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THE TURBULENT SHEAR FLOW AROUND A ROTATING CYLINDER IN A QUIESCENT FLUID (AN INTERPRETATION OF THE TURBULENT FIELD EXPRESSED IN WAVE NUMBER SPACE)

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ABSTRACT

To obtain a physical insight into the processes controlling the structure of the shear flow around a rotating cylinder in a quiescent fluid, the spatial cross correlation equations and corresponding spectrum equations were derived using the rotating cylindrical coordinate system and, its structure was discussed. Some terms in the spectrum equations can be explained by the same concept used in the theory of isotropic turbulence, but most terms are characteristic of a rotating shear flow. In particular, the re-distributive transport function and the two-component transfer are clarified. A figure is obtained to depict the physical interpretation of the terms of the spectrum equations.

NOMENCLATURE

(See Figure 1. Suffixes do not obey the summation convention.)

- U: mean velocity component in the ϕ-direction relative to the rotating cylinder
- $\mathbf{k}$: wave number vector
- $\mathbf{r}$: metric vector

Suffixes 1,2,3: r-, ϕ- and y-directions, respectively: the coordinate system (x_1, x_2, x_3) correspond to (r, ϕ, y), and fluctuating velocities (u_1, u_2, u_3) correspond to (w, u, v).

Two-point correlations between points A and B are defined as follows.

- $Q_{i,j}$: two-point double correlation between two velocity components $= \overline{u_{iA} u_{jB}}$
- $S_{i,jk}$, $S_{ij,k}$: two-point triple correlation between three velocity components $= \overline{u_{iA} u_{jB} u_{kB}}$, $\overline{u_{iA} u_{jA} u_{kB}}$, respectively.
- $K_{p,}$: two-point correlation between a velocity component and pressure $= \overline{p_A u_{iB}}$
- $\psi_{i,j}$: energy spectraum tensor

The separation between two points is written

$r_s = r_B - r_A$, $\phi_s = \phi_B - \phi_A$, $y_s = y_B - y_A$

Other

- . : differential operator

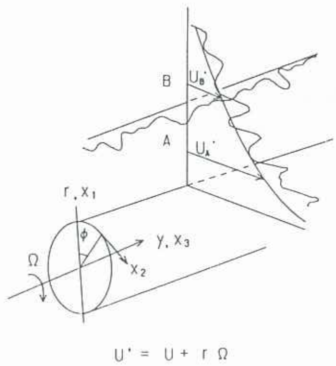


Fig. 1 Flow field and coordinate system

INTRODUCTION

The consideration of turbulent field based on the turbulent energy equation and the Reynolds stress equation has been established as the most effective method (Rotta, 1972). However, since a characteristic of turbulence is multi-scale structure in time and space, only one-point statistics, i.e., turbulent energy or Reynolds stress, is insufficient to understand the turbulent flow. Space double correlation of two-point statistics and their spectrum are important as the next step. However, the quantity to be measured is great and measurement is difficult, and so little study has been achieved (Favre 1965, Lai 1989). For rotating flows of practical importance, only the present authors have reported any results of investigations (Nakamura, et al., 1988a,b).

Rotta(1972) regarded a correlation as a kind of non-preserved quantity carried by fluid motion, e.g., a chemical quantity which mixes with fluid. The present study is concerned with $Q_{1,1}$, $Q_{2,2}$ and $Q_{3,3}$ related directly to one point energy and their spectrum.

SPECTRAL EQUATIONS AND RELATED CONSIDERATIONS

Two-Point Correlations

We consider the following two-point correlation equations for three components:

$$D_{1,1}+\left(\frac{U_B}{r_A+r_s}-\frac{U_A}{r_A}\right)Q_{1,1,\phi_s}+\frac{1}{r_A}(S_{11,1}r_A)_{,r_A}+\frac{1}{\rho}\frac{1}{r_A}(r_AK_{1,p})_{,r_A}+\frac{1}{\rho}\frac{1}{(r_A+r_s)}[(r_A+r_s)K_{p,1}]_{,r_A}-\frac{2U_A}{r_A}Q_{2,1}$$

I II III① III② III②′ V①

$$-\frac{2U_B}{r_A+r_s}Q_{1,2}-\frac{S_{1,22}}{r_A+r_s}-\frac{S_{22,1}}{r_A}+\frac{1}{\rho}\left(\frac{K_{p,2}}{r_A+r_s}-\frac{K_{2,p}}{r_A}\right)_{,\phi_s}-2\Omega(Q_{2,1}+Q_{1,2})+\frac{1}{\rho}(K_{p,3}-K_{3,p})_{,y_s}+E_{1,1}=0\ , \tag{1}$$

V① V② V③ VI① VI② VII

$$D_{2,2}+\left(\frac{U_B}{r_A+r_s}-\frac{U_A}{r_A}\right)Q_{2,2,\phi_s}+\frac{1}{r_A}(S_{12,2}r_A)_{,r_A}+Q_{1,2}r_A\left(\frac{U_A}{r_A}\right)_{,r_A}+Q_{2,1}(r_A+r_s)\left(\frac{U_B}{r_A+r_s}\right)_{,r_s}+\frac{2U_A}{r_A}Q_{1,2}$$

I II III① IV IV′ V①

$$+\frac{2U_B}{r_A+r_s}Q_{2,1}+\frac{S_{2,12}}{(r_A+r_s)}+\frac{S_{12,2}}{r_A}-\frac{1}{\rho}\left(\frac{K_{p,2}}{r_A}-\frac{K_{2,p}}{r_A+r_s}\right)_{,\phi_s}+2\Omega(Q_{1,2}+Q_{2,1})+E_{2,2}=0\ , \tag{2}$$

V① V② V③ VI① VII

$$D_{3,3}+\left(\frac{U_B}{r_A+r_s}-\frac{U_A}{r_A}\right)Q_{3,3,\phi_s}+\frac{1}{r_A}(S_{13,3}r_A)_{,r_A}-\frac{1}{\rho}(K_{p,3}-K_{3,p})_{,y_s}+E_{3,3}=0\ . \tag{3}$$

I II III① VI② VII

Here, $D_{i,j}$ and $E_{i,j}$ are the inertial and viscous terms, respectively. The terms of correlation equations are classified into two different types. In one the terms become zero as A coincides with B, and have no corresponding ones in a turbulent energy equation, namely terms I and II. In the other the terms have corresponding ones in the turbulent energy equation.

Spectral Equations

It is often convenient to understand the transfer of energy in the wave number space using spectral equations, i.e., Fourier transform of the correlation equations.

A pair of Fourier transforms is expressed by the symbols $\mathcal{F}(*)$ and $\mathcal{F}^{-1}(*)$. Symbols of Fourier transforms of correlations are defined by Eqs. 4(a)-(c):

$$Q_{i,j}(\boldsymbol{r},\boldsymbol{r}_A)=\mathcal{F}\,\phi_{i,j}(\boldsymbol{k},\boldsymbol{r}_A)\ , \tag{4a}$$

$$S_{i,jk}(\boldsymbol{r},\boldsymbol{r}_A)=\mathcal{F}\,\phi_{i,jk}(\boldsymbol{k},\boldsymbol{r}_A)\ , \tag{4b}$$

$$K_{i,p}(\boldsymbol{r},\boldsymbol{r}_A)=\mathcal{F}\,\pi_{i,p}(\boldsymbol{k},\boldsymbol{r}_A)\ . \tag{4c}$$

The definitions of Fourier transforms of some terms containing differential operators are described by Eqs.(5)-(9) after Batchelor (1953):

Term I ;

$$D_{i,j}=\mathcal{F}\,\Upsilon_{i,j}(\boldsymbol{k},\boldsymbol{r}_A)\ . \tag{5}$$

Term II ;

$$\left(\frac{U_B}{r_A+r_s}-\frac{U_A}{r_A}\right)(Q_{i,j})_{,\phi_s}=\mathcal{F}\,H_{i,j}(\boldsymbol{k},\boldsymbol{r}_A)\ . \tag{6}$$

Term V①, (i=1, 2);

$$(-1)^i\left(\frac{2U_A}{r_A}Q_{-i+3,i}+\frac{2U_B}{r_A+r_s}Q_{i,-i+3}\right)$$

$$=(-1)^i\left(\frac{2U_A}{r_A}\mathcal{F}\,\phi_{-i+3,i}+\frac{2U_B}{r_A+r_s}\mathcal{F}\,\phi_{i,-i+3}\right)$$

$$=\mathcal{F}\,\Theta_{i,i}(\boldsymbol{k},\boldsymbol{r}_A)\ . \tag{7a}$$

Term V②, (i=1, 2):

$$(-1)^i\left(\frac{S_{i,(-i+3)2}}{r_A+r_s}+\frac{S_{(-i+3)2,i}}{r_A}\right)$$

$$=(-1)^i\left(\frac{1}{r_A+r_s}\mathcal{F}\,\phi_{i,(-i+3)2}+\frac{1}{r_A}\mathcal{F}\,\phi_{(-i+3)2,i}\right)$$

$$=\mathcal{F}\,\Pi_{i,i}(\boldsymbol{k},\boldsymbol{r}_A)\ . \tag{7b}$$

Term V③. (i=1, 2):

$$\frac{1}{\rho}(-1)^i\left[\frac{1}{r_A+r_s(i-1)}(K_{2,p})_{,\phi_s}-\frac{1}{r_A+r_s(-i+2)}(K_{p,2})_{,\phi_s}\right]$$

$$=\frac{1}{\rho}(-1)^i\left[\frac{1}{r_A+r_s(i-1)}(\mathcal{F}\,\pi_{2,p})_{,\phi_s}\right.$$

$$\left.-\frac{1}{r_A+r_s(-i+2)}(\mathcal{F}\,\pi_{p,2})_{,\phi_s}\right]=\mathcal{F}\,\Lambda_{i,i}(\boldsymbol{k},\boldsymbol{r}_A)\ . \tag{7c}$$

Term VI①. (i=1. 2):

$$(-1)^i(2\Omega)(Q_{2,1}+Q_{1,2})$$

$$=(-1)^i(2\Omega)\mathcal{F}(\phi_{2,1}+\phi_{1,2})$$

$$=\mathcal{F}\,\Delta_{i,i}(\boldsymbol{k},\boldsymbol{r}_A)\ , \tag{8a}$$

Term VI②. (i=1. 3):

$$(-1)^{(1/2)(i-1)}\frac{1}{\rho}(K_{p,3}-K_{3,p})_{,y_s}$$

$$=(-1)^{(1/2)(i-1)}\frac{1}{\rho}[\mathcal{F}(\pi_{p,3}-\pi_{3,p})]_{,y_s}$$

$$=\mathcal{F}\,\Lambda_{i,i}(\boldsymbol{k},\boldsymbol{r}_A)\ . \tag{8b}$$

Term VII :

$$E_{i,j}(\boldsymbol{r},\boldsymbol{r}_A)=\mathcal{F}\,\Psi_{i,j}(\boldsymbol{k},\boldsymbol{r}_A)\ . \tag{9}$$

In isotropic turbulence, spectral equations do not have the integral symbols. From the above definitions, spectral equations for the present problem are obtained as follows:

$$\Upsilon_{1,1}+H_{1,1}+\frac{1}{r_A}(r_A\phi_{11,1})_{,r_A}+\frac{1}{\rho}\frac{1}{r_A}(r_A\pi_{1,p})_{,r_A}+\frac{1}{\rho}\mathcal{F}^{-1}\left\{\frac{1}{r_s+r_A}[(r_s+r_A)K_{p,1}]_{,r_A}\right\}+\Theta_{1,1}+\Pi_{1,1}+\Lambda_{1,1}+\Delta_{1,1}+\Phi_{1,1}+\Psi_{1,1}=0\ , \tag{10}$$

I II III① III② III②′ V① V② V③ VI① VI② VII

$$\Upsilon_{2,2}+H_{2,2}+\frac{1}{r_A}(r_A\phi_{12,2})_{,r_A}+\phi_{1,2}r_A\left(\frac{U_A}{r_A}\right)_{,r_A}+\mathcal{F}^{-1}\left[(r_s+r_A)\left(\frac{U_B}{r_s+r_A}\right)_{,r_s}Q_{2,1}\right]+\Theta_{2,2}+\Pi_{2,2}+\Lambda_{2,2}+\Delta_{2,2}+\Psi_{2,2}=0\ , \tag{11}$$

I II III① IV IV′ V① V② V③ VI① VII

$$\gamma_{3,3} + H_{3,3} + \frac{1}{r_A}(r_A\phi_{13,3})_{,r_A} + \Phi_{3,3} + \Psi_{3,3} = 0 . \quad (12)$$

I II III① VI② VII

[Terms I] In general, these are named inertial terms and play the role of a cascade which transfers energy from a lower to a higher wave number in the wave number space.

[Terms II] It is known that terms II represent spectral transfer of energy due to the distortion by the mean shear in the wave number space.

[Terms III] Terms III① are regarded as turbulent diffusion in the direction of r_A. The integrals of III① and III② over the whole value of r_A are zero. Thus, these terms do not contribute to whole fields of energy flow. Lumley(1964) states that the scale of turbulence is small near the wall and large far from it, and if the diffusion occurs in the direction away from the wall, its effect brings about the energy transfer from a higher to a lower wave numbers; these terms have an inverse function to the inertial terms. In the present flow, the same is also true.

[Terms IV] Terms IV are named the production terms. However, we cannot find the term corresponding to these terms in correlation equations for the mean flow. The mean-flow energy is feeded into eddies of a particular size. It is considered that the feeding of the mean flow energy to turbulence at a one-point may have an effect on the profile of spectrum $\psi_{2,2}$.

[Terms V①] These terms do not occur in Cartesian coordinate system. They appear as the negative term by centrifugal force (metric force) in eq.(1) and as the positive term by metric force in eq.(2). Using eq.(7a), the sum of $\Theta_{1,1}$ and $\Theta_{2,2}$ becomes,

$$\mathfrak{F}\Theta_{1,1} + \mathfrak{F}\Theta_{2,2} = \left(\frac{2U_B}{r_A+r_s} - \frac{2U_A}{r_A}\right) Q_{2,1} - \left(\frac{2U_B}{r_A+r_s} - \frac{2U_A}{r_A}\right) Q_{1,2} . \quad (13)$$

When the separation r_s is zero, the next relation is obtained.

$$\int \left[\Theta_{1,1}(\mathbf{k}, r_A) + \Theta_{2,2}(\mathbf{k}, r_A) \right] d\mathbf{k} = 0 . \quad (14)$$

Since $\Theta_{1,1}$ and $\Theta_{2,2}$ seem to be different in sign in most of the wave-number space, they contribute to the exchange of energy between r- and ϕ-directions at the same wave number. However eq.(14) does not impose so severe restriction on this energy exchange as the following relation for the spectra of pressure correlation,

$$\Gamma_{1,1}(\boldsymbol{k}) + \Gamma_{2,2}(\boldsymbol{k}) + \Gamma_{3,3}(\boldsymbol{k}) = 0 , \quad (15)$$

in Cartesian coordinate system; eq.(15) controls the energy exchange between three directions at the same wave number and the terms in this equation can be called re-distribution terms.

Equation (14) states that the sum of $\Theta_{1,1}$ and $\Theta_{2,2}$ serves as transport of energy between wave numbers and its overall contribution is zero, in the same way as the spectra of the inertial terms for isotropic turbulence, in which the following relation is satisfied:

$$\int \Xi_{i,i}(\mathbf{k})\, d\mathbf{k} = 0, \quad (i=1, 2, 3), \quad (16)$$

where $\Xi_{i,i}$ are the spectra of the inertial terms. These terms are called transport terms.

In short, the terms $\Theta_{1,1}$ and $\Theta_{2,2}$ play a role similar to re-distribution of energy between r- and ϕ-directions, and at the same time the sum of them or residual of re-distribution works as a transport of energy in wave number space. We name here the terms with this kind of function as re-distributive trasport terms.

[Terms V②] These terms do not occur in Cartesian coordinate system as well. Using eq.(7b), for $\Pi_{1,1}$ and $\Pi_{2,2}$ the following relation is obtained.

$$\int \Pi_{1,1}(\boldsymbol{k}, r_A) d\boldsymbol{k} + \int \Pi_{2,2}(\boldsymbol{k}, r_A) d\boldsymbol{k} = 0 . \quad (17)$$

Thus, these terms have the same functions between $\psi_{1,1}$ and $\psi_{2,2}$ as $\Theta_{1,1}$ and $\Theta_{2,2}$ by both the centrifugal force (r-component) and the metric force (ϕ-component).

[Terms V③] These terms represent the contribution of pressure and are characteristic in the present flow. Using eq.(7c), the following relation is obtained:

$$\int \Lambda_{1,1}(\boldsymbol{k}, r_A) d\boldsymbol{k} + \int \Lambda_{2,2}(\boldsymbol{k}, r_A) = 0 . \quad (18)$$

These terms have also the re-distributive transport functions between $\psi_{1,1}$ and $\psi_{2,2}$.

[Terms VI①] It is considered that these terms represent the transfer of energy between $\psi_{1,1}$ and $\psi_{2,2}$ by Corioli's force of fields $2\Omega\overline{uw}$(Tritton, 1978). Since from eq.(8a) the sum of $\mathcal{F}\Delta_{1,1}$ and $\mathcal{F}\Delta_{2,2}$ is zero, it follows from Fourier inverse-transform that

$$\Delta_{1,1}(\boldsymbol{k}, r_A) + \Delta_{2,2}(\boldsymbol{k}, r_A) = 0 . \quad (19)$$

Hence, the function of these terms is the transfer of energy between $\psi_{1,1}$ and $\psi_{2,2}$ at the same wave number. This is similar to the role of spectrum of pressure correlation $\Gamma_{i,i}$, in which energy is re-distributed between three components, whereas in the present flow energy is re-distributed between two components by these terms.

[Terms VI②] Since from eq.(8b) the sum of $\mathcal{F}\Phi_{1,1}$ and $\mathcal{F}\Phi_{3,3}$ is zero, it follows from Fourier inverse-transform that

$$\Phi_{1,1}(\boldsymbol{k}, r_A) + \Phi_{3,3}(\boldsymbol{k}, r_A) = 0 . \quad (20)$$

The role of these terms is the transfer of energy between $\psi_{1,1}$ and $\psi_{3,3}$ at the same wave number and is similar to the role of spectrum $\Delta_{i,i}$.

The re-distributive transport of energy occurs between $\psi_{1,1}$ and $\psi_{2,2}$, on the other hands two-component transfer of energy occurs either between $\psi_{1,1}$ and $\psi_{2,2}$ or between

$\psi_{1.1}$ and $\psi_{3.3}$. It has been confirmed that re-distributive transport function and two -component transfer do not occur between the other spectra, e.g., $\psi_{1.2}$ and $\psi_{3.1}$ (Ueki et al, 1992).

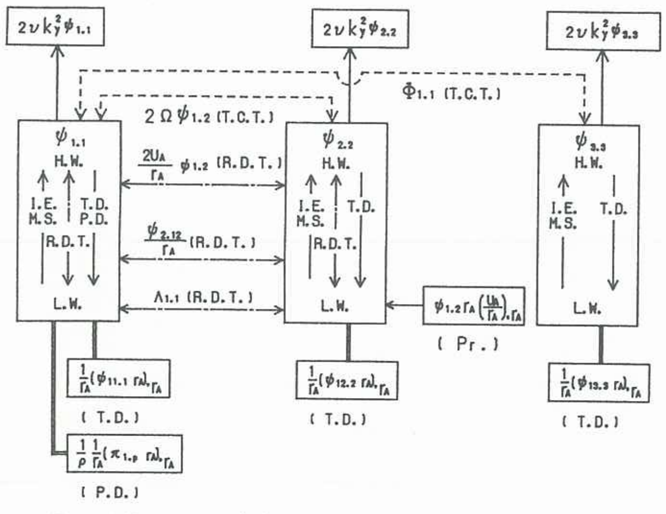
[Term Ⅶ] These terms are composed of viscous diffusion and dissipation terms.

<u>Energy Flow Schema in Wave Number Space</u>

The flow of energy in the wave number space is shown in Fig. 2 using the typical terms of eqs.(10)-(12). The solid line indicates the flow of energy which is explained by the past theory for Cartesian coordinate system. Other terms than the production and viscous terms are positioned with no respect to the value of wave numbers. The arrows in the figure show the direction of energy flow. In what follows, the figure for $\psi_{1.1}$ is mainly explained.

Firstly, $\psi_{2.2}$ is produced in low wave numbers by $\psi_{1.2} r_A (U_A/r_A)_{,rA}$ (= Pr. in the figure, or term Ⅳ in eq.(11)). Energy is transferred from a low wave number to a high wave number by inertial term $T_{1.1}$ and by mean shear term $H_{1.1}$. Energy is dissipated by viscous term $2\nu k_y^2 \psi_{1.1}$. Although there are a lot of dissipation term in this flow, in Fig. 2 a typical ones are shown. The characteristic flow of energy for the present flow is shown by broken lines (T.C.T.) and chain lines (R.D.T.).

Energy flows directly between $\psi_{1.1}$ and $\psi_{2.2}$ by re-distributive transport function for $\Theta_{1.1}$, $\Pi_{1.1}$ and $\Lambda_{1.1}$. Two-component transfer between $\psi_{1.1}$ and $\psi_{2.2}$ occurs by



I.E., M.S.: Inertial effect and mean shear effect, respectively
Pr.: Production
T.C.T.: Two-component transfer
R.D.T.: Re-distributive transport
T.D., P.D.: Turbulent diffusion and pressure diffusion, respectively
L.W., H.W.: Low wave number and high wave number, respectively

Fig.2 Flow of energy in the wave number space

Coriolis force of field $2\Omega \psi_{1.2}$ and two-component transfer between $\psi_{1.1}$ and $\psi_{3.3}$ occurs due to the spectrum of pressure correlation $\Phi_{1.1}$. The turbulent diffusion term and pressure diffusion term shown by the double solid line mean the transfer of energy from one point to another in metric space $\mathbf{r}_A$.

CONCLUDING REMARKS

In an attempt to understand the role of each term of the correlation equations for rotating flows through the spectral tranformation, we obtained the following conclusions.

(1) Spectral equations for rotating flows are derived. Some terms have the same meanings as ones in Cartesian coordinate system.

(2) New terms which do not exist in the Cartesian coordinate system occur and have characteristic functions. It is obvious that the centrifugal terms Ⅴ①, the metric terms Ⅴ② and the pressure terms Ⅴ③ work as a re-distributive transport function between $\psi_{1.1}$ and $\psi_{2.2}$, and that two-component transfer occurs between $\psi_{1.1}$ and $\psi_{2.2}$ due to terms Ⅴ① of Coriolis force of field and between $\psi_{1.1}$ and $\psi_{3.3}$ due to the pressure terms Ⅵ②.

(3) The block-diagram of each spectral tensor was shown by the figure and the mutual correlations were clarified.

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