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AN EXPERIMENTAL STUDY ON THE DYNAMIC BEHAVIOUR OF A HYDRODYNAMIC JOURNAL BEARING

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This paper proposes a new method to measure the dynamic coefficients of journal bearings. Utilising the transfer functions of vibrating responses to the exciting forces, all coefficients can be found through the operation of a least-square-error estimator. Experiment results are also presented.

NOMENCLATURE

b^s_{ij} — s=1,2,3; i,j=x,y the damping coefficient of sth bearing
B_{ij} — i,j=x,y , dimensionless damping coefficient
C — bearing clearance , m
C_b — $=W/C\Omega$, N.s/m
C_k — =W/C, N/m
D — diameter of bearing, m
e , ε — eccentricity (m) and eccentricity ratio ε=e/C
F — exciting or dynamic force , N
h , H — film thickness, m ; H=h/C
J — transverse inertia moment of the shaft , kg.m^2
L — width of the bearing , m
k^s_{ij} — s=1,2,3; i,j=x,y the stiffness coefficient of sth bearing
K_{ij} — i,j=x,y , dimensionless stiffness coefficient
m — shaft mass , kg
p , P — pressure , N/m^2 ; P=pC/μU dimensionless
p_s — groove pressure
R — radius of the bearing , m
$S=\left(\frac{R}{C}\right)^2\frac{\mu NDL}{W}=\frac{\mu NDL}{W\psi^2}$ — Sommerfeld number
W — Static load , N
x, y, Z — coordinates, m
X — variables vector
μ — lubricant viscosity, N.s/m^2
φ, ϕ_0 — angular coordinate and attitude angle, rad
ψ — =C/R clearance ratio
ω — frequency , rad/s
Ω — rotational speed of the journal, rad/s
ρ — lubricant density, kg/m^3

INTRODUCTION

The journal bearing theory has been studied extensively but there are still many unresolved topics, such as pressure distribution, cavitation, boundary conditions, oil whirl and critical speed etc. which need further investigation. Excessive vibration of turbogenerator rotor bearing system can cause catastrophic failures. The degree of vibration is greatly influenced by the dynamic coefficients of the bearing, which dominate the performance of a fluid film journal bearing. Many measurement techniques have been developed but the results still differ from one another. This paper attempts to investigate experimentally the bearing characteristics and to find a suitable method to measure the dynamic coefficients of the bearings.

CHARACTERISTICS OF JOURNAL BEARING

A journal bearing with two axial grooves shown in Fig.1 has its characteristics defined in terms of the Sommerfeld number, attitude angle, 4 stiffness and 4 damping coefficients. These characteristics can be calculated by solving the following dimensionless Reynolds equation :

$$\frac{\partial}{\partial\phi}\left(H^3\frac{\partial P}{\partial\phi}\right)+\frac{\partial}{\partial Z}\left(H^3\frac{\partial P}{\partial Z}\right)=\frac{6}{\psi}\frac{\partial H}{\partial\phi}+\frac{12}{\psi^2}\left(\frac{\dot{h}}{U}\right) \quad (1)$$

The boundary condition used in this work are:

(i) $P(\phi,0)=P(\phi,L/R)=0$
(ii) $P(0,Z)=P(\phi_e,Z)=0$
(iii) $P(\phi,Z)=P_s$ at the grooves (2)

The Finite Element Method was employed to solve the equation. Integrating the pressure P in the x direction will yield the fluid film force F_x and in the y direction F_y. Perturbing x , y , $\dot{x}$ and $\dot{y}$ yield the corresponding force changes ΔF_x and ΔF_y and the dynamic coefficients are given by:

$$k_{xx}=\frac{\Delta F_x}{\Delta x},\ k_{xy}=\frac{\Delta F_x}{\Delta y},\ k_{yx}=\frac{\Delta F_y}{\Delta x},\ k_{yy}=\frac{\Delta F_y}{\Delta y}$$
$$b_{xx}=\frac{\Delta F_x}{\Delta\dot{x}},\ b_{xy}=\frac{\Delta F_x}{\Delta\dot{y}},\ b_{yx}=\frac{\Delta F_y}{\Delta\dot{x}},\ b_{yy}=\frac{\Delta F_y}{\Delta\dot{y}} \quad (3)$$

In order to compare with the literature, the dimensionless

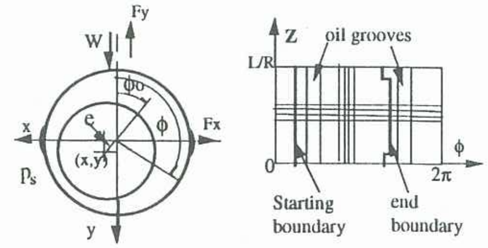

Fig. 1 Hydrodynamic bearing and oil film coordinate system

coefficients are defined as:

$$K_{ij}=\frac{C}{W}\,k_{ij}=k_{ij}/C_k\ ,\ B_{ij}=\frac{C\Omega}{W}\,b_{ij}=b_{ij}/C_b\quad i\,,j=x\text{ or }y \qquad (4)$$

Fig.2 shows the calculated stiffness coefficients K and damping coefficients B, compared with those by Someya [1].

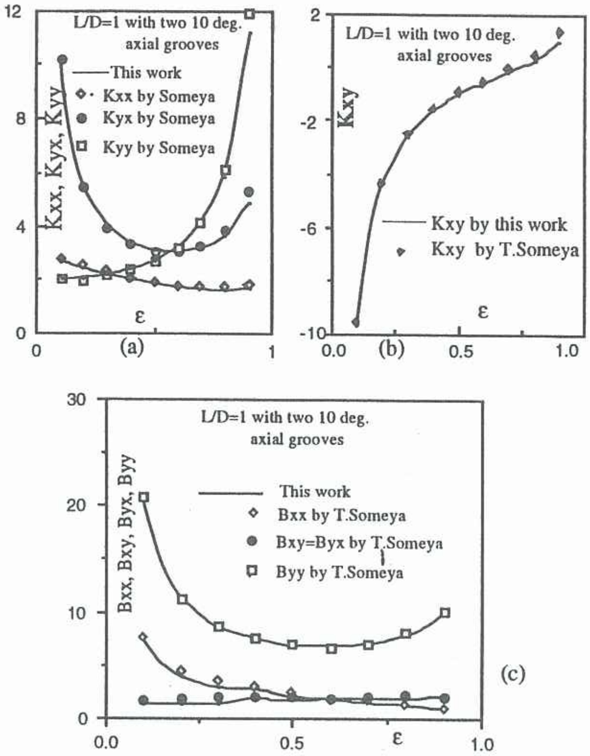

Fig.2 Calculated bearing characteristics compared with Someya [1]

IDENTIFICATION OF THE DYNAMIC COEFFICIENTS

The existing journal bearing rig has three fluid film journal bearings, each takes different geometric parameters and their dynamic characteristics coupled to each other. There is no present method that can be applied to estimate the dynamic coefficients of these three bearings.

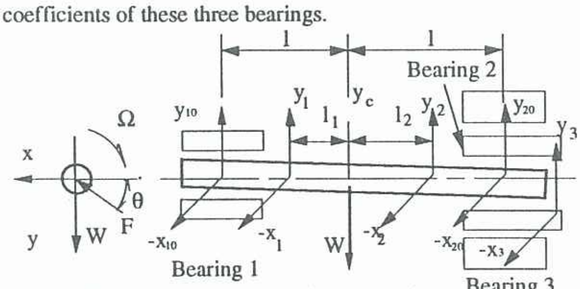

Fig . 3 Bearing system in the test rig

For the dynamic system shown in Fig.3 , its Lagrangian equation can be written as:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{X}}\right)-\frac{\partial T}{\partial X}-\Sigma F=0$$

where the system kinetic energy : $T=\frac{1}{2}\dot{X}^T M \dot{X}$

and $\frac{\partial T}{\partial X}=0,\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{X}}\right)=M\ddot{X},\ \Sigma F=Q_0F-B\dot{X}-KX$

Then the system mathematical model is governed by:

$$M\ddot{X}+B\dot{X}+KX=Q_0F \qquad (5)$$

where : $X=[\,x_1\,,y_1\,,x_2\,,y_2\,,x_3\,,y_3\,]^T$

The equivalent mass matrix M, damping matrix B, stiffness matrix K and force factor matrix Q_0 are shown in the Appendix.

Fourier transform was performed to obtain the frequency domain model:

$$[\,K-\omega^2M+j\,\omega B\,]\frac{X(j\omega)}{F(j\omega)}=Q_0 \qquad (6)$$

Here :

$$\frac{X(j\omega)}{F(j\omega)}=\left[\frac{x_1(j\omega)}{F(j\omega)}\ \frac{y_1(j\omega)}{F(j\omega)}\ \cdots\ \cdots\ \frac{x_3(j\omega)}{F(j\omega)}\ \frac{y_3(j\omega)}{F(j\omega)}\right]^T$$

$$=[H^{r\,T}+j\,H^{r\,T}] \qquad (7)$$

and $H^r=(H_{x1}^r,H_{y1}^r,H_{x2}^r,H_{y2}^r,H_{x3}^r,H_{y3}^r)$

$H^i=(H_{x1}^i,H_{y1}^i,H_{x2}^i,H_{y2}^i,H_{x3}^i,H_{y3}^i)$

superscript r and i mean the real and imaginary parts respectively.

At frequency ω_k, the above transfer function can be expressed as a matrix :

$$H_k=\begin{bmatrix} H_{2k-1;1} & H_{2k-1;2} & \ldots\ldots & H_{2k-1;6} \\ H_{2k;1} & H_{2k;2} & \ldots\ldots & H_{2k;6}\end{bmatrix}$$

Exciting the shaft n (n≥2) times with different frequencies yield n sets of linear equations (6) which are rearranged as:

$$A\,Z=Q \qquad (8)$$

where the coefficient matrix Z has the form:

$$Z=\begin{bmatrix} k_{xx}^1\,k_{xx}^2\,k_{xx}^3\,k_{xy}^1\,k_{xy}^2\,k_{xy}^3\,b_{xx}^1\,b_{xx}^2\,b_{xx}^3\,b_{xy}^1\,b_{xy}^2\,b_{xy}^3 \\ k_{yx}^1\,k_{yx}^2\,k_{yx}^3\,k_{yy}^1\,k_{yy}^2\,k_{yy}^3\,b_{yx}^1\,b_{yx}^2\,b_{yx}^3\,b_{yy}^1\,b_{yy}^2\,b_{yy}^3\end{bmatrix}^T$$

The least-square-error estimator of Z is:

$$\hat{Z}=(A^TA)^{-1}A^TQ \qquad (9)$$

The excitation force F can be any kind of dynamic force, and need not necessarily be a harmonic force. If the rotor is excited harmonically only twice, the above estimator (9) reduces to the traditional harmonic excitation method [2]. If it is a pulse excitation which involves a wide range of frequency characteristics , the method can yield all coefficients directly. Previously, other researchers [4] have to use the regression method to determine the coefficients which takes long calculation time, and the results are easily affected by the initial coefficients values.

EXPERIMENT

Fig.4 shows the diagrammatic view of the bearing test rig. Its basic data are:

200 mm diameter x 200 mm length for hydrodynamic bearings
280 mm diameter x 200 mm length for hydrostatic bearing
Clearance ratio (C/R) of the hydrodynamic bearing: 0.075%
Shaft speed: 10 --- 4000 rpm (driven by a 16 kw DC motor)
Static load : 0---120 kN
Dynamic force : 0--- 28 kN , harmonic

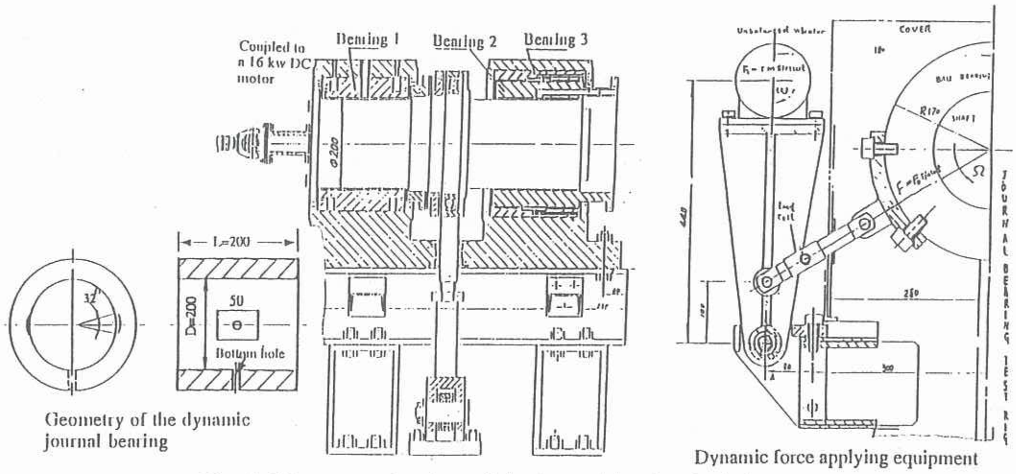

Fig. 4 Diagrammatic view of the journal bearing test rig

Excitation frequency: 0--- 60 Hz

Lubricant : ISO 15 lubricating oil

Two Kulite Semiconductor XTM-1900 pressure transducers and two Bently Nevada probe 33103 proximitors are buried in the hollow shaft and connected to the amplifiers through a 24 channel slip ring. Thirty eight thermocouples are buried around two hydrodynamic journal bearings. Static load is applied to the shaft by an air bellows through a lever and a ball bearing in the middle of the shaft. The dynamic force was generated by an electric vibrator and transferred to the shaft through a connecting rod and the ball bearing, whose mass is negligible compared with the rotor. The exciting force angle θ is 30^o. All signals during the experiment were sampled by a Mac IIcx computer with one 833 kHz and one 142 kHz A/D converters.

Fig.5 shows the pressure and gap distribution curves measured at the mid plane of bearing 1. The least-square-error parameter estimate method was employed to identify the eccentricity ratio ε and the attitude angle ϕ_o. Fig.6 presents a measured gap curve and its estimated eccentricity ratio and the attitude angle. From Fig.6, the estimated ε and ϕ_o by the least-square-error method provided the curve of best fit from the measurement. The measured ε and ϕ_o, as shown in Fig.8 , agree well with the theoretical calculation.

To measure the dynamic coefficients, the shaft was vibrated 4 times at different frequencies. At each frequency when the response was stable, the exciting force and static load from two force transducers and the shaft vibrating displacements from 6 proximitors were sampled and stored. The Fast Fourier Transfer (FFT) was performed to convert these time domain signals to frequency domain and the six transfer functions were found at each vibrating frequency. These transfer functions were then substituted into equation (8) and the coefficients calculated through the least-square-error estimator (9). Fig.8 shows the data processing procedure. The exciting force (in kN) and some vibrating displacements at frequency 26.5 Hz. are shown in Fig.9. The measured coefficients are tabulated in Table 1 and 2.

The measured coefficients K_{xx}, K_{yy} , B_{xx} and B_{yy} correlated fairly well with the theory. But some cross coefficients as B_{xy} and B_{yx} differ significantly from the calculation. At this stage, it is difficult to say that the theory needs modification. Further investigation indicated that if the shaft is excited at only one direction, Eq.(8) is ill-conditioned (the condition number $\|A\|\,\|A^{-1}\| >> 1$) which means a little error in the transfer functions will yield big discrepancy in the coefficients [6]. Since the rotor mass is much smaller than the bearing load and the vibrator frequency is limited to 60 Hz, the inertial item ω^2M in Eq.(7) is much less than the exciting force F. Changing the exciting frequency will not affect the system model significantly as it makes system model Eq.(8) either ill-conditioned (in this work, $\|A\|\,\|A^{-1}\|> 10^5$) or almost linearly dependent at different exciting forces. To improve the condition, another exciter is needed to exert dynamic force at different direction. Secondly, the bearing clearance ratio (C/R=0.075%) at this step is very small. To keep the oil film force linear the vibrating amplitude should be very small ($\leq$ 10% of C), and the available proximitors can not measure such a small amplitude vibration accurately due to small signal to noise ratio. Raising the vibrating amplitude to obtain sufficient measurement accuracy will be in conflict with the assumption of linearized oil film forces and it can produce large error. To eliminate the error, the clearance ratio should be increased to a common value such as 0.15%.

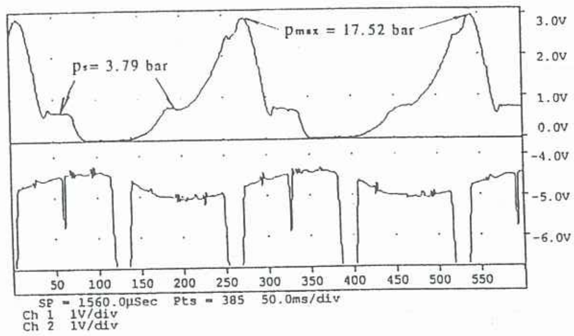

Fig.5 Measured pressure and gap distribution around bearing

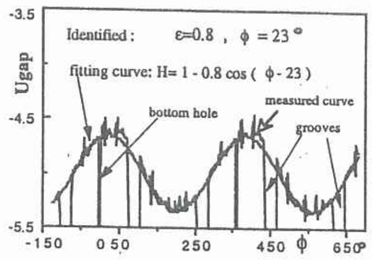

Fig 6 Measured gap curve and ε and ϕ_o identification

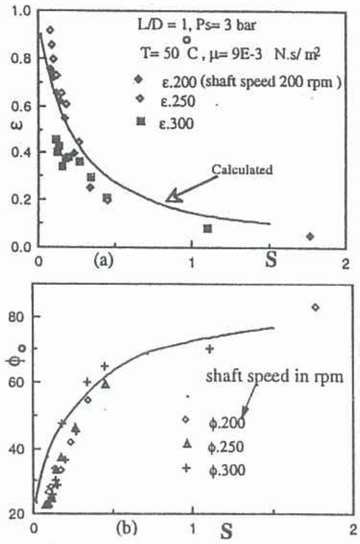

Fig.7 Measured eccentricity ratio ε and attitude angle ϕ_o

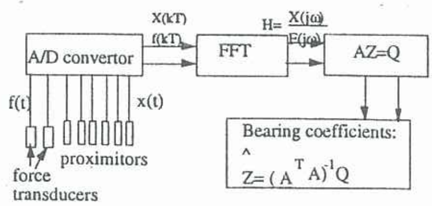

Fig. 8 Measurement of the dynamic coefficients

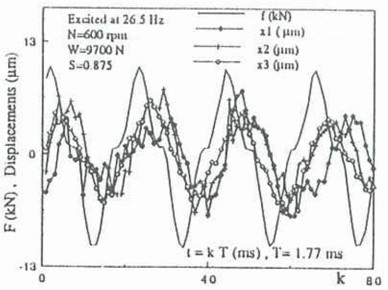

Fig. 9 Exciting force and corresponding responses

Table 1 Measured coefficients at lower eccentricity

Bearing # 1	N=600rpm	W=9700 N	S=0.5968	T= 50 °C
ps=3 bar	Excited frequencies: 21.2, 26.5 , 31.0 and 32.9 Hz			
stiffness	kxx	kxy	kyx	kyy
MN/m	606	-349	194	214
theory	475	-774	565	242
damping	bxx	bxy	byx	byy
MN.s/m	7.17	-2.67	-1.77	15.5
theory	9.02	0.48	0.48	18.5

Table 2 Measured coefficients at higher eccentricity

Bearing # 1	N=200rpm	W=22720 N	S=0.0832	T= 50 °C
ps=3 bar	Excited frequencies: 21.2 , 26.3 , 31.3 and 34.7 Hz			
stiffness	kxx	kxy	kyx	kyy
MN/m	837	422	-482	901
theory	548	-26.6	986	1148
damping	bxx	bxy	byx	byy
MN.s/m	6.34	-2.05	-3.66	88.9
theory	20.4	27.0	27.0	90.7

CONCLUSION

1. The experimental static characteristics of the journal bearing agree with the theoretical calculation based on the Reynolds equation and Reynolds boundary condition .
2. This work derived a least-square-error estimator for all bearing coefficients, which can be used in any type of excitation force.
3. As an initial investigation, the measuring accuracy of the dynamic coefficients in this paper is fair. To improve the accuracy, the clearance ratio should be increased to a larger value and two-direction excitation should be adopted in future work.

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APPENDIX MATRICES IN EQ(5)

$$Q_0 = [l_2\cos\theta/a,\ l_2\sin\theta/a,\ l_1\cos\theta/a,\ l_1\sin\theta/a, 0, 0]^T$$
$$= [\ q_1, q_2, \ ...., q_6\ ]^T$$

$$M=\frac{1}{a^2}\begin{bmatrix} ml_2^2+J & 0 & ml_1l_2-J & 0 & 0 & 0 \\ 0 & ml_2^2-J & 0 & ml_1l_2-J & 0 & 0 \\ ml_1l_2-J & 0 & ml_1^2+J & 0 & 0 & 0 \\ 0 & ml_1l_2-J & 0 & ml_1^2+J & 0 & 0 \\ 0 & 0 & 0 & 0 & a^2m_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & a^2m_3 \end{bmatrix}$$

$$K=\begin{Bmatrix} b^2k^1_{xx}+d^2k^2_{xx} & b^2k^1_{xy}+d^2k^2_{xx} & -bck^1_{xx}-dek^2_{xx} & -bck^1_{xy}-dek^2_{xy} & adk^2_{xx} & adk^2_{xy} \\ b^2k^1_{yx}+d^2k^2_{yx} & b^2k^1_{yy}+d^2k^2_{yy} & -bck^1_{yx}-dek^2_{yx} & -bck^1_{yy}-dek^2_{yy} & adk^2_{yx} & adk^2_{yy} \\ -bck^1_{xx}-dek^2_{xx} & -bck^1_{xy}-dek^2_{xy} & c^2k^1_{xx}+e^2k^2_{xx} & c^2k^1_{xy}+e^2k^2_{xx} & -aek^2_{xx} & -aek^2_{xy} \\ -bck^1_{yx}-dek^2_{yx} & -bck^1_{yy}-dek^2_{yy} & c^2k^1_{yx}+e^2k^2_{yx} & c^2k^1_{yy}+e^2k^2_{yy} & -aek^2_{yx} & -aek^2_{yy} \\ adk^2_{xx} & adk^2_{xy} & -aek^2_{xx} & -aek^2_{xy} & a^2(k^2_{xx}+k^3_{xx}) & a^2(k^2_{xy}+k^3_{xy}) \\ adk^2_{yx} & adk^2_{yy} & -aek^2_{yx} & -aek^2_{yy} & a^2(k^2_{yx}+k^3_{yx}) & a^2(k^2_{yy}+k^3_{yy}) \end{Bmatrix}$$

$$B=\begin{Bmatrix} b^2b^1_{xx}+d^2b^2_{xx} & b^2b^2_{xy}+d^2b^2_{xx} & -bcb^1_{xx}-deb^2_{xx} & -bcb^1_{xy}-deb^2_{xy} & adb^2_{xx} & adb^2_{xy} \\ b^2b^1_{yx}+d^2b^2_{yx} & b^2b^1_{yy}+d^2b^2_{yy} & -bcb^1_{yx}-deb^2_{yx} & -bcb^1_{yy}-deb^2_{yy} & adb^2_{yx} & adb^2_{yy} \\ -bcb^1_{xx}-deb^2_{xx} & -bcb^1_{xy}-deb^2_{xy} & c^2b^1_{xx}+e^2b^2_{xx} & c^2b^1_{xy}+e^2b^2_{xx} & -aeb^2_{xx} & -aeb^2_{xy} \\ -bcb^1_{yx}-deb^2_{yx} & -bcb^1_{yy}-deb^2_{yy} & c^2b^1_{yx}+e^2b^2_{yx} & c^2b^1_{yy}+e^2b^2_{yy} & -aeb^2_{yx} & -aeb^2_{yy} \\ adb^2_{xx} & adb^2_{xy} & -aeb^2_{xx} & -aeb^2_{xy} & a^2(b^2_{xx}+b^3_{xx}) & a^2(b^2_{xy}+b^2_{xy}) \\ adb^2_{yx} & adb^2_{yy} & -aeb^2_{yx} & -aeb^2_{yy} & a^2(b^2_{yx}+b^3_{yx}) & a^2(b^2_{yy}+b^3_{yy}) \end{Bmatrix}$$

in which:

$a = l_1 + l_2$ $\quad b = l + l_2$ $\quad c = l - l_1$

$d = l - l_2$ $\quad e = l - l_1$ $\quad$ see also Fig. 3