_p - P_f(L,t)(A_t - A_p) + A_h P_1 \\ -\mu[V_r(L,t) - V_t(L,t)] = 0 \end{aligned} \tag{30}$$

(c) The equilibrium equation of the flow is:

$$\begin{aligned} (A_t - A_r)V_f(L,t) = (A_t - A_p)V_t(L,t) \\ +(A_p - A_r)V_r(L,t) \end{aligned} \tag{31}$$

For different situations of valves, eqns.(29) through (31) will be different.

Initial Conditions

Let t be zero, when the polish rod is located at the lowest point and the traveling valve is open, i.e. P=P_f.

The initial conditions of eqns.(1) through (6) are given by:

$$\begin{aligned} V_r(X,0) &= 0 \\ V_t(X,0) &= 0 \\ V_f(X,0) &= 0 \\ Q_r(X,0) &= A_r[\rho_r g(L-X) - \rho_f gL - P_0] \\ Q_t(X,0) &= \rho_t g(A_h - A_t)(L-X) \\ &+ (\rho_f gL + P_0)A_t - A_h P_1 \\ P_f(X,0) &= \rho_f gX + P_0 \end{aligned} \tag{32}$$

If the preceding initial conditions are used, then the computer program need to run only for three pumping cycles to damp out effectivelly the start-up transients.

Solutions and Analyses

To obtain the numerical solution of eqns.(1) through (6), the finite difference method was used.
To simplify the analysis, we assume that the operation of sucker rod pumping systems is normal and simple rod string is used.
Operating the computer program, inputing the known data of the sucker rod pumping systems, one can get the values of Q_r,Q_t, P_f,V_r,V_t,V_f at any X, then draw the dynamographs of any position. The results can be called predicting of systems.
Fig.4 shows the effects of the dynamic viscosity of the liquid column. Here μ=0, N=-12, L=1400, P_0=0 and R=1.2.
As the value of η increases, the corresponding dynamograph will be "fat". This means that in a case where η is large, the work done by the pumping unit is also larger.
One interesting effect of ω is shown in Fig.5. Here μ=0, η=0.025, L=1400, P_0=500000 and R=1.2. Similar regularity is depicted by Doty and Schmidt[4], but obviously, the results are more accurate. For comparing the results of three mathematical models, the computer programs for one-dimensional vibration model and two-dimensional vibration model were used. Fig.6 shows the results obtained from the three mathematical models. Here (a), (b) and (c) represent:

three-dimensional vibration model of the sucker rod string, tubing and liquid column;

two-dimensional vibration model of the sucker rod string and liquid column;
one-dimensional vibration model of the sucker rod string respectively.

From curve(b) one can find that the time when standing valve is open, shifts to an earlier. Because in (b) it is assumed that the tubing is anchored. Curve(b), however, is closer to curve(a) than curve(c), because the vibration of liquid column in (c) is ignored. This becomes specially important for analysing performance of rod pumps lifting heavy oils from the well.

Conclusion

The new mathematical model of three-dimensional analysing of the vibration of sucker rod string, tubing and liquid column provides useful information on the design and operation of sucker pumping installations. The results indicate that the new model is an improvement over the existing one-dimensional and two-dimensional models. The new model can be used as a powerful tool for predicting the behavior of sucker rod pumping systems. It is also a potential tool for well site diagnosis.

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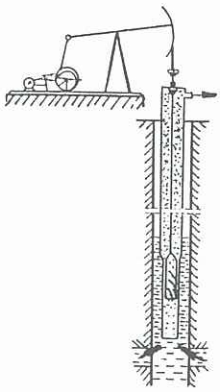

Fig. 1

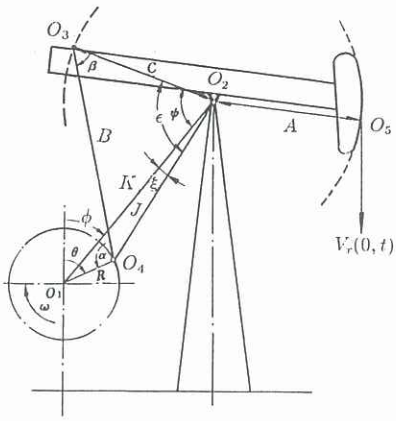

Fig. 2

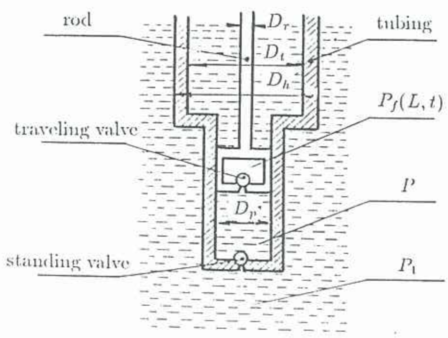

Fig. 3

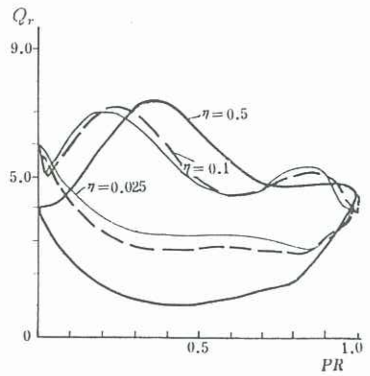

Fig. 4

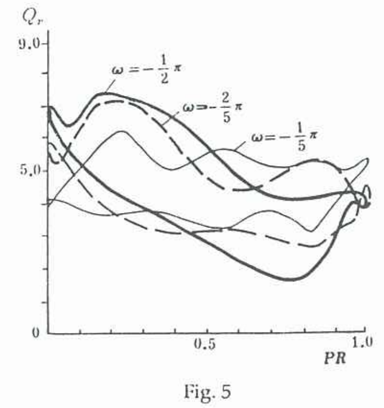

Fig. 5

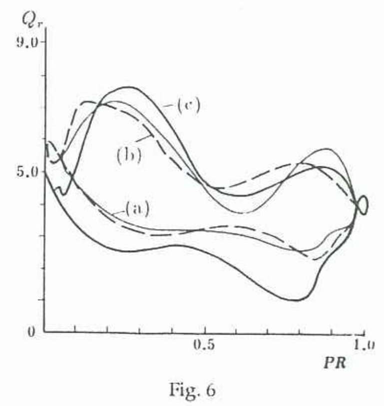

Fig. 6