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Abstract

The purpose of this paper is to present unified spectral models for the three-component velocity fluctuations. A general spectral model is presented which can be matched to velocity spectra observed in flows developed over any kind of upwind terrain and in wind tunnel generated shear flows.

For velocity spectra observed over flat, smooth and uniform (FSU) terrain, the model simplifies sufficiently to allow evaluation of the coefficients and the exponents. Two models can be recognized: a "blunt" and a "pointed" model for which the unknown coefficients can be evaluated based on the integral requirement and matching in the inertial subrange once the turbulence ratios $S_a/U*$ are specified. "Regional" parameters such as the horizontal variance, S_a, and turbulence integral scale, L_x^a, should not be used for nondimensionalizing the velocity spectra. Instead "local" parameters such as the shear velocity, $U*$, and height above the surface, z, must be preferred.

Introduction

Theoretical and statistical analysis of wind loads on buildings and structures requires knowledge of velocity spectra in the atmospheric boundary layer, developed over a variety of upwind terrain conditions. In the wind engineering community one deals primarily with near-neutral conditions so that the effects of surface heat flux and of the convective boundary layer will not be considered in this article.

All analytical forms of normalized spectra have been developed for spectral densities at all frequencies, except for the viscous subrange. Most spectral models are interpolation expressions between a high-frequency asymptote and a low-frequency asymptote, and are primarily based on observations in the atmospheric surface layer developed over FSU terrain. However it has become evident that these models do not represent the observations over terrain with upwind roughness elements or upwind topographic features.

One-Dimensional Velocity Spectrum

One-dimensional spectra of the fluctuating velocity components are the one-dimensional Fourier transforms of the longitudinal and transverse space correlations. In the high-wave number (equilibrium) range, the spectral densities vary uniquely with wavenumber, k, dissipation, ϵ, and viscosity, ν. The low-wave number (energy containing) range, spectral densities are governed by the gain of turbulent energy from the mean flow and the loss of energy to the smaller eddies. For equilibrium turbulence (gain = loss) in a turbulent boundary layer the velocity spectrum in this range must vary uniquely with the friction velocity, $U*$, and the macroscale which is proportionally to height, z. For large Reynolds numbers ($R_T = U^*z/\nu$) an intermediate wave number range exists where both scaling laws are equally valid (Tennekes and Lumley, 1972). Asymptotic matching of these two laws leads to the inertial subrange where the spectral densities are governed by the Kolmogorov law

$$S_a(k) = a_a \epsilon^{2/3} k^{-5/3} \qquad (1)$$

where a_a is a universal constant ($a_u = 0.5$, $a_v = a_w = 0.67$).

<u>Spectra Over FSU Terrain</u>

Consider flow in the surface layer developed over FSU terrain for which the logarithmic velocity law applies. Frequencies of the time spectra obtained from velocity fluctuations observed at a fixed point are related to wavenumbers according to Taylor's hypothesis as follows:

$$n = Uk/2\pi \qquad (2)$$

Replace the dissipation, ϵ, by the normalized dissipation ϕ_ϵ which is defined as

$$\phi_\epsilon = \frac{dissipation}{production} = \frac{\epsilon}{\overline{uw}dU/dz} = \frac{\epsilon K z}{U*^3} \qquad (3)$$

where K is the von *Kármán* constant 0.4. Substituting (2) and (3) in the Kolmogorov law (1), one obtains

$$\frac{nS_a(n)}{U*^2\,\phi_\epsilon^{2/3}} = A_a f^{-2/3} \qquad (4)$$

where $A_a = a_a(2\pi K)^{-2/3}$ and $f = nz/U$. Accordingly the logarithmic velocity spectra presented in this normalized form collapse in the inertial subrange. However in the low-frequency range the slightly stable spectra vary not only with f but also with the Monin-Obukhov stability parameter, z/L (Kaimal et al., 1972). For neutral air with $\phi_\epsilon = 1$ and $z/L = 0$, the general logarithmic spectrum for both frequency ranges takes the following form

$$\frac{nS_a(n)}{U*^2} = F_a(f) \qquad (5)$$

which in the inertial subrange ($f > 1$) becomes

$$\frac{nS_a(n)}{U*^2} = A_a f^{-2/3} \qquad (6)$$

The spectrum of the velocity fluctuations observed at a fixed point can be obtained from the autocorrelation function as follows.

$$S_a(n) = 4\int_0^\infty R_a(\tau)cos(2\pi n\tau)d\tau \quad (7)$$

where $R_a(\tau)$ is the autocorrelation coefficient for time delay τ. For low frequencies ($n \rightarrow 0$) the spectrum function $S_a(0)$ becomes

$$S_a(0) = 4T_aS_a^2 \quad (8)$$

where T_a is the integral time scale and S_a^2 is the variance. This requires that for low frequencies the logarithmic spectrum function, $nS_a(n)$, must be proportional to n, while in the inertial subrange $nS_a(n)$ must be proportional to $n^{-2/3}$. Consequently a graphical representation of $nS_a(n)$ versus n must exhibit these asymptotes with a maximum in between. Of many possibilities three of the simplest and most popular interpolation expressions which obey these requirements are:

$$\frac{nS_a(n)}{U*^2} = \frac{Af}{(1+Bf)^{5/3}} \quad (9)$$

and

$$\frac{nS_a(n)}{U*^2} = \frac{Af}{(1+Bf^{5/3})} \quad (10)$$

and

$$\frac{nS_a(n)}{U*^2} = \frac{Af}{(1+Bf^2)^{5/6}} \quad (11)$$

A general expression for these three models is (Olesen et al. 1984)

$$\frac{nS_a(n)}{U*^2} = \frac{Af}{(1+Bf^\alpha)^\beta} \quad (12)$$

Many researchers have used these forms to describe observed spectra with varying values of the coefficients A and B. These coefficients influence the position of the spectrum function, while its shape is determined by the exponents α and β.

Spectra Over Complex Terrain

For flow over terrain with non-homogeneous surface roughness and/or with topographic variations (complex terrain) no simple relationship exist between the mean flow, the turbulence and the surface roughness. As the terrain conditions increase in complexity the turbulence is no longer in equilibrium with the mean wind profile and parameters derived from it such as the roughness length, z_o, and the shear velocity, $U*$, do not adequately describe the turbulence.

The idea that horizontal turbulence is affected by upwind terrain features was first introduced by Panofsky et al. (1977). Large scale eddies primarily generated by upwind topography are oriented in a horizontal plane and therefore contribute significantly to the variances of the horizontal turbulence, but not much to the variance of the vertical component (Panofsky et al., 1978 and 1980). The vertical velocity fluctuations adjust quickly to the local surface conditions and bear the characteristics of the "local" terrain. In contrast the peak wave lengths of the horizontal turbulence components are much longer than those of the vertical component and thus require a longer fetch to adjust to changes in terrain. Since the contribution of these large scale eddies to the horizontal velocity variances dominates that of the small scale fluctuations (inertial subrange), the variances of the horizontal components bear primarily the characteristics of a long upwind fetch ("regional" terrain). Based on these observations one must conclude that for near-neutral flow over complex terrain, the high-frequency range of the horizontal spectra and the entire vertical spectrum are usually in equilibrium with the local terrain. The low-frequency parts of the u and v spectra are affected by the upwind terrain characteristics and therefore are representative of the "regional" terrain. These spectra exhibit a significant increase in low-frequency energy relative to the corresponding spectra observed over FSU terrain (Panofsky et al., 1982). In order to adjust for these increased spectral densities, Teunissen (1980) proposed a spectral model of the following form

$$\frac{nS_a(n)}{U*^2} = \frac{Af}{(C+Bf^\alpha)^\beta} \quad (13)$$

Here C was taken to be less than one which has the effect of increased spectral densities in the low-frequency range and of a larger value of the spectral peak. However for flow over complex terrain with significant topographic features well upwind from the observation site, this model is inadequate since the slope of the low-frequency asymptote is often less than one (Panofsky et al., 1982). Also the u and v spectra often have a broader shape when compared to the FSU spectra. Consequently for complex terrain an adequate spectral model may be of the following form

$$\frac{nS_a(n)}{U*^2} = \frac{Af^\gamma}{(C+Bf^\alpha)^\beta} \quad (14)$$

Obviously the FSU spectral models (9, 10 and 11) and the Teunnissen model (13) are special cases of this generalized model.

Modeling Criteria

The general spectral model (14) is subject to the following requirements:

1. The spectral slope in the inertial subrange ($f > 1$) must be consistent with (6)

$$\alpha\beta - \gamma = 2/3 \quad (15)$$

2. The variation of the spectral densities in the inertial subrange must follow (6) requiring that

$$A/B^\beta = A_a = \begin{cases} 0.27 \; for \; a = u \\ 0.36 \; for \; a = v \; and \; w \end{cases} \quad (16)$$

3. The observed spectral density at low frequencies (say $f = 10^{-4}$) must be equivalent to

$$S_\ell = Af^\gamma / C^\beta \quad (17)$$

4. The observed spectral peak should occur at

$$f_m = \left(\frac{1.5\gamma C}{B}\right)^{1/\alpha} \quad (18)$$

5. The observed spectral density at f_m must be

$$S_m = \frac{A(1.5\gamma C/B)^{\gamma/\alpha}}{[C(1+1.5\gamma)]^\beta} \quad (19)$$

6. The integral of (14) must be equivalent to $(S_a/U*)^2$.

For the last requirement it is automatically assumed that the spectrum function continues according to (6). In the atmosphere the inertial subrange extends approximately three decades to relatively low spectral values. Beyond this point, whether or not the spectral function drops off with the same slope as the inertial subrange or drops off faster as is the case in the viscous subrange is immaterial as far as requirement 6 is concerned.

Consider FSU terrain for which the exponent $\gamma = 1$ and $C = 1$, Olesen et al. (1984) recognize two models (9 and 10) for which requirement 6 leads to:

a. The "blunt" model (9) with $\alpha = 1$ and $\beta = 5/3$

$$S_a/U* = \left[\frac{1.5A}{B}\right]^{0.5} \tag{20}$$

Together with requirement 2 this relation will allow us to calculate A and B provided $S_a/U*$ is known:

$$A = A_a B^{5/3} \text{ and } B = \left[\frac{(S_a/U*)^2}{1.5A_a}\right]^{1.5} \tag{21}$$

b. The "pointed" model (10) with $\alpha = 5/3$ and $\beta = 1$

$$S_a/U* = [A(3.088/B)^{0.6}]^{0.5} \tag{22}$$

which combines with requirement 2 to:

$$A = A_a B \text{ and } B = \left[\frac{(S_a/U*)^2}{1.967A_a}\right]^{2.5} \tag{23}$$

Similarly for model (11)

$$S_a/U* = [A(4.117/B)^{0.5}]^{0.5} \tag{24}$$

which combines with requirement 2 to:

$$A = A_a B^{5/6} \text{ and } B = \left[\frac{(S_a/U*)^2}{2.029A_a}\right]^{3} \tag{25}$$

With the equilibrium values of the turbulence ratios $S_a/U*$ of 2.5, 2.0 and 1.25 for $a = u, v$ *and* w respectively, coefficients A and B for the three models and the corresponding values of f_m and S_m (requirements 4 and 5) can be evaluated (Table I). Plots of these spectral models show the subtle differences between model 9 and 10, while model 11 is even more pointed than model 10. In order to normalize the logarithmic spectra with their respective variances, values of coefficient A should be divided by $(S_a/U*)^2$ =6.25, 4, and 1.56. However it must be understood that the horizontal variances are "regional" parameters which can not be expected to satisfy the universality of the Kolmogorov law in the inertial subrange for terrain categories other than FSU. Tieleman (1991) compared models 9 and 10 with full-scale spectra and other models. For observed neutral u spectra over true FSU terrain (Meijer, 1989) the pointed model (10) provides the best fit, while for perturbed upwind terrain (Levitan, 1991) the blunt model (9) is preferred. Consequently for neutral air in the atmospheric surface layer the following spectral models are recommended:

Table I
Coefficients and Properties for FSU Spectral Models

		Model (9)	Model (10)	Model (11)
u-comp.	A	252.6	128.28	118.7
	B	60.62	475.09	1485
	f_m	0.0247	0.0316	0.0318
	S_m	1.357	1.621	1.435
v-comp.	A	53.76	27.3	-
	B	20.16	75.84	-
	f_m	0.0744	0.095	-
	S_m	0.869	1.037	-
w-comp.	A	5.13	2.604	-
	B	4.92	7.232	-
	f_m	0.305	0.389	-
	S_m	0.340	0.405	-

FSU Terrain   Perturbed Terrain

$$\frac{nS_u(n)}{S_u^2} = \frac{20.53f}{1+475.1f^{5/3}} \qquad \frac{nS_u(n)}{S_u^2} = \frac{40.42f}{(1+60.62f)^{5/3}} \tag{26}$$

$$\frac{nS_v(n)}{S_v^2} = \frac{6.83f}{1+75.84f^{5/3}} \qquad \frac{nS_v(n)}{S_v^2} = \frac{13.44f}{(1+20.16f)^{5/3}} \tag{27}$$

$$\frac{nS_w(n)}{S_w^2} = \frac{1.67f}{1+7.23f^{5/3}} \qquad \frac{nS_w(n)}{S_w^2} = \frac{3.28f}{(1+4.92f)^{5/3}} \tag{28}$$

If the upwind terrain is more complex, the turbulence ratios $S_u/U*$ and $S_v/U*$ are expected to be somewhat larger than 2.5 and 2.0 respectively, while above the surface layer these ratios are expected to be smaller. For specific empirical relations for the variation of $S_u/U*$ with height above the surface layer, the reader is referred to Harris and Deaves (1980). New values of the coefficients A and B can be calculated from (21), (23) and (25), indicating that for increasing values of $S_u/U*$, the magnitude of the logarithmic spectral peak increases while simultaneously shifting to lower values of f.

Over non-homogeneous terrain the observed u and v spectra at low wave numbers are functions of the upwind topography and vary from site to site. The horizontal spectra cannot be described with a universal relation, since the shape of these spectra in the low-frequency range appears to be influenced in an unpredictable fashion by upwind terrain conditions. The spectra over complex terrain are characterized by a low-frequency asymptote whose slope, γ, is less than unity, by a higher value for the spectral peak, and with the low and high-frequency asymptotes further apart than predicted by the FSU terrain models (26, 27 and 28). In order to match the observed complex terrain spectra to the general spectral model (14), it is required to specify $S_m(19)$, $S_\ell(17)$ and γ in order to calculate coefficients A, B, and C and the exponents α and β. In general, the integral criterion cannot be used since the turbulence ratio $S_a/U*$ for the horizontal components varies greatly with the character of the upwind terrain. Results of this matching procedure, including those of the Rock Springs complex terrain spectra (Panofsky et al., 1982) are presented by Tieleman (1991).

Near-neutral logarithmic velocity spectra are often described in terms of the ratio f/f_m or n/n_m, where f_m is the reduced frequency for which $nS_a(n)$ exhibits its maximum value. In order to transform the general spectral model (14) in this format the following substitutions for the constants A, B and C need to be made:

$$A = S_m/f_m^\gamma,\ B = B'/(A'f_m^\alpha) \text{ and } C = C'/A',$$

which leads to

$$\frac{nS_a(n)}{U*^2} = S_m(f/f_m)^\gamma \left[\frac{A'}{C' + B'(f/f_m)^\alpha}\right]^\beta$$

The requirements for this format are equivalent to the previous 6 requirements with the above substitutions for coefficients A, B and C.

Comparison of Spectral Models

In the past, the Davenport spectral model (Davenport, 1961) was generally used in the wind engineering community to represent the u-spectra. However this model has a logarithmic spectral peak which is too high for the FSU terrain category, and decreases too rapidly in the low-frequency range ($\gamma = 2$). Moreover this model cannot satisfy the similarity condition (6) in the inertial subrange.

The von *Kármán* model was considered to be a better model to describe the observed u spectra. This model is similar to the models (9, 10 and 11) but instead was derived for u-spectra observed in wind tunnel flows behind grids. In this model the logarithmic spectral density $nS_a(n)$ is normalized with the corresponding variance, S_a^2, (in grid flow $U*$ vanishes), and the wave length is normalized with the turbulence integral scale, L_x^a, which is the only available scaling length. For atmospheric flows, S_a and L_x^a are "regional"parameters affected by the characteristics of the upwind terrain and not suitable as scaling variables in the inertial subrange, the latter being in equilibrium with the "local"terrain. Consequently with the choice of S_a and L_x^a as scaling variables, universality of the normalized velocity spectra in the inertial subrange as required by the Kolmogorov law (6) cannot be expected.

Possible exceptions are surface layer flows developed over true FSU terrain and turbulent shear layers developed in long test-section wind tunnels. Also observed integral scales vary with height and terrain roughness and their observed values exhibit excessive scatter, more reasons to reject this parameter as length scale. Tieleman (1991) shows in detail how the von *Kármán* spectrum functions together with the empirical relations (ESDU, 1974) for the integral scale are incompatible with the universal Kolmogorov law (6). For all these reasons velocity variances and turbulence integral scales are not suited as scaling variables for velocity spectra, instead the choice of $U*$ and z must be preferred.

Conclusions

A general spectral model with 3 coefficients and 3 exponents has been presented which is subject to a number of requirements. For true FSU and slightly perturbed upwind terrain the general spectral model can be simplified to the point that the remaining unknowns (2 coefficients) can be calculated. Two models emerge: the pointed model for true FSU terrain and the blunt model for slightly perturbed terrain. For more complex terrain the situation is not so simple, several elements of the spectral model need to be specified. Spectral models should be developed in such a manner that they reduce to the universal Kolmogorov law in the high-frequency range. One should avoid normalization of the spectral models with "regional"parameters (S_a and L_x^a) unless one deals with true FSU upwind terrain. In all other situations only "local"parameters such as $U*$ and z must be preferred for the normalization process.

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