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COMPUTATION OF HIGH SPEED FLOWS USING AN EXPLICIT SPATIAL MARCHING ALGORITHM

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ABSTRACT

The paper describes the salient features of an explicit spatial marching algorithm for high speed flows being developed by the author and its application to compute hypersonic flow past a compression corner. The procedure solves the Reduced Navier-Stokes equations in generalised coordinates and uses the Five-stage Runge-Kutta technique to advance the solution spatially. Unlike the other spatial marching procedures the present one retains the time derivative in the governing equations. Using this algorithm an unseparated hypersonic flow at Mach 14.1 past a 15^0 corner is computed. This is a frequently used test case studied experimentally by Holden and Moselle. The computed wall pressure and heat transfer rates show reasonable agreement with the experiments. The pressure and Mach number contours indicate that the method is capable of capturing the features of a hypersonic interaction.

INTRODUCTION

Spatial Marching procedures have been extensively used in computing supersonic and hypersonic flows (Lawrence et al., 1989, Korte and McRae, 1988, Siclari and Del Gudice, 1990, Harvey III et al., 1991, Chitsomboon et al., 1988, Chang and Merkele, 1989). For these high speed flows with a dominant direction, it seems very appropriate to use the Reduced Navier-Stokes (*RNS*) equations which are obtained by dropping the viscous derivatives in the flow direction. Further, these flows are characterised by the fact that any upstream influence in them is limited to subsonic portions inside thin boundary layers. Hence the Spatial Marching methods, wherein one marches from a station with x or ξ = constant to the next downstream station, are the appropriate ones to be employed. It may be pointed out that such methods are almost an order of magnitude faster than the Time Marching techniques. Many Spatial Marching methods have been developed in literature and most of these have been implicit in nature. But from the point of view of modern computer architecture involving parallel processors, explicit schemes seem to be very desirable. It is well recognised that the explicit schemes are easily implemented and they render themselves readily to parallel processing. In fact, there are a few explicit Spatial Marching algorithms in use (Korte and McRae, 1988, Siclari and Del Gudice, 1990, Srinivas, 1992).

The present author has been developing an explicit procedure based on the Jameson - Schmidt scheme (Jameson and Schmidt, 1984). A special feature of the method is that it retains the time derivative ,dW/dt, in the governing equation whereas it is dropped in the other methods. Now the term Δt is interpreted as a relaxation parameter. An earlier version of this scheme was used to compute the interaction between a shock wave and a supersonic boundary layer (Srinivas, 1992). The results were encouraging but the solution seemed very oscillatory near the shocks.

The present work is an extension of the previous one. Now the algorithm makes use of a five-stage Runge-Kutta procedure with three evaluations of dissipative terms instead of a four stage-scheme. The previous work was characterised by the fact that repeated downstream marches were carried out in order to take into account the separation of flow. But in the present work an unseparated flow is considered and hence only one sweep is performed from upstream to downstream.

The test case chosen is that of a hypersonic flow at Mach 14.1 past a compression corner (15^0). This problem has been experimentally studied by Holden and Moselle (1969) and has been a test case in many computational studies (Lawrence et al., 1989, Korte and McRae, 1988, Rudy et al., 1989). This example provides a good benchmark to validate the computational methods for their ability to compute hypersonic flows and hypersonic interactions.

In this paper the marching algorithm and the governing equations are discussed next. This is followed by a discussion of the computed results.

MARCHING ALGORITHM

The broad features of the algorithm used to advance the solution spatially are described in this section. For details one is referred to Srinivas (1992).

Governing Equations

The governing equations are the Reduced Navier Stokes equations in two dimensions and for a spatial marching in the ξ - direction are given by,

$$\frac{\partial W}{\partial t} + \frac{\partial F}{\partial \xi} + \frac{\partial G}{\partial \eta} - \frac{\partial G_v}{\partial \eta} = 0 \qquad (1)$$

where,

$$W = \{\rho\ ,\rho u\ ,\rho v\ ,e\}^T \tag{2}$$

In the above equation ρ is density, u and v are the velocity components in the x and y directions, and e is total energy expressed as,

$$e = \left(\frac{p}{\gamma - 1} + \rho\frac{u^2 + v^2}{2}\right) \tag{3}$$

The inviscid flux terms F, G and the viscous flux terms in the η direction, G_v are given by

$$F = \frac{1}{J}\{\ \rho U,\ \xi_x p+\rho uU,\ \xi_y p+\rho vU,\ (e+p)U\ \}^T$$

$$G = \frac{1}{J}\{\ \rho V,\ \eta_x p+\rho uV,\ \eta_y p+\rho vV,\ (e+p)V\ \}^T$$

$$G_v = \frac{1}{J}\{\ \eta_x\bar{F} + \eta_y\bar{G}\ \}^T \tag{4}$$

The various terms in eqn. 4 are given by,

$$\bar{F} = \{\ 0,\ \sigma_{xx},\ \tau_{xy},\ u\sigma_{xx}+v\tau_{xy}+q_x\ \}^T$$

$$\bar{G} = \{\ 0,\ \tau_{xy},\ \sigma_{yy},\ u\tau_{xy}+v\sigma_{yy}+q_x\ \}^T$$

$$\sigma_{xx} = 2\frac{\mu}{3}(2\eta_x u_\eta - \eta_y v_\eta)$$

$$\sigma_{yy} = 2\frac{\mu}{3}(2\eta_y v_\eta - \eta_x u_\eta) \tag{5}$$

$$\tau_{xy} = \mu(\eta_y u_\eta + \eta_x v_\eta)$$

$$q_x = -k\eta_x T_\eta \quad ; \quad q_y = -k\eta_y T_\eta$$

Pressure Splitting

In accordance with the spatial marching procedure, the pressure term in the x - momentum equation is split as,

$$p^+ = \omega p \quad ; \quad p^- = (1-\omega)p \tag{6}$$

where ω is a function of the stream wise Mach number , M_ω and is given by

$$\omega = \frac{\sigma\gamma M_\xi^2}{1+(\gamma-1)M_\xi^2}\ ; \quad M_\xi < 1$$

$$\omega = 1\ ; \qquad M_\xi \geq 1 \tag{7}$$

where σ is a safety factor and is equal to 0.7 in the present study.

In the above equation the term p^{-} accounts for the upstream effect. There are many ways of handling this term and one such is described in Srinivas (1992). In the present example considered there is no separation of flow and as such the upstream effect will be very limited. Accordingly this term is dropped.

Marching Algorithm

If now a suitable discretisation Q is assumed for the spatial derivatives in the eqn. 1, we have

$$\frac{dW}{dt} + Q(F,G) = 0 \tag{8}$$

This equation is integrated using the Jameson - Schmidt scheme (Jameson and Schmidt, 1984) which is a modified form of Runge - Kutta technique and is given by

$$W^{(0)} = W^n$$

$$W^{(1)} = W^{(0)} - \alpha^{(1)}\Delta t Q(W^{(0)})$$

----- (9)

$$W^{(k)} = W^{(0)} - \alpha^{(k)}\Delta t Q(W^{(k-1)})$$

$$W^{n+1} = W^{(k)}$$

where 'k' denotes the number of stages used in the Runge - Kutta method and is 5 in the present work.

Spatial Discretisation

In the present work the η - derivatives are discretised using the central differences as

$$G_\eta = \frac{G_{i,j+1} - G_{i,j}}{2\Delta\eta} \tag{10}$$

For the ξ - derivatives, two point back ward differences are used --

$$F_\xi^+ = \frac{F_{i,j}^+ - F_{i-1,j}^+}{\Delta\xi} \tag{11}$$

where F^+ denotes that p^+ has been substituted for pressure in the calculation of the flux term F in eqn. 4.

The procedure needs artificial dissipation for shock handling and for convergence. These terms follow Srinivas (1992) and Swanson and Radespiel (1991) and are calculated at stages 1,3 and 5 in the Runge-Kutta processing of the solution (see eqn. 9).

The marching procedure is carried out as follows. Starting conditions i.e, the freestream conditions are prescribed at the upstream station. Then the solution is marched downstream by performing adequate number of iterations at every station (based on eqn. 8) till the *rms* change in density for the station is less than 10^{-4}.

RESULTS AND DISCUSSION

The geometry of the test case is shown in Fig.1 and is that of a hypersonic flow over a 15^0 compression corner.

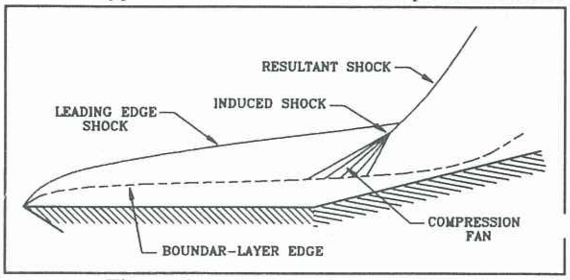

Figure 1, Geometry of the test case.

The flow conditions correspond to one of the cases experimentally studied by Holden and Moselle (1969) and are as follows: M_∞=14.1, T_∞=72.2K, Re_l=1.04x10^5, γ=1.4, l=0.439m, T_w=297K and Pr=0.72. Re_l is the freestream Reynolds number based on the distance between the leading edge and the compression corner. The flow involves a very strong shock and an unseparated laminar flow. Since the freestream static temperature is low (72.2 K) there are no significant real gas effects even though the freestream Mach number is high (14.1). The flow however involves complicated inviscid and viscous interactions. The leading edge shock interacts with the compression shock and produces a stronger resultant shock. Heat transfer rate and pressure rise sharply following the compression corner and the boundary layer thins down. Thus the flow is challenging from a computational point of view.

Computations were started from the leading edge of the geometry with a very fine grid in the flow direction consisting of 100 points between x=0 and 0.1m. In the region between x=0.1m and 1m, 150 uniformly spaced grid points were used (see Fig. 2 which shows the grid from x=0.4m to x=1m). In the flow normal direction the grid spacing was non-uniform with 45 points and the placing of the first point close to the wall was 1x10^{-4}m.

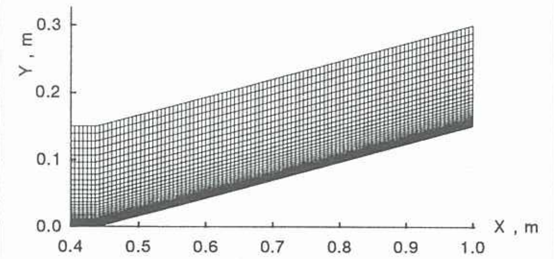

Figure 2, Computational Grid.

The distribution of the wall pressures expressed as C_p ($p_w/0.5\ \rho_\infty V_\infty^2$) is shown in Fig.3 together with the experimental data of Holden and Moselle (1969). There is a good agreement in general. But in regions upstream of interaction, pressure is over predicted in the present computations. This feature is present in most of the computational results for this problem (Lawrence et al., 1989, Korte and McRae, 1988, Rudy et al., 1989).

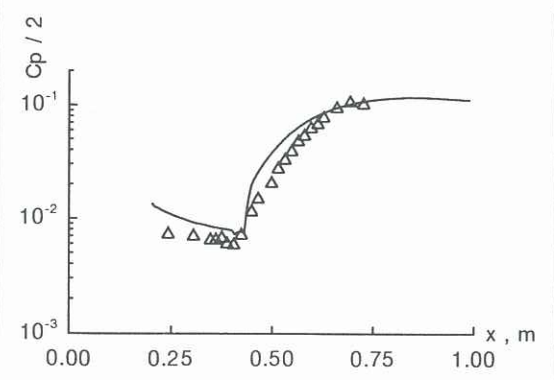

Figure 3, Wall Cp distribution, ____ , present, Δ, Holden and Moselle (1969).

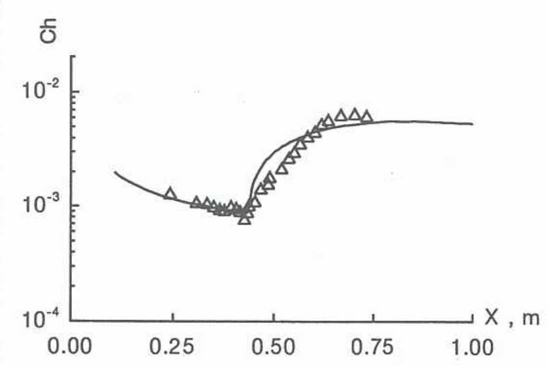

Figure 4, Distribution of wall heat transfer coefficient, ____, present, Δ, Holden and Moselle (1969).

The distribution of wall heat transfer coefficient Ch (defined as $q'/\rho_\infty U_\infty\ (H_\infty - H_w)$ where q' is the heat transfer rate, H is enthalpy and ∞ denotes the freestream conditions) shown in Fig.4. seems to exhibit a better trend. But any agreement in the Ch values in regions upstream of the corner should be considered fortuitous. Downstream of the corner pressure and heat transfer coefficients rise somewhat sharply in the present computations and exhibit a typical first order behaviour.

A detailed computational study by Rudy et al.(1989) has revealed that substantial three dimensional effects are present in the experimental results of Holden and Moselle. In fact, Rudy et al. use a three dimensional code with an angle of attack correction to obtain results that are in excellent agreement with the experimental ones. The present results were obtained with a two dimensional code without any angle of attack correction thus accounting for the discrepancy between computed and experimental results.

Figures 5 and 6 show the pressure and Mach number contours as computed in the present study.

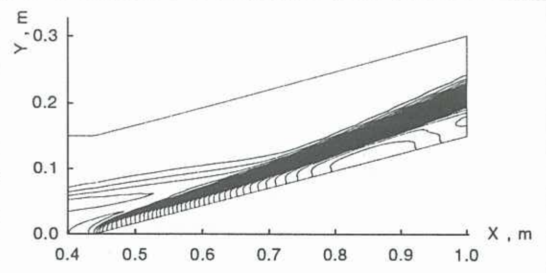

Figure 5, Pressure contours.

Only the contours on the ramp part of the geometry are shown. The formation of the induced shock at the compression corner and the interaction of this shock with the leading edge shock giving rise to the resultant shock are clearly seen. However, the expansion wave forming out of this interaction is diffused. But it is to be noted that the pressure contours are free of oscillations.

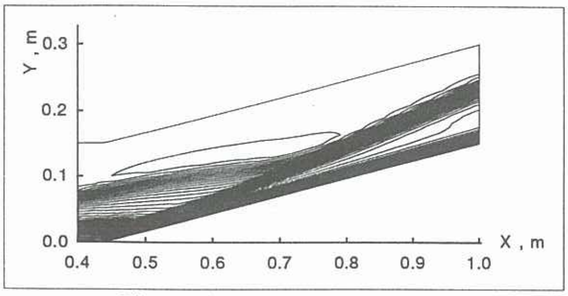

Figure 6, Mach number contours.

The pressure distribution and heat transfer rate predictions could perhaps be improved using a higher order method. The possibilities of improving the order of accuracy of the present procedure will be considered in future.

CONCLUSIONS

The hypersonic flow over a 15^0 compression ramp is computed using an explicit spatial marching procedure developed by the author. Computed results are found to be in reasonable agreement with the experimental ones.

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