

WAVE FORCES ON A SHIP RUNNING IN QUARTERING SEAS - A SIMPLIFIED CALCULATION METHOD

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ABSTRACT

A simplified method to calculate wave induced forces acting on a ship is developed on the basis of Ohkusu's theory (Ohkusu, 1986) to analyze broaching. The method can be regarded as a limit of a strip method. Numerical results from this method more closely correspond to the experimental results than the Froude-Krylov Force on its own.

NOMENCLATURE

AE	aft end of the ship
B	ship breadth
B(x)	breadth of the section
c	wave celerity
d	ship draught
d(x)	draught of the section
FE	fore end of the ship
g	gravity acceleration
H	wave height
k	wave number
L	ship length
S(x)	area of the section
S _o (x)	added mass of the section
t	time
U	ship velocity
W	ship weight
ζ _w	wave amplitude
λ	wave length
ξ _w /λ	relative position of ship to wave
ρ	water density
φ	velocity potential
χ	heading angle
ω	wave frequency
ω _e	encounter frequency

INTRODUCTION

A ship travelling in quartering seas with a high forward speed may suffer an uncontrolled yaw behaviour, known as broaching. This can occur when the wave induced yaw moment exceeds that available from the rudder. Therefore, in order to determine whether broaching will occur it is necessary to be able to predict the wave forces. (Motora et al., 1981 and Renilson, 1982) Since in this situation the wave encounter frequency is very low, it was initially assumed that the Froude-Krylov components are dominant. (Wahab and Swaan, 1964) Later measurements of the

wave forces for a towed model in quartering seas, however, revealed that this is incorrect as the Froude-Krylov components alone does not predict the wave forces with sufficient accuracy. (Motora, et al., 1981) Since then a number of investigations have been carried out in an attempt to improve the theoretical prediction of the wave forces. Terao (1980) and Yoshino, et al. (1988) applied Chapman's high-speed slender body theory to this problem, and Ohkusu (1986) developed a new slender body theory under the assumption that the encounter frequency is very low, resulting in calculated values those correspond well with the experimental results. This paper presents a simplified calculation method based on Ohkusu's theory, which has been utilized to analyze broaching. (Umeda and Renilson, 1992)

CALCULATION METHOD

As can be seen in Fig.1, two coordinate systems are used: wave fixed with origin at a wave trough 0, the ξ axis in the direction of wave travel; and body fixed with origin at the centre of gravity G, the x axis pointing toward the bow, the y axis to starboard and the z axis downwards.

The ship is assumed to travel with a certain heading angle χ and a constant forward velocity U. For simplicity, the fluid is assumed to be inviscid and incompressible, and the effect of surface tension can be disregarded. Further, we assume small parameters as follows:

$$\begin{aligned} B/L, d/L &= O(\epsilon) & (1) \\ \omega_e &= \omega - (2\pi/\lambda)U \cos \chi = O(\epsilon) & (2) \\ z_1/\lambda &= O(\delta) & (3) \end{aligned}$$

where ε and δ are much smaller than 1. Here the incident wave z₁ is defined by the following velocity potential,

$$\begin{aligned} \phi_1 \exp(i\omega_e t) &= ig\zeta_w/\omega \\ &\cdot \exp\{-kz + i(kx \cos \chi - k y \sin \chi - \omega_e t)\} \end{aligned} \quad (4)$$

where k is ω²/g.

The diffraction potential in the near field is defined by a singular per-

turbation method based on Equations (1)-(3) as $\phi_D \exp(i\omega_e t)$ that ϕ_D satisfies

$$\nabla^2 \phi_D = 0 \quad (5)$$

$$\frac{\partial}{\partial z} (\phi_D) = -z_1 \frac{\partial^2}{\partial z^2} (\phi_s) \quad \text{on } z = 0 \quad (6)$$

$$\frac{\partial}{\partial n} (\phi_D) = -\frac{\partial}{\partial n} (\phi_1) \quad \text{on the body} \quad (7)$$

where n denotes the outward normal on the ship's hull surface. (Ohkusu, 1986) Here ϕ_s indicates the potential due to the ship running in calm water. (Tuck, 1964) We note that the non-homogeneous term of Equation (6) which Ohkusu does not include should be included by the principle of perturbation method.

Using

$$\phi_D' = \phi_D + z_1 \frac{\partial}{\partial z} (\phi_s) \quad (8).$$

Equations (5)-(7) are transformed as follows:

$$\nabla^2 \phi_D' = 0 \quad (9)$$

$$\frac{\partial}{\partial z} (\phi_D') = 0 \quad \text{on } z = 0 \quad (10)$$

$$\frac{\partial}{\partial n} (\phi_D') = -\frac{\partial}{\partial n} (\phi_1) + z_1 \frac{\partial}{\partial z} (U \frac{\partial}{\partial x} (n)) \quad \text{on the body} \quad (11).$$

Then, for the sake of simplicity, the following assumptions are made:

$$\frac{\partial}{\partial n} (\phi_1) = \frac{\partial}{\partial n} (\phi_1(z=d/2)) \quad (12)$$

$$\frac{\partial}{\partial z} \left\{ U \frac{\partial}{\partial x} (n(z=d/2)) \right\} = 0 \quad \text{on the body} \quad (13).$$

The former indicates that the effect of wave velocity can be approximated at the averaged depth. The latter assumes a hull form with wall sides and a flat bottom. As a result of these approximations, the following conditions are obtained:

$$\nabla^2 \phi_D' = 0 \quad (14)$$

$$\frac{\partial}{\partial z} (\phi_D') = 0 \quad \text{on } z = 0 \quad (15)$$

$$\frac{\partial}{\partial n} (\phi_D') = -\frac{\partial}{\partial n} (\phi_1(z=d/2)) \quad \text{on the body} \quad (16).$$

Since this boundary value problem of ϕ_D' is identical to that of a swaying cylinder in infinite fluid, it is easily solved by numerical methods, such as a boundary element method.

Then the horizontal wave force acting on the section of the ship, $w_w(x)$, is obtained as follows: (Ohkusu, 1986)

$$\begin{aligned} w_w(x) / \{ \rho \exp(i\omega_e t) \} &= i\omega \int \phi_1 n_D dS - U \int \frac{\partial}{\partial x} (\phi_D') n_D dS \\ &+ \int \left(\frac{\partial}{\partial y} (\phi_D') \frac{\partial}{\partial y} (\phi_1) \right. \\ &\quad \left. + \frac{\partial}{\partial z} (\phi_D') \frac{\partial}{\partial z} (\phi_1) \right) n_D dS \\ &+ \int \left(\frac{\partial}{\partial y} (\phi_1) \frac{\partial}{\partial y} (\phi_1) \right. \\ &\quad \left. + \frac{\partial}{\partial z} (\phi_1) \frac{\partial}{\partial z} (\phi_1) \right) n_D dS \\ &+ U \int \frac{\partial}{\partial x} (z_1 \frac{\partial}{\partial z} (\phi_1)) n_D dS \\ &- \int \frac{\partial}{\partial y} (z_1 \frac{\partial}{\partial z} (\phi_1)) \\ &\quad \cdot \frac{\partial}{\partial y} (\phi_1) n_D dS \\ &- \int \frac{\partial}{\partial z} (z_1 \frac{\partial}{\partial z} (\phi_1)) \\ &\quad \cdot \frac{\partial}{\partial z} (\phi_1) n_D dS \quad (17) \end{aligned}$$

where the integrals are carried out along the contour of the section under the

still water surface, C . The first term, which is the lowest-order one, is the Froude-Krylov force. Other are of the order which is higher than the lowest-order by ϵ . The second term represents the hydrodynamic lift due to velocities of wave particles. The third and fourth terms represent the interaction derived from square terms of the Euler pressure equation. The other three terms are derived from the non-homogeneous free surface condition.

For practical purpose, calculating the terms after the third one are too complex because they require solving the additional boundary value problem of ϕ_s . Thus the first two terms only are calculated.

Finally, the wave induced sway force Y_w and the wave induced yaw moment N_w are obtained as follows:

$$\begin{aligned} Y_w(\xi_0/\lambda, U, \chi, t) / \{ \exp(i\omega_e t) \} &= \rho g \zeta_w k \sin \chi \int c_1(x) S(x) e^{-k_d(x)/2} \\ &\quad \cdot \text{sink}(\xi_0 + x \cos \chi) dx \\ &+ \zeta_w \omega \sin \chi \int \rho S_y(x) e^{-k_d(x)/2} \\ &\quad \cdot \text{sink}(\xi_0 + x \cos \chi) dx \\ &- \zeta_w \omega U \sin \chi \left[\rho S_y(x) e^{-k_d(x)/2} \right. \\ &\quad \left. \cdot \text{cosk}(\xi_0 + x \cos \chi) \right] \quad (18). \end{aligned}$$

$$\begin{aligned} N_w(\xi_0/\lambda, U, \chi, t) / \{ \exp(i\omega_e t) \} &= \rho g \zeta_w k \sin \chi \int c_1(x) S(x) e^{-k_d(x)/2} \\ &\quad \cdot x \text{sink}(\xi_0 + x \cos \chi) dx \\ &+ \zeta_w \omega \sin \chi \int \rho S_y(x) e^{-k_d(x)/2} \\ &\quad \cdot x \text{sink}(\xi_0 + x \cos \chi) dx \\ &+ \zeta_w \omega U \sin \chi \int \rho S_y(x) e^{-k_d(x)/2} \\ &\quad \cdot \text{cosk}(\xi_0 + x \cos \chi) dx \\ &- \zeta_w \omega U \sin \chi \left[\rho S_y(x) e^{-k_d(x)/2} \right. \\ &\quad \left. \cdot x \text{cosk}(\xi_0 + x \cos \chi) \right] \quad (19). \end{aligned}$$

Here the integrals are carried out from the aft end, AE, to the fore end, FE, of the ship and

$$c_1(x) = \sin(kB(x)/2 \sin \chi) / (kB(x)/2 \sin \chi) \quad (20)$$

$$S_y(x) = \int_C \phi_D' n_D dS \quad (21).$$

This is identical to the strip method where the two dimensional added mass and wave damping at zero-frequency are used. Since the coefficients are independent of frequency, this can be directly applied to transient motions, such as broaching.

NUMERICAL RESULTS AND DISCUSSION

In order to compare the proposed method with existing captive model experiments (Matora, et al., 1981), it was applied to the small Japanese fishing vessel with principal particulars given in Table I. The calculation was carried out for $\omega_e = 0$, namely, $U/c = 1/\cos \chi$, while U/c in the experiments was 0.9. For the sway force, results from the present method are similar to the Froude-Krylov force and Ohkusu's theory but slightly smaller than the measured results, Fig. 2. As to the yaw moment, the Froude-Krylov component is much smaller than the measured results in amplitude, Fig. 3. The present method and Ohkusu's theory both

predicts the amplitude of measured values well, however, a small discrepancy still exists in the prediction of the phase. Since the present method agrees with Ohkusu's theory, the third and fourth terms of Equation (17) do not have a significant role. The measured amplitude of the yaw moment drastically increases as can be seen in Fig.5. The present method, that includes the hydrodynamic lift, explains this increasing tendency, while the Froude-Krylov component alone does not.

In conclusion, for the critical wave length, the present method more closely corresponds to the wave induced yaw moment obtained experimentally than the Froude-Krylov component on its own. This well allows a more accurate prediction of when broaching will occur.

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Table I Principal particulars of the ship

Length B.P.	L	7.14	[m]
Breadth	B	1.87	[m]
Draught fore	d_f	0.260	[m]
Draught midship	d_m	0.3835	[m]
Draught aft	d_a	0.507	[m]
L.C.B. (aft)	1/L	0.0842	
Block coefficient	C_B	0.471	

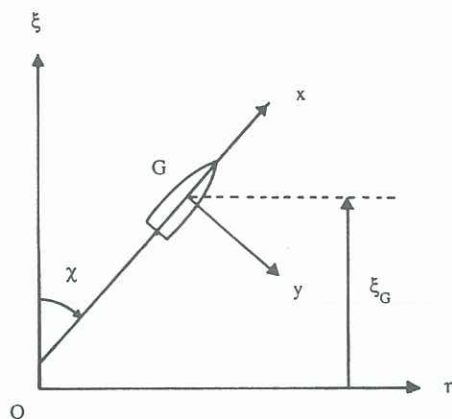


Fig.1 Coordinate systems

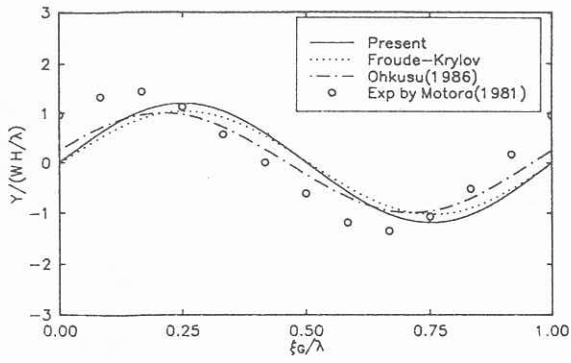


Fig. 2 Wave induced sway force ($\chi = 30^\circ$)

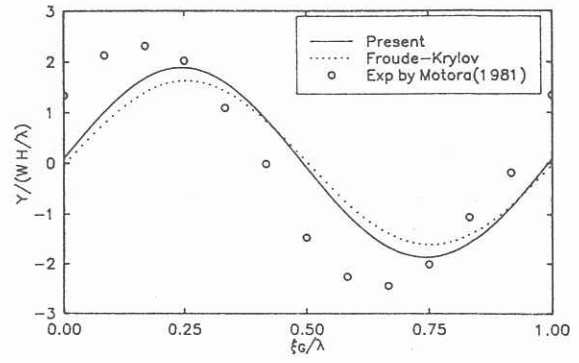


Fig. 4 Wave induced sway force ($\chi = 45^\circ$)

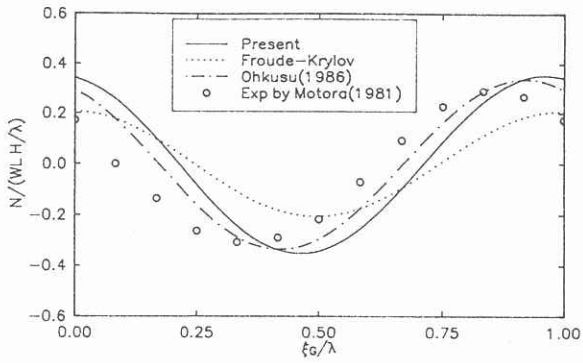


Fig. 3 Wave induced yaw moment ($\chi = 30^\circ$)

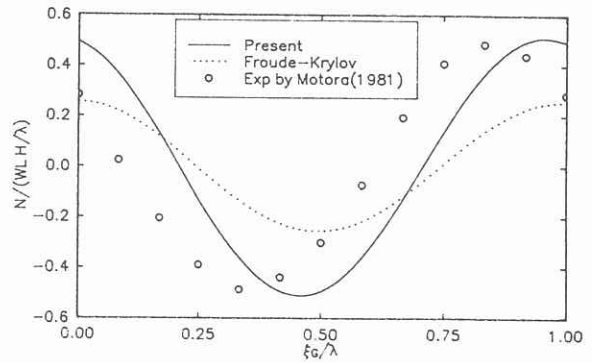


Fig. 5 Wave induced yaw moment ($\chi = 45^\circ$)