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AXISYMMETRIC VORTEX BREAKDOWN IN AN ANNULUS

Naftali TSITVERBLIT and Eliezer KIT

Dept of Fluid Mechanics & Heat Transfer
Faculty of Engineering, Tel-Aviv University
Ramat Aviv 69978, ISRAEL

Abstract

The steady axisymmetric Navier-Stokes equations for the flow in a cylindrical container and in an annulus have been solved by the continuation method, being a standard tool of numerical methods for bifurcation problems. By alteration of the boundary condition for vorticity at the axis, the transition of the flow structure, including the separation bubble, being present in a conventional container, to that in an annulus with an infinitesimal inner radius was proceeded. By further continuation in the ratio between inner and outer radii the flow pattern in the annulus was computed. It was shown that even for very high values of this ratio a recirculation region occurred at the inner wall of the annulus. The transition to a nonsteady motion through supercritical Hopf bifurcation was studied by following the time development of a small axisymmetric disturbance introduced to the full or linearized system of equations.

1. Introduction

In recent years, great attention has been attracted to studies of the vortex breakdown phenomenon in a container with rotating endwall. The axisymmetric steady and oscillating flow in such a geometry was tested experimentally and computed numerically; good agreement was found between the results (see, e.g. Escudier 1984, Lugt and Abboud 1987, Lopez 1990, Lopez and Perry 1991, Daube 1991, Tsitverblit 1992). By applying the method of continuation to solve the steady Navier-Stokes equations in a stream function-vorticity formulation Tsitverblit (1992) was able to compute the unstable steady solution and to show that the transition to oscillating flow occurred through supercritical Hopf bifurcation.

It might be interesting, therefore, to compute the flow in the same confined geometry, but by applying a different boundary condition for the azimuthal vorticity at the axis, which actually corresponded to the axis replacement by a solid rod of an infinitesimal diameter. In this configuration of a container, the numerical boundary condition for vorticity ξ at the axis, r = 0, expressed through stream function ψ, may be presented in the form $\xi = \tau\, 0.5\, \frac{\partial^3\psi}{\partial r^3}$. By variation of the continuation parameter τ from 0 to 1 one can follow the flow structure modification continuously from a conventional container with a "fluid" axis to a container with "solid" axis. In such a configuration of an annulus type, the influence of viscosity should be enhanced in the region of the separation bubble and therefore could be better assessed. From theoretical considerations, the behavior of inviscid swirled flow in an annulus and in a conventional container might be different. In particular, Keller and Egli (1985) showed that when the approaching supercritical flow is a potential vortex, a transition to another supercritical flow with a cavity of finite size was expected.

By further applying continuation to the ratio between the internal and the external radii of the annulus, solution can be obtained in the whole range from 0 to 1, or from a container with a "solid" axis to a straight channel.

The detailed results describing the flow patterns, the region of vortex breakdown existence , the axisymmetrical stability margin and the disturbance evolution as a function of time are presented for a conventional container and for an annulus while its radii ratio is varied.

2. Description of numerical approach and results.

2.1 *Mathematical formulation*

In the present study the same mathematical and numerical formulation as described in Tsitverblit (1992) was adopted. For the geometry of a closed container of the annulus type illustrated in Fig. 1 the vorticity-stream function formulation of the steady axisymmetric Navier-Stokes equations in cylindrical coordinates (r,θ,z) with appropriate boundary conditions was considered.

The equations were rendered nondimensionally using the following scales:

lengthscale d for the radial r-coordinate, where d is the width of the channel;

lengthscale H for the axial z-coordinate, where H is the height of the channel;

velocity scale ν/d, where ν is the kinematic viscosity.

The choice of the vertical lengthscale was dictated by the need to have aspect ratio $\gamma = d/H$ as an explicit parameter in the equation so that continuation in it, if necessary, would be possible. The Reynolds number was defined as $Re = \frac{\Omega \cdot d \cdot r_e}{\nu}$ where Ω is the angular velocity of the rotating endwall and r_e was the external radius of an annulus. This definition yields the usual definition of Reynolds number for a conventional container $Re = \frac{\Omega \cdot d^2}{\nu}$.

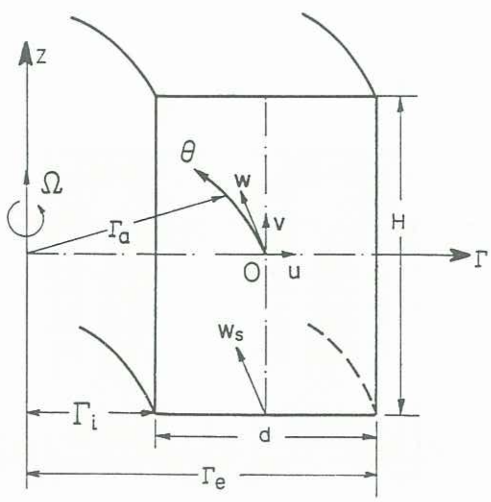

Fig. 1 Coordinate system for annulus.

To allow for the variable grid sizes, the following stretching functions were incorporated into governing equations following Tsitverblit (1992): $r = x - a{\cdot}\sin(2\pi x)$ and $z = y - b{\cdot}\sin(2\pi y)$, where r and z ranged from 0 to 1 and a and b were chosen from the following intervals: $a \in |0;0.13|$, $b \in |0;0.15|$.

After discretization the steady axisymmetric Navier-Stokes equations with boundary conditions constitute a system of nonlinear algebraic equations which can be written as $\mathbf{f}(\mathbf{x},Re,\gamma,R,\tau) = 0$ where Reynolds number Re, aspect ratio γ, radii ratio R, and continuation parameter τ, incorporated in the boundary condition for vorticity at inner wall are variable parameters and $\mathbf{x}$ stands for a vector of unknowns of finite dimension. In most of the runs, Reynolds number served as a continuation parameter upon which a steady solution was continued from zero at Re = 0 (for more details see Tsitverblit 1992). The stability of the computed steady solutions and the transition to axisymmetric developed oscillating flow were simulated by introduction of initial disturbances into the equation $\mathbf{M}\left(\frac{\partial \mathbf{x}}{\partial t}\right) + \mathbf{f}(\mathbf{x},Re,\gamma,R,\tau) = 0$ where $\mathbf{M}$ is the matrix reflecting the fact that the conservation equation did not depend explicitly on time. The linearized form of this equations can be written as $\mathbf{M}\left(\frac{\partial \mathbf{x}'}{\partial t}\right) + \mathbf{x}'{\cdot}\mathbf{f}_{\mathbf{x}}(\mathbf{x}_0,Re,\gamma,R,\tau) = 0$ where $\mathbf{f}_{\mathbf{x}}(\mathbf{x}_0,Re,\gamma,R,\tau)$ is the Jacobian matrix , $\mathbf{x}'$ is the deflection from the steady solution $\mathbf{x}' = \mathbf{x} - \mathbf{x}_0$ and $\mathbf{x}_0$ is the steady solution for the chosen set of parameters Re, γ, R and τ. The implicit method was used to solve these equations in both linear and nonlinear cases.

The main bulk of computation was performed for the container with an aspect $\gamma = 2$ and stretching parameters: a = 0.117, b =0.128. Three modifications of the geometry of the container were considered: a) a conventional container, b) a container with a solid rod of an infinitesimal diameter placed at the container axis, c) an annulus geometry. The only difference between cases a) and b) was the boundary condition for vorticity at the axis. As it was mentioned above the continuation parameter τ was varied from 0 to 1, thus allowing to transfer the flow conditions continuously from case a) (a conventional container) to case b) (container with a solid rod at the axis).

The transition from case b) to case c) (an annulus geometry) was achieved by further application of continuation to the ratio R between the internal and the external radii of the annulus. By such a procedure the solution could be obtained in the whole range of R from 0 to 1. When this ratio is very close to zero (for the results presentation we choose a value of 0.01) the flow pattern in the annulus should be very close to that of the case b). It was verified that for the fixed selected values of τ and R the same results could be obtained by continuation in Reynolds number Re and in the computations all these approaches were used. Most of the calculations were conducted on a grid 25x25 which gives very reasonable results when stretching was introduced in the discretizied equations in the same way as it was done in Tsitverblit (1992) and the optimal stretching parameters were chosen in accordance with his careful calculations. Verification of the numerical scheme performance was done on the grid 53x53.

2.2 *The flow structure in conventional container, in container with an axis replaced by solid rod and in annulus with very small radii ratio R = 0.01*

2.2.1 Steady flow and interval of vortex breakdown existence.

The most significant result which follows from the comparative analysis of these three cases at the same value of Re is a very close similarity of the flow pattern for all these modifications in both regions: outer flow region and the region of close vicinity of the axis where vortex breakdown occurs. To illustrate this the streamlines of the secondary flow in the container computed at Re = 2000 are shown in Fig. 2. It is worth to note that the instant of the vortex breakdown appearance depends slightly on the boundary condition at the axis as it can be seen from the Table 1.

Using the continuation in τ, it was also checked that the transition from the "fluid" axis (conventional container) to "solid" axis was very smooth and the modification of the separation bubble during this transition was unnoticeable. It was conjectured by previous authors (cf. Lopez 1990) that although the flow in the container as a whole was viscous, the emergence of the separation bubbles and their flow structure are essentially phenomena of inviscid nature. It can be seen from the Table 1 that introduction of viscous effects directly to the region of vortex breakdown expands only slightly the range of existence.

TABLE I: Range of vortex breakdown existence and flow stability margin.

	Conventional container	Annulus			
		$r_a = 0.5$	$r_a = 0.501$	$r_a = 1$	$r_a = 5$
Re_{min}	1460-1470	1410-1420	1460-1470	1520-1540	2900-2920
Re_{max}	3140-3160	3260-3280	3320-3340	4250-4300	> 200000
Re_{cr}	2530-2550	2530-2550	2530-2550	2100-2125	2880-2900

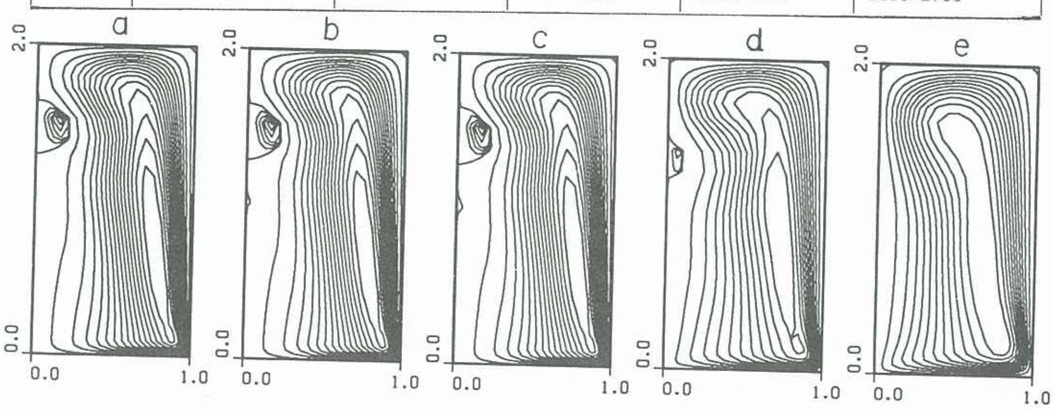

Fig. 2 Streamline contours for Re = 2000 in a container: a) conventional, b) with a "solid" axis and in an annulus with: c) $r_a = 0.501$, d) $r_a = 1$, e) $r_a = 5$.

2.2.2 Transition to unsteady flow: instability and pulsating flow in a container.

Daube (1991) showed that the flow in the vicinity of Re_{cr} behaved according to the assumption that transition occurred through supercritical Hopf bifurcation which was also confirmed by the detailed numerical computation performed by Tsitverblit (1992). This assumption allows one to use the linearized system of equations to find the stability margin for small axisymmetric disturbances. In the bottom line of Table 1 the results of stability margin calculation are presented. There is practically no influence on the stability of the boundary condition at the container axis. It is important to note that in the frame of this work only instability to axisymmetrical disturbances can be assessed. Fortunately, the experiments of Escudier(1984) showed that in the range of parameters where the vortex breakdown in a container was usually investigated there was only a very short interval at relatively high values of aspect ratio γ = 3.1-3.5 where a clear deflection from axial symmetry occurred and the first sign of non-steady motion was according to Escudier "a precession of the lower breakdown structure". One can speculate that in this range of parameters the instability starts to develop inside the recirculation zone of the flow i.e. the separation bubble. The very existence of a region of an unsteady vortex breakdown in a container controlled by three-dimensional instability seems to be very important since the flow in a container is usually considered to be an example of a strictly axisymmetrical vortex breakdown.

At the first sight it is unclear why the linear stability does not depend on the boundary condition at the axis as obtained in the present computations for different types of container flow (the bottom line in Table 1). It seems therefore reasonable to assume that the development of pulsating flow in a container is due mainly to instabilities in the flow region external to the recirculation zone. Thus, the swirled flow in the vicinity of the container axis is forced essentially by a pulsating external flow.

The results of stability computations for a flow in a container with "fluid" and "solid" axes are shown in Fig. 3.

The azimuthal velocity variations, obtained from the solution of linearized system of equations for Re = 2530 < Re_{cr} and Re = 2550 > Re_{cr} are presented in Figs. 3a,b and Figs. 3c,d correspondingly. The decay and growth of the disturbances is very slow. The same values obtained from the solution of the full nonlinear system of equation for Re = 3000 are shown in Fig. 3e,f. The initial nondimensional disturbance was ϵ = 0.001. Almost identical behavior of the solution in container with "fluid" and "solid" axis indicates that there is only small influence of the boundary condition at the axis on the transition from the steady to oscillating axisymmetric vortex breakdown.

2.3 *Vortex breakdown in an annulus (radii ratio R $\propto$ 0.5).*

As it was explained above the flow patterns in the annulus can be computed by continuation in radii ratio R starting from the container flow with "solid" axis (R = 0). In such a way one can follow the transition of separation bubbles while the curvature of the annulus gradually decreases. The interesting result which was obtained from this type of computations was that even for high values of R $\simeq$ 0.95 a steady separation bubble can emerge in the flow. Most of our calculations were performed for the fixed values of R and by continuation in Re. For the presentation we have chosen R = 0.33 and R = 0.82 (the corresponding nondimensional radii of the annulus defined as $r_a = (r_i + r_e)/2d$ were $r_a = 1$ and $r_a = 5$).

It worth to note that annulus is widely used as an instrument in different fields of fluid mechanics for basic research as well as for technological applications (see e.g. Kato & Phillips 1969 and Gelfgat et al. 1972) and it is therefore important to know the flow structure in such an apparatus. Although the real flow in the annulus is usually turbulent, it was shown in Kit & Mazor (1990) that the main features of the flow structure and especially the form of the secondary circulation cell could be captured quite satisfactorily by laminar flow calculations. The range of existence of the steady vortex breakdown obtained for an annulus with $r_a = 5$ was Re $\geq$ 2900. The stability calculations for this case revealed that the flow is unstable in the whole range.

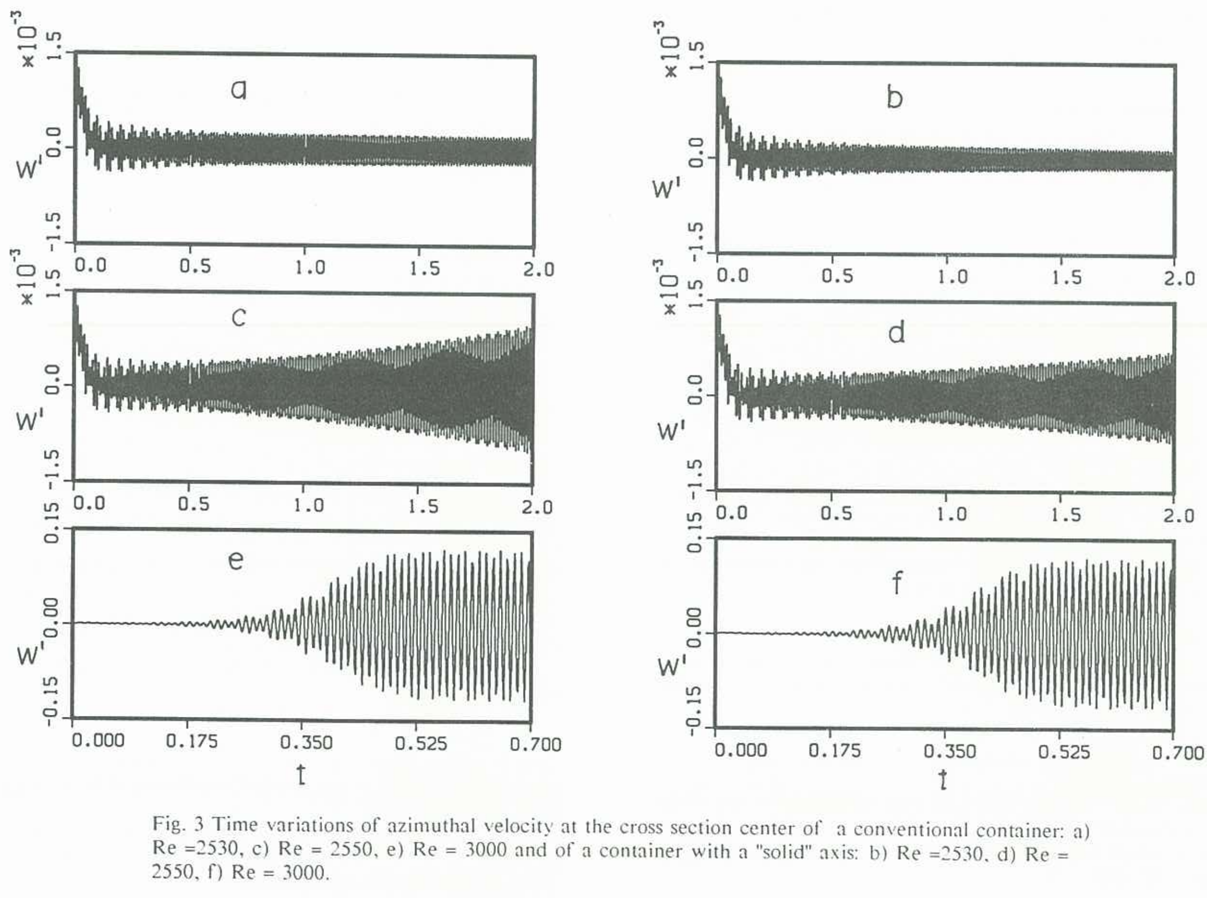

Fig. 3 Time variations of azimuthal velocity at the cross section center of a conventional container: a) Re =2530, c) Re = 2550, e) Re = 3000 and of a container with a "solid" axis: b) Re =2530, d) Re = 2550, f) Re = 3000.

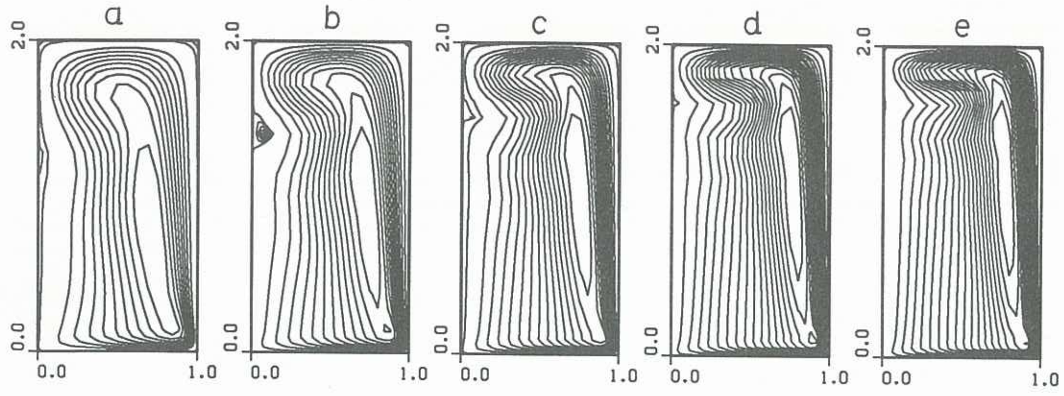

Fig. 4 Streamline contours in an annulus with $r_a = 1$ for a)Re = 1600, b)Re = 2250, c)Re = 3000, d)Re = 3500, e)Re = 4250.

The situation is different for the annulus with nondimensional radius $r_a = 1$. In this case the range of existence of steady separation bubbles at the inner wall of the annulus is $1570 \leq Re \leq 4300$ and the instability occurs for the first time at Re = 2125. Consequently, there is an interval of Reynolds numbers where the axisymmetrical steady vortex breakdown exists in the annulus flow. There are no experimental data for an annulus flow in this range of parameters, thus a possibility of three-dimensional instability cannot be excluded. However, since the intensity of the flow in the separation bubble becomes lower when compared to conventional container for the same Reynolds number and the axisymmetrical instability is shifted to the lower Reynolds number, it can be assumed that the transition to unsteady oscillating flow in annulus occurs through axisymmetrical instability and not by three-dimensional instability. In Fig. 4, the streamline contours of the secondary flow are shown for several Reynolds numbers chosen from the interval of the vortex breakdown existence. The azimuthal velocity computed for Re = 2075 and Re =2125 from linearized system of equations and for Re =2500 from full system, is presented as a function of time in Fig. 5

3. Concluding remarks

The present investigation revealed that a steady separation bubble at the inner wall of the annulus could occur when the nondimensional radius of curvature was finite. By continuation in the radii ratio, steady solutions were obtained for an annulus with relatively high nondimensional radius of curvature, which indicates of a clear recirculation zone related to the vortex breakdown at the inner wall of an annulus. When this radius is relatively small, there exists a finite interval of Reynolds numbers where this separation bubble is stable. When the inner radius becomes infinitesimally small, the annulus modifies to a container with a no-slip boundary condition at the axis. The comparison of the flow structure in this configuration and in a conventional container shows that the influence of the boundary condition at the axis is relatively small. The existence region of vortex breakdown is slightly wider for the "solid" axis. There is almost no difference in the critical Reynolds number where the flow becomes unstable through a supercritical Hopf bifurcation.

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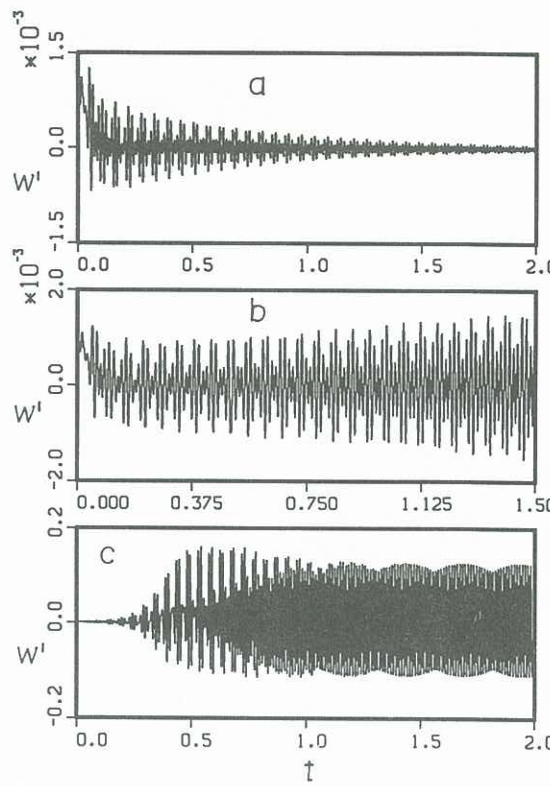

Fig. 5 Time variations of azimuthal velocity at the cross section center of an annulus with $r_a = 1$. a) Re = 2075 b) Re = 2125 c) Re = 2500

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