

## HYDROMECHANICS OF MOORED OFFSHORE STRUCTURES : PREDICTION OF ENVIRONMENTAL LOADS, MOTION RESPONSES AND MOORING FORCES

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### ABSTRACT

Recent advances in numerical hydrodynamics now enable complete analytical treatment of the behaviour of some complex hydraulic structures. In this paper we describe a method of using wind, wave and current data, to develop analytical model to predict the behaviour of offshore structures moored in shallow and confined rough waters. The model is employed to select suitable mooring systems and predict mooring forces and motion responses of such offshore structures, taking into account both cost of the moorings and safety of the structure. The effects of nonlinear viscous drag and wave drift damping on the mooring system behaviour are considered. Mooring lines are simulated by lumped masses in which current drag forces and elastic properties are included. A method of reducing motions and highly peaked tension forces is also proposed.

Hydrodynamic forces of the structure are obtained by the 3D Green's function/integral equation method in which the irregular frequencies are removed by imposing an interior surface boundary condition.

Application is made for the mooring designs of a large floating dock, a storage tanker and a catamaran. Calculations are compared with subsequent measurements for the dock. Model experiments for two of the structures are underway and results will be published in due course.

### 1. INTRODUCTION

An offshore structure, moored in irregular waves, simultaneously experiences low frequency horizontal motions as well as wave frequency motions. In horizontal motions, restoring forces are only provided by the mooring line's stiffness which determines the system's horizontal natural frequency modes. Since mooring forces are usually small compared to the mass of the structure, natural periods in these modes will be large in comparison with wave periods. Furthermore, because damping forces are also small at low frequencies, the horizontal motion of the structure becomes large when the period of slow varying forces approaches the natural period. In shallow water depth, the mooring system is characterized by relatively small displacement capability when a catenary chain is adopted. The mooring system becomes stiff under the action of low frequency motions. When superimposed by wave frequency motions, the structure will shudder and mooring forces increase suddenly and may exceed safe levels. In the long term this leads to excessive line fatigue. To alleviate this problem, we suggest a method by which clump weights are inserted into the mooring lines at a certain distance from the anchor.

According to our calculations, this can lead to large reduction in both low frequency motion and maximum tension.

For accurate estimation of motion responses and mooring forces, the wave drift damping (Wichers(1982)) and viscous drag forces need to be taken into account.

### 2. THE EQUATION OF MOTION FOR A MOORING SYSTEM

Under the simultaneous actions of wind, current and random seawaves, the motion of the moored structure may be represented by the following second order time-dependent differential equations originally suggested by Van Oortmerssen (1976).

$$\sum_{k=1}^6 [(M_{kj} + m_{kj}) \ddot{\eta}_j + \int_{-\infty}^t K_{kj}(t-\tau) \dot{\eta}_j(\tau) dt + C_{kj} \dot{\eta}_j] = F_{wk}(t) - F_{mk}(t) + F_{wk} + F_{vdk}(t) \quad j, k = 1, 2, \dots, 6 \quad (1)$$

Where

$C_{kj}$ : hydrostatic restoration force coefficient,

$F_{mk}$ : nonlinear mooring loads,

$F_{wk}$ : wind force,  $F_{vdk}$ : viscous drag force,

$F_{wk}$ : total wave exciting force,

$K_{kj}$ : retardation function,  $m_{kj}$ : added mass matrix,

$M_{kj}$ : mass matrix of structure,

$\eta_j$ : displacement of structure; one dot on top means velocity and two for acceleration, and  $t$  is the time.

$K_{kj}$  is time dependent and can be written in terms of the frequency dependent damping coefficient  $B_{kj}$  through the Fourier transform

$$K_{kj} = \frac{2}{\pi} \int_0^{\infty} B_{kj}(\omega) \cos(\omega t) d\omega \quad (2)$$

The frequency-independent added mass  $m_{kj}$  is determined by

$$m_{kj} = A_{kj}(\omega') + \frac{1}{\omega'} \int_0^{\infty} K_{kj}(t) \sin(\omega' t) dt \quad (3)$$

where  $A_{kj}(\omega')$  is frequency-dependent added mass at some arbitrary frequency  $\omega'$ .

Since the unknowns  $\ddot{\eta}_j$ ,  $\dot{\eta}_j$  and  $\eta_j$  vary nonlinearly with time, the above equation of motion is solved iteratively. In order to prevent a shock load at the start of simulation, the ramp function of type  $(1.0 - e^{-\delta t})$ , where  $\delta$  is a small negative quantity, is applied. The equation is then solved for the acceleration according to Newmark (1959) method.

In equation (1), mooring forces are computed using lumped mass technique (Chen and Wang (1991)); which accounts for current drag forces and mooring line's elasticity.

### 3. EXTERNAL FORCES

#### 3.1 Wind forces

Mean wind forces, which may be assumed steady in the analysis of large moored offshore structure, are calculated from:

$$\begin{aligned} F_{W1} &= \rho_a V_w^2 C_{W1}(\theta_w) A_T \\ F_{W2} &= \rho_a V_w^2 C_{W2}(\theta_w) A_L \\ F_{W6} &= \rho_a V_w^2 C_{W6}(\theta_w) A_T L_{PP} \end{aligned} \quad (4)$$

where

- $A_L$ : projected windage lateral area,
- $A_T$ : projected windage transverse area,
- $C_{W1}$ ,  $C_{W2}$  and  $C_{W6}$  are wind forces and moment coefficients in surge, sway and yaw directions respectively.
- $L_{PP}$ : structure's length,
- $V_w$ : wind velocity,
- $\theta_w$ : wind direction angle,
- $\rho_a$ : mass density of the air.

#### 3.2 Viscous drag forces

Steady viscous drag forces on the submerged hull are calculated in analogous way to wind forces, where effective current velocity  $V$  is given by

$$V = \sqrt{V_C^2 + V_2^2} \quad (5)$$

in which

$$\begin{aligned} V_1 &= V_C \cos \beta_C + V_{W1} - \dot{\eta}_1 \\ V_2 &= V_C \sin \beta_C + V_{W2} - \dot{\eta}_2 \\ \theta_C &= \arctan(V_2/V_1) \end{aligned}$$

$V_C$  is the average current speed,  $\beta_C$  is the current heading angle.  $V_{W1}$  and  $V_{W2}$  are the wave velocity components in surge and sway directions, and  $\dot{\eta}_1$  and  $\dot{\eta}_2$  are the surge and sway structure's velocity components, respectively.

Furthermore, due to the structure's rolling motion, the nonlinear damping moments caused by cross-flow on the bottom and the sides are calculated by using the method of Kaplan(1982).

#### 3.3 Wave exciting forces and wave drifting forces

The two parameters ( $H_S$  and  $T_z$ ) modified Pierson-Moscovitz formula is used to represent the irregular seastate in spectral form. The significant wave height  $H_S$  accounts for the generating wind speed and the average zero up-crossing wave period  $T_z$  accounts for the spread of seawaves energy  $S(\omega)$  over the frequency range( $\omega$ ). This type of spectrum is particularly suitable for offshore work.

$$\frac{S(\omega)}{H_S^2 T_z} = \frac{4\pi^2}{(\omega T_z)^5} \exp\left\{-\frac{16\pi^3}{(\omega T_z)^4}\right\} \quad (6)$$

The time representation of the wave surface can then be written as

$$\zeta(t) = \sum_{n=1}^N a_n \sin(\omega_n t + \theta_n) \quad (7)$$

where

$a_n$  and  $\omega_n$  are the individual wave amplitude and wave frequency, respectively.  $N$  is the number of wave frequencies and  $\theta_n$  is the random phase angle. The time-dependent local wave envelope  $\lambda(t)$  and frequency  $\omega(t)$  are calculated respectively from

$$\lambda(t) = \sqrt{\zeta^2(t) + \eta^2(t)} \quad (8)$$

$$\omega(t) = \frac{d}{dt} [\arctan(\frac{\zeta(t)}{\eta(t)})] \quad (9)$$

where  $\eta(t)$  is the Hilbert transform of  $\zeta(t)$ .

The time dependent first order wave exciting force and second order wave drift force are calculated, respectively, from

$$F^{(1)}(t) = F^{(1)}(\omega_n) \cos(\omega_n t + \theta_n - \varepsilon_n) \quad (10)$$

$$\text{and } F^{(2)}(t) = \lambda^2(t) \{F^{(2)}(0, \omega(t)) - b(\omega(t)) \dot{\eta}\} \quad (11)$$

where

$F^{(1)}(\omega_n)$  and  $\varepsilon_n$  are the amplitude and phase of the first order frequency dependent wave exciting force.  $F^{(2)}(0, \omega)$  is the zero forward speed drift force transfer function.  $b(\omega)$  is referred to as the wave drift damping which is obtained from the derivatives of the drift force in relation to the forward speed.  $\dot{\eta}$  is the hull speed in the particular motion mode.

### 4. IRREGULAR FREQUENCY REMOVAL

The frequency-dependent hydrodynamic and wave exciting forces are obtained using a specially developed package utilizing the 3-D Green's function/boundary integral technique. It is necessary, however, to remove erroneous results at and in the neighborhood of irregular frequencies that occur at the eigen-frequencies associated with the adjoint interior potential problem. We extend Ohmatsu's(1983) 2D based Combined Integral Equation Method C.I.E.M., which he applied for deep water, to the 3D shallow water case. The combined integral equations to be solved for the unknown velocity potential  $\phi(P)$  are

$$-2\pi\phi_i(P) + \iint_{S_H} \phi_j(Q) \frac{\partial G_i(P, Q)}{\partial n_Q} dS(Q) = \iint_{S_H} n_j(Q) G_i(P, Q) dS(Q) \quad (12a)$$

$$\iint_{S_H} \phi_j(Q) \frac{\partial G_i(P, Q)}{\partial n_Q} dS(Q) = \iint_{S_H} n_j(Q) G_i(P, Q) dS(Q) \quad (12b)$$

$$\iint_{S_H} \phi_j(Q) \frac{\partial G_i(P, Q)}{\partial n_Q} dS(Q) = \iint_{S_H} n_j(Q) G_i(P, Q) dS(Q) \quad (12c)$$

$$\iint_{S_H} \phi_j(Q) \frac{\partial G_i(P, Q)}{\partial n_Q} dS(Q) = \iint_{S_H} n_j(Q) G_i(P, Q) dS(Q) \quad (12d)$$

where  $P$  is the field point  $(x, y, z)$ ,  $Q$  is the source point  $(\xi, \eta, \zeta)$ .  $S_{F_i}$  is the interior free surface.  $G_i(P, Q)$ ,  $G_{i_x}(P, Q)$  and  $G_{i_y}(P, Q)$  are the appropriate Green's function and its derivatives.  $n_j(Q)$  is the direction of the outward normal to the body surface  $S_H$ .

The above equations are solved numerically by dividing the submerged structure's surface  $S_H$  into a number of small facets. This results in a system of non-square overdetermined

complex equations which we solve by least squares orthonormalizing procedure.

Successful application of this method is demonstrated by the typical results shown in Figures 1 and 2 below, where comparison is made with results using the original Green Integral Equation Method (G.I.E.M.).

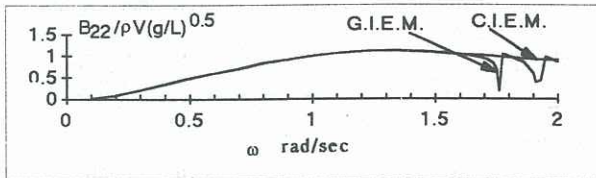


Fig. 1. Sway damping coefficient on floating dock.

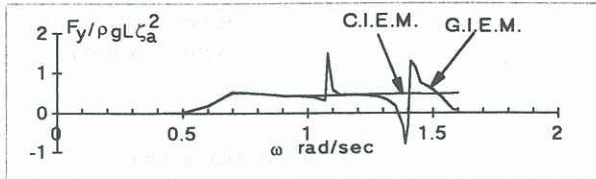


Fig 2. Drift force in sway on 216m tanker.

## 5. APPLICATIONS

### 5.1 216m storage tanker

A fully loaded storage tanker is moored in offshore location in 30m water depth. Mooring system (Fig.3) is to withstand environmental forces comprising wind speed (22kn), current surface speed (3.0kn) and a sea state ( $H_s=3.0m$ ,  $T_z=6.3s$ ); all assumed to act in the starboard-port direction.

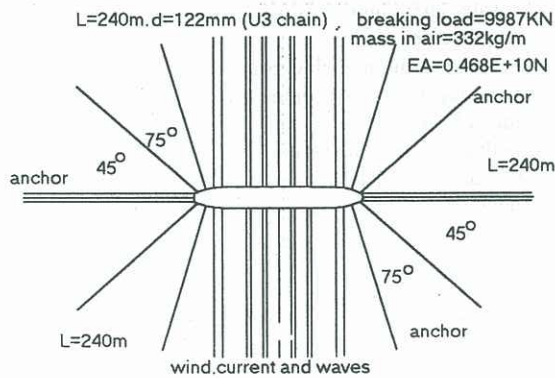


Fig. 3 Mooring system for the storage tanker

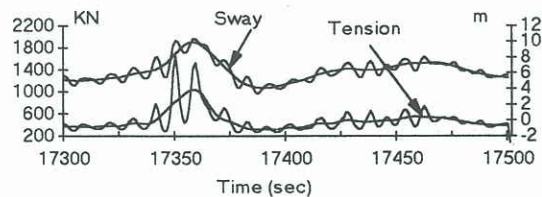


Fig. 4. Swaying motion and tension time series

Under the action of above environmental forces, mooring lines attached perpendicularly to the starboard side will experience largest tension. Applying the numerical model, computed sway motion and tensions in time series and spectral form are given in Fig. 4 and 5 respectively. The trend in the behaviour of the tension force follows that of the low

frequency large amplitudes sway motion. However, large peaks in the mooring lines tension forces are clearly shown to be the result of coupling between low frequency and the wave frequency motions. Accordingly, wave frequency dependent tension forces should not be ignored; see Fig. 5.

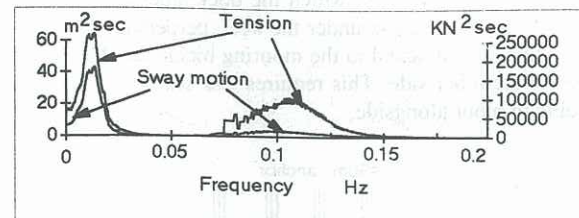


Fig. 5. Swaying motion and tension spectra

### 5.2 119m catamaran ferry

The catamaran is to be moored in 30m water depth. The environmental conditions comprise steady wind of 19knots, surface speed current 1.0knots and random seas of different  $H_s$  but all with  $T_z=7.0s$ . The mooring arrangement consists of four mooring lines: two lines are attached to the bow and two to the stern, all being parallel to the fore-and-aft direction of vessel. These are manufactured from 78mm(U2) steel chains of  $EA=0.1911E+10N$  and a breaking load of 3159kN. All the chains have the same length of 200m and moored at a level of 7.78m above water line. The horizontal distance between anchor and upper points is 185.0m. Results of calculations of surge wave drift damping, surge motion variance and tension maxima and tension variance are shown in Figs. 6, 7 and 8, respectively. It is evident that the importance of including wave drift damping in the prediction method increases with the significant wave height.

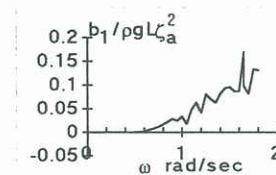


Fig.6. Surge wave drift damping on catamaran

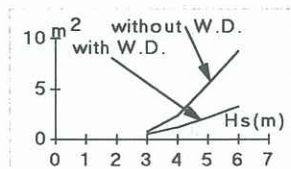


Fig 7. Effect of wave drift damping on surge motion var.

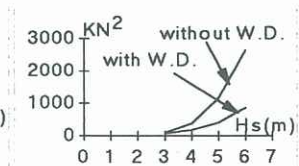
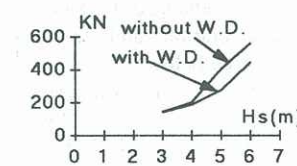


Fig. 8 Effect of wave drift damping on maximum tension and tension variance.

### 5.3 220 meters floating dock

This dock, moored in 17m of water depth offshore in a restricted searoom, is almost continuously used in ship-repairing and maintenance operations. When fully loaded with maximum size ship, the dock is required to remain operational with acceptable level of motion amplitudes and that its moorings should safely withstand the environmental conditions estimated as: wind velocity = 5knots; current speed = 3.0knots and seastate of  $H_s = 1.5m$ ,  $T_z = 6.32s$ . All the forces are assumed to be coming simultaneously in the abeam direction. The mooring arrangement is shown in Fig.9. To minimize heel due to strong wind, chains are attached to

the dock at a level of 3.8m above keel.  $y_0=82.5\text{m}$  (horizontal distance between upper and anchor points of side chains).

With this type of open moorings, however, because of the presence of the chains, ships are not able to moor alongside the dock. An alternative arrangement would be to adopt "cross moorings" in which the dock sides are fastened by chains and then cross under the keel, perpendicular to the dock's axis, and attached to the mooring blocks on the seabed opposite the other side. This requires less searoom and ships are able to moor alongside.

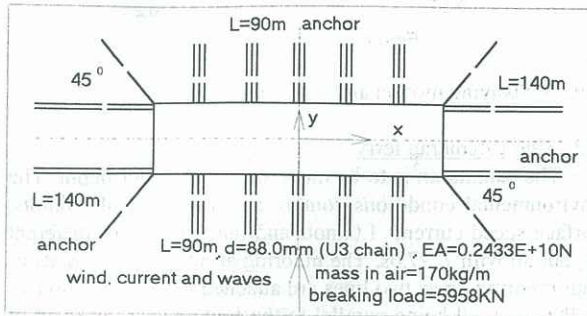


Fig. 9. Open mooring arrangement for the dock.

For this dock, which is moored within a confined searoom, transverse motion can be reduced by introducing clump weights into the mooring lines. Higher peak mooring loads are also reduced. The effects on motions and tension forces by the introduction of a clump weight, with different mass, at 72m along each of the side chains from the anchor point are summarized below:

clump weight tonnes	0	10	20	30
mean sway (m)	3.16	1.68	1.11	0.83
max. sway (m)	5.87	4.78	3.50	3.01
max. tension (kN)	2304.01	529.44	517.38	606.25
mean tension (kN)	114.38	231.07	357.86	486.01
$\sqrt{\text{tension var.}}$ (kN)	36.3	22.63	19.4	21.15

The above calculations, carried out for the "open mooring" case, demonstrate that introduction of clump weights will minimize motions and reduce highly peaked mooring forces, but the mean tensions are higher.

With 10 tonnes clump weight, attachment points of side chains are moved to the keel and  $y_0$  changed to 84.0m. Comparison between motions, acceleration and tension maxima of the two systems is summarized below.

	open mooring	cross mooring
maximum sway (m)	3.55	3.91
max/min roll (deg)	5.14/-4.80	4.73/-5.31
max/min accn. ( $\text{m/s}^2$ )	1.09/-1.18	1.20/-1.21
max tension (kN)	389.04	563.96
$\sqrt{\text{tension var.}}$ (kN)	45.48	57.88

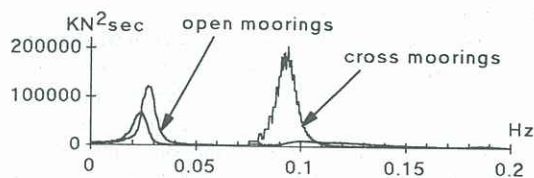


Fig. 10. Tension spectra (10 tonnes clump weight)

Tension forces seem to be strongly affected by the type of moorings but the maximum motion and acceleration are only marginally better in open moorings. Tension spectra of side chains are shown in Fig. 10, where it is indicated that

tension forces in cross moorings occur with larger amplitudes than in open moorings in the wave frequency region. This result is of important significance since the implication is that the fatigue life of the chains in the cross mooring is much less when compared with open mooring.

Recordings of roll motion of the dock with "cross moorings" and estimates of prevailing weather conditions during storms gave the following data.

wind velocity, knots	10.0	25.0	25.0	28.0	11.0
current speed, knots	3.0	5.0	-	5.0	3.0
$H_s$ , m	1.0	1.5	1.83	2.1	2.6
max. roll ampl. deg	1.5	4.5	2.0	5.0	4.0

It is to be noted that  $H_s$  has been estimated from observed wave height and wind speed is calculated at the center of pressure from quoted data. The direction being roughly abeam to the dock. Correlation between these observed values and calculations is, nevertheless, good.

## 6. CONCLUSIONS

Using wind, wave and current data, a numerical model has been developed which will reliably predict the behaviour of moored offshore structures. The model can be used in the design of mooring system. Results of applications made here show that:

- When a catenary chain system is to be adopted for the mooring of large structures in shallow waters, the effect on tension forces of wave frequency motions should be taken into account.

- The introduction of appropriately designed clump weights into the mooring lines can reduce the mean and slow varying motions of the structure and high peak mooring loads. In the same time, the shudder problem, experienced with shallow water chain moorings is alleviated and excessive line fatigue is prevented.

- Because of unfavorable coupling between motions of chain/ship attachment point in the wave frequency region, the amplitudes of tension forces in "cross mooring" system will be larger than those in "open mooring" system.

- For accurate determination of various hydrodynamic coefficients in the equations of motion of moored structures using the Green's function/integral equation method, it is necessary to remove erroneous results caused by the presence of irregular frequencies. For this we recommend the use of the "Combined Integral Equation Method" because of its simplicity and ease of implementation.

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