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THE EFFECTS OF LATENT HEAT IN AN IMMERSED VAPOUR BUBBLE

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ABSTRACT

A semi-empirical approach is used to investigate the change of pressure in a vapour bubble in water. The equation for the change of pressure in a vapour bubble is formulated within the framework of classical thermodynamics. The properties of the vapour, such as internal energy and specific volume at various temperatures and pressures, are derived from the relevant engineering steam tables. The principles of energy conservation and potential flow are applied to give a second equation which relates the growth rate of the bubble to the surrounding fluid. These two equations are solved simultaneously. Results show that the pressure and temperature inside a vapour bubble increase sharply as the bubble collapses. The calculated half periods of the bubble give good agreement with the measured time interval for the growth of bubbles. The mechanism for the formation of a vapourised bubble and the significance of heat transfer will be discussed.

INTRODUCTION

The behaviour of immersed vapour bubbles is a significant factor to be considered in many thermo-fluid studies. In the experimental study of a vapour bubble, the growth and collapse of the bubble is initiated by a high energy input such as an electrical spark discharge or laser energy. The best recording method for such a transient event is to capture the images of the bubble evolution by high speed photography. The growth rate of the bubble, measured from these images, is the main source of information for analysing the behaviour of the bubble. The pressure and temperature in the bubble and the associated process of vapourisation and condensation are the significant factors which govern the growth and collapse of the bubble.

Many researchers have assumed that the pressure in the vapour bubble is either constant or followed simple gas laws (Kumar & Booker (1991), Vokurka (1985)). These simple assumptions for the vapour in the bubble produce flow models in which the pressure is determined by the surrounding liquid.

Fujikawa, et al. (1980) carried out a very elaborate study on the collapse of a cavity bubble in which the interactions between vapour/gas inside the cavity bubble and liquid outside were considered. Approximation formulae were used for the condensation of vapour, the thermal boundary layer and constant specific heats. The gases and vapours in the bubble are assumed to follow the perfect gas law. Computational results show features of rebound of the bubble and demonstrate that the vapour pressure and temperature inside the bubble fluctuates with the process of condensation and vapourisation. It is evident that the pressure in the vapour bubble will influence the pressure field of the liquid phase; and it follows that the equations of motion for liquid and the vapour bubbles are coupled.

The assumption that vapours follow simple gas laws has neglected the energy contribution due to latent heat. However, these simple gas laws are convenient to use on vapours in which their thermodynamic properties are not known in detail. In the case of water vapour, a more accurate model for the behaviour of vapour can be built from the information available in the steam tables.

This paper presents the formulation, within the framework of classical thermodynamics, for the fluctuation of pressure in a vapour bubble. The properties of the vapour are derived from engineering steam tables. Through the use of energy conservation, the results of interaction between the liquid phase and the bubble are illustrated in terms of the evolution of the bubble and the pressure increase in it.

FORMULATION

Consider the energy and work done in a vapour bubble as its volume changes. The work done by the bubble is given by $P_v\, dV$, $\rho_v V du_g$ is the change of the internal energy of the vapour, and the change in the internal energy for the condensation of vapour (assuming that the condensed liquid is mixed with the surrounding liquid) is given by $dm_L[u_g - u_L]$. If Q denote the rate of heat transfer into the bubble, the First Law of thermodynamics for control mass analysis gives:

$$P_v\, dV + \rho_v V\, du_g(P_v) - dm_L[u_g(P_v) - u_L(T)] = Q\, dt \tag{1}$$

The conservation of mass requires that

$$dm_L + d(\rho_v V) = 0 \tag{2}$$

Let us assume that the only heat transfer is through the mixing of the condensed vapour with the surrounding water (i.e. Q = 0). At this point, it may be convenient to define a pressure function H which has a dimension of pressure and is dependent on the vapour pressure and the temperature of the surrounding liquid.

$$H(P_v, T) = \rho_v[u_g(P_v) - u_L(T)] \tag{3}$$

Here H is a function of a property of saturated steam and is conveniently expressed in terms of either the vapour pressure or temperature of saturation.

Thus Equations (1), (2) and (3) give,

$$P_v \, dV = - d\{VH(P_v,T)\} \tag{4}$$

The integration of the above equation will provide an expression which relates the volume of the bubble to the pressure in it. The change of volume can also be analysed from the hydrodynamics of the system. Consider an experimental situation in which the water depth is 'd' and a bubble is generated at 'h' above the rigid floor. The boundary condition on the free surface is linearized so that requirement for constant pressure on the free surface is approximated by making the horizontal velocity component zero. The boundary condition on the floor is simply a vanishing velocity component normal to the floor. The flow of water generated by the rate of change of volume of a bubble, dV/dt, is similar to a source (or sink) which has a strength equal to this rate of volume change. Thus, by using a series of positive and negative images of point sources (or sinks), the velocity potential, valid only outside the bubble, will satisfy these boundary conditions.

It has been shown by Soh (1992) that the kinetic energy of the fluid around the bubble is given by

$$[KE] = \frac{\rho}{2} \frac{(\dot{V})^2}{4\pi} \left[\frac{1}{R(t)} + \frac{\pi}{2h} \cot\left(\frac{\pi h}{d}\right) \right] \tag{5}$$

The work done by the bubble on the fluid domain is the integration of pressure in the bubble with respect to its volume and with the help of Equation (4) it is expressed as

$$-\int_0^V P_v \, dV = V \, H(P_v,T) \tag{6}$$

There is a rise of the free surface, in volume, equal to the volume of the bubble. The work done by water on the atmosphere of constant pressure P_{atm} is P_{atm} V.

The conservation of energy is assumed. By neglecting the potential energy due to the rise of the free surface, the algebraic sum of the kinetic energy and all work done is a constant. This gives the equation

$$\frac{\rho}{2} \frac{(\dot{V})^2}{4\pi} \left[\frac{1}{R(t)} + \frac{\pi}{2h} \cot\left(\frac{\pi h}{d}\right) \right] + V \, H(P_v,T) + V \, P_{atm}$$

$$= V_{max} \, H(P_v^*,T) + V_{max} \, P_{atm} \tag{7}$$

The right hand side of Equation (7) corresponds to the situation where the bubble's volume is at its maximum (i.e. dV/dt = 0 and $V=V_{max}$) and the pressure in the bubble at this instant is P_v^*. Note that the volume $V = \frac{4\pi}{3} R^3$.

It can be shown that the Rayleigh equation for a spherical bubble (Raleigh (1917)) can be recovered from Equation (7) by replacing H with a constant vapour pressure P_v (equal to P_v^*), and replacing the rate of change of volume by $4\pi R^2$ dR/dt..

In addition to Equation (7) the relationship between V and H is required. This can be derived from rearranging Equation (4) and integrating the resulting expression with respect to volume to give

$$\ln\left[\frac{V}{V_{max}}\right] = - \int_{H^*}^{H} \frac{dH}{H + P_v} \tag{8}$$

Equation (8) will result in a tabulation of V/V_{max} against the values of H and H^*. Equations (7) and (8) can be numerically solved for V as a function of time and from the ratio V/V_{max}, the values of H are obtained by inverting Equation (8). It follows that P_v (and hence the temperature in the bubble, T_v) can be calculated by inverting Equation (3).

COMPUTATIONS AND DISCUSSIONS

It is evident that by making dV/dt zero, Equation (7) yields

$$\frac{V_b}{V_{max}} = \frac{H^* + P_{atm}}{H + P_{atm}} \tag{9}$$

where V_b is the value of V at dV/dt = 0 and H^* corresponds to the condition at $V = V_{max}$. A trivial solution is $V_b = V_{max}$, however Equation (8) and Equation (9) (replacing V with V_b) yields another solution. This solution is denoted by V_{min} and it corresponds to the minimum volume of the bubble before rebound takes place. In other words, the volume of the bubble will not be reduced to zero -- this agrees with experimental observations. It must be noted that the analysis by Rayleigh (1917), which assumes constant vapour pressure, will produce a result which allows the bubble to collapse to zero volume.

In the absence of heat transfer, the growth phase of the bubble is a mirror image of the collapse phase. This means that the bubble will grow from a finite small volume V_{min}. This is an approximation for the formation of a vapour bubble from a shock of high energy input (e.g. spark discharge) which brings about the local boiling of water.

In the cinematographic study of vapour cavity bubbles, the minimum volume (i.e. V_{min}), and the maximum volume (i.e. V_{max}) of the bubble can be measured. The ratio V_{min}/V_{max} will replace V_b/V_{max} in Equation (9) and V/V_{max} in Equation (8). These two equations now hold the implicit relationship between $[P_v]_{max}$ (pressure at maximum volume) and $[P_v]_{min}$ (pressure at minimum volume). In the experimental situation, a bubble is generated by local heating of the fluid (such as by spark discharge and laser). High speed cine-photography is used to capture the images. On the assumption of axisymmetry, the volume of the bubble can be measured at any time. It follows that the ratio of the maximum to the minimum volume will give the minimum and maximum pressures in the bubble according to the solution of Equations (8) and (9).

Equation (7) is integrated using the Fourth Order Runge-Kutta method. The pressure terms in (7) are a function of volume and are calculated from the inversion of Equation (9). It must be noted that the pressure function H is derived from property data for saturated steam from Steam Tables (1970) and hence the process of interpolation is required to emulate the functional relationship between pressures and volumes (given in (9)) as required in Equation (7).

The plot of volume ratio, V/V_{max} and pressure in the bubble against time are shown in Figures 1 and 2. The sharp rise of pressure is a feature which agrees with the study by Fujikawa, et al. (1980). It must be noted that the peak pressure will be somewhat reduced if convective and conductive heat transfer are allowed to take place. A further examination of the temperature in the bubble shows that it increases from a temperature of about 8^oC which is below the temperature of the surrounding liquid (20^oC) to almost 100^oC (see Figure 3). This pressure and temperature in the bubble could be much higher if it has sufficient energy to collapse into smaller volume. An evidence for the existence of high pressure and high temperature in the bubble also come from the phenomenon of luminesence. As suggested by Neppiras and Noltingk (1950), at the collapse of the bubble, the highly compressed gases in the bubble become incandescent. The generation of high temperature in the bubble in such a short time will give rise to large temperature gradient across the vapour-liquid interface and the rate of heat transfer across the bubble surface can no longer be ignored.

The calculated results, assuming adiabatic conditions, are symmetrical about the the point where the volume is the maximum. In other words, the process of growth of bubble from minimum volume to its maximum volume is reversed as the bubble collapses. Thus the times taken by the bubble to grow and to collapse are the same but will vary according to the situations such as the depth of water and the height of the bubble above the rigid floor. These times can be compared with those measured in cinematographic study by Soh and Yu (1992). Experimental data for bubble generated at various distances from rigid floor and free surface were recorded for comparison with the calculated results. Figure 4 summarises the comparison among these three time durations: the calculated time, the duration for the bubble to grow to its maximum, t_g, and the duration for it to collapse to its minimum volume, t_c. The calculated times have a better agreement with the measured t_g. With the exception of four cases, the calculated time is slightly shorter than t_g. It is interesting to note that this agreement does point to the fact that the time for the bubble to grow is related to three nondimensional parameters: d/R_{max}, h/R_{max} and the volume ratio, V_{min}/V_{max}. The volume ratio is especially important as it depends on the energy input to the bubble.

The measured times for the bubble to collapse from its maximum volume (i.e. t_c) is much longer than the duration of bubble growth, and the calculated results could imply that the effect of heat transfer has become important in the collapse phases. This speculation is further strengthened by the observation that in a series of rebounds (as many as six rebounds have been observed) the maximum volume attained in each rebound is smaller than the previous one -- an indication of reduced in energy in the bubble.

CONCLUDING REMARKS

The adiabatic process, including vapourisation and condensation which is modelled in this work, will give rise to pressure changes in the bubble and will also show that the bubble will collapse in to a small volume before rebound occurs. This is in agreement with experiments (Wong et al.(1989), Soh and Yu (1992)).

The ratio between the maximum and minimum pressure could be lower if the effect of heat convection is included in the analysis; however, it is not possible at this stage as the order of magnitude of the coefficient of the heat transfer is not known. So far there is no direct measurements of pressure and temperature in the bubble to allow the amount of heat transfer to be estimated.

The present model assumes bubbles grow from a small initial volume. This turns out to be an approximation for bubbles which grow from a local heating. This high energy heating initiates an unsteady thermodynamic process for a very short time. For spark discharge it is estimated to be about 15 microseconds.

The formation of a cavitation bubble in hydraulic machines is due to the reduction of pressure and does not involve initial heating at high temperature. On examination of Equation (1), it is evident that the vapourisation of water requires energy and this can only be achieved through heat transfer from the surrounding water. In other words, dV in Equation (1) will be positive if the initial vapour temperature falls below that of the surrounding liquid by at least a small fraction of a degree. Further study is required to investigate this process of heat transfer.

REFERENCES

FUJIKAWA, S and AKAMATSU, T (1980), Effects of the non-equilibrium condensation of vapour on the pressure wave produced by the collapse of a bubble in a liquid, J. Fluid. Mech., 97, part 3, 481-512.

GIBSON, D C (1968), Cavitation adjacent to plane boundaries, Proc. Third Australasian Conference in Hydraulics and Fluid Mechanics, Institution of Engineer, Australia, 210-214.

KUMAR, A and BOOKER, J F (1991), A finite element cavitation algorithm, Trans. ASME, April, 113, 276-286.

NEPPIRAS, E A and NOLTINGK, B E (1950), Cavitation produced by ultrasonics, Proc. Phys. Soc. (London) 63B, 674-685.

RAYLEIGH, LORD (1917), On the pressure developed in a liquid during the collapse of a spherical void, Phil. Mag. 34, 94-98.

SOH, W K (1992) An energy approach to the study of cavitation bubbles near compliant surfaces, Applied Mathematical Modelling, 16, May 263-268.

SOH, W K and YU, C F (1992), A scale model study of underwater explosion , Report for research contract, Materials Research Lab, Defence Science and Technology Organization, Australia.

VOKURKA, K (1985), On the assumption of a uniform pressure field in the bubble interior, Acta Technica Csav, 30, no. 5, 585-593.

UK STEAM TABLES IN SI UNITS 1970, United Kingdom Committee on the Properties of Steam, Edward Arnold, London

WONG, K C, SOH, W K and BLAKE, J R (1989), High speed visualization of vapour cavity near boundaries, Tenth Australasian Fluid Mechanics Conference, Institution of Engineers (Aust.), Melbourne, December, 227 - 230.

NOMENCLATURE

d =	Depth of water
dt =	The time increment.
h =	Height of bubble above rigid floor
H =	Pressure function as defined in equation (3)
H^* =	Pressure function corresponds to $V = V_{max}$
m_L =	The mass of liquid condensed from the vapour in the bubble
P =	Pressure
P_{atm} =	Pressure on free surface
P_v =	Vapour pressure in a bubble
$[P_v]_{max}$	= Vapour pressure at maximum volume
$[P_v]_{min}$	= Vapour pressure at minimum volume
Q =	Rate of heat transfer to the vapour bubble
R =	Radius of bubble
t_c =	Measured period for bubble to collapse to its minimum
t_{calc} =	Computed half period of the bubble
t_g =	Measured time for bubble to grow to its maximum
T =	Temperature in the surrounding liquid
T_v =	Temperature in the bubble
u_g =	The specific internal energy of the vapour
u_L =	The specific internal energy of the surrounding liquid
V =	Volume of the bubble
Vmax =	Maximum volume of the bubble
Vmin =	Minimum volume of the bubble
ρ_v =	The density of the vapour in the bubble;

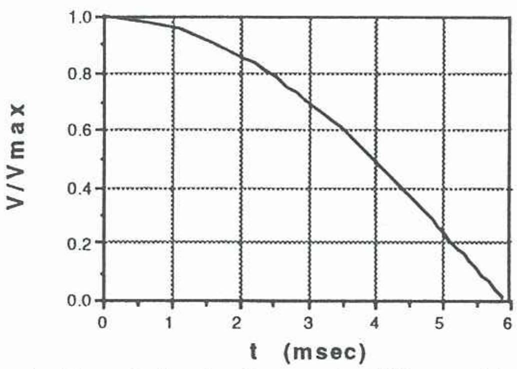

Figure 1 The calculated volume ratio V/V_{max} with respect to time.

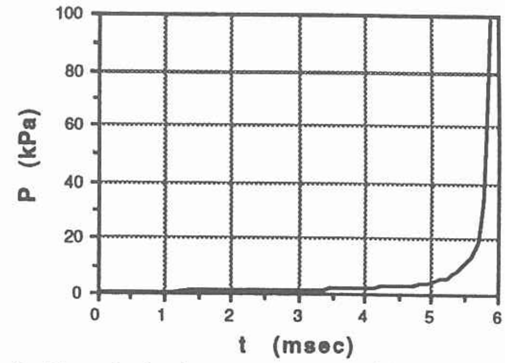

Figure 2 The calculated vapour pressure with respect to time .

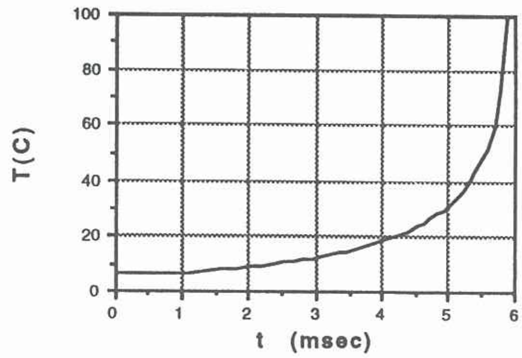

Figure 3 The calculated temperature in the bubble with respect to time.

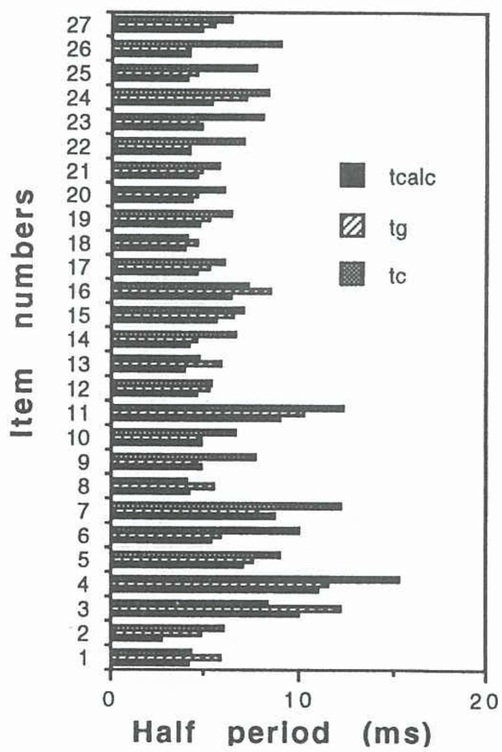

Figure 4 Comparison between calculated time (Tcalc) with the duration of growth of a bubble (Tg) and duration of collapse (Tc)