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COMPUTER STUDY ON THE REBOUND OF A VAPOUR CAVITY BUBBLE

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ABSTRACT

This paper investigates the dynamics of a vapour cavity bubble above a horizontal rigid surface. The collapse and rebound of the bubble is modelled by a boundary integral technique. Tabulated properties of the vapour is used to calculate the thermodynamics process inside the bubble. The results demonstrate the relationship between the initial conditions and the pulsations of the vapour bubble.

INTRODUCTION

Vapour cavity bubble is generated in the flow of liquid where the local pressure reaches below the vapour pressure. As the bubble migrates to high pressure region it collapses rapidly. This is the phenomenon of cavitation which occurs in many aspects of fluid flows in hydraulic machinery such as hydraulic valves, and pumps. Experimental investigations have shown that the collapse of a bubble in the vicinity of a rigid surface is accompanied by a high speed water jet which is formed from the far side of the bubble. This liquid jet treads through the bubble and impinges on the rigid surface (Gibson (1968), Soh & Yu (1991)). This continuous impingement of high momentum micro-jet is believed to be the main cause of cavitation damage on surfaces.

In the case of underwater explosion, rapid local boiling of water due to high energy input generates a large vapour bubble which has a displacement compatible to that of its target ship. The growth and collapse of this bubble impost adverse distribution of bending moment in the hull and thus cause structural damage to the ship. The water jet which follows the collapse of the bubble will generate high impact on the hull.

Most computational and analytical researches have been carried out with the assumption of either a constant vapour pressure inside of the bubble (Blake, Taib and Doherty (1986), Kucera & Blake (1988), Taib (1985)) or pressure change following simple gas laws (Kumar & Booker (1991)). These assumptions ignore the process of energy transfer in the vapour in which latent heat is an important factor.

A computer simulation of a cavitation vapour bubble is carried out by using the boundary integral equation technique to show the collapse and rebound phases of the bubble above a rigid surface.

The release and the absorption of latent heat in the bubble will effect the pressure and the temperature of the vapour, the dynamics of the bubble and therefore the hydrodynamics of the flow around the bubble. In this regard the thermodynamic process inside of the bubble can not be ignored (Soh & Shervani Tabar (1992)).

A computer model which incorporates empirical thermodynamics properties of the vapour is used to describe the thermodynamic process inside of the bubble. The simulation of condensation and vaporisation of water is brought about by a formulation which is derived from the First Law in Thermodynamics for control mass analysis (Soh & Shervani Tabar (1992)).

This paper focuses on the relationship between of the initial conditions and the dynamics of the vapour bubble. The initial conditions consist of the following parameters: maximum size of the bubble (R_m), vapour pressure in the bubble when it is at its maximum size (P_{vs}), the ambient temperature (T) and pressure (P_o) and the distance of the bubble from a rigid surface.

FORMULATION

The Energy Equation

The energy equation (First Law of classical thermodynamics for control mass) for vapour in a cavitation bubble is given by :

$$P_v\, dV + \rho_v V\, du_g(P_v) - dm_L[u_g(P_v) - u_L(T)] = Q\, dt \tag{1}$$

where V is the volume of the bubble; P_v is the vapour pressure, $u_g(P_v)$ is the specific internal energy of the vapour; $u_L(T)$ is the specific internal energy of the ambient liquid; m_L is the mass of the condensed liquid in the bubble and Q is the heat transfer into the bubble.

The equation for conservation of mass is:

$$dm_L + d(\rho_v V) = 0 \tag{2}$$

By assuming no heat transfer, the thermodynamic process in the bubble (from equations (1) and (2)) becomes:

$$P_v\, dV + d\{\rho_v V[\, u_g(P_v) - u_L(T)]\} = 0 \tag{3}$$

The Hydrodynamics Equation:

The surrounding liquid of the bubble is assumed to be a potential flow domain (Ω). If ϕ is considered as the velocity potential, the Green's integral formula (Wrobel & Brebbia (1980)) is given by:

$$C(p)\,\phi(p) + \int_S \phi(q)\,\frac{\partial}{\partial n}\left[\frac{1}{|p-q|}\right] dS = \int_S \frac{\partial\phi(q)}{\partial n}\,\frac{1}{|p-q|}\, dS \tag{4}$$

where S is assumed to be a piece-wise smooth surface of the bubble, p is the any given point in the domain Ω, q is a point on S and C(p) is 2π when p is on the surface S and 4π otherwise.

The unsteady Bernoulli's equation, in Lagrangian form, is given by:

$$\frac{D\phi}{Dt} = \frac{P_o - P_v}{\rho} + \frac{1}{2}\,|\nabla\phi|^2 + g\,z \tag{5}$$

where P_o is the ambient pressure and P_v is the vapour pressure in the bubble.

DISCRETIZED APPROXIMATION

The surface of the vapour bubble is approximated by N elements. In each of these element the potential and its normal derivative are constant (see Figure 1). The mid-points of the elements are designated as collocation points p_i and q_j are points on the surface S. Thus for p_i to lie on the surface of the bubble, the integrals in equation (4) are sectionalized to give the following expression:

$$2\pi\,\phi_i + \sum_{j=1}^{N} \phi_j \int_{S_j} \frac{\partial}{\partial n}\left[\frac{1}{|p_i - q_j|}\right] dS = \sum_{j=1}^{N} \frac{\partial}{\partial n}[\phi_j] \int_{S_j} \frac{1}{|p_i - q_j|}\, dS \tag{6}$$

Denoting the velocity component normal to the surface by ψ, equation (6) becomes a system of linear equations in the form:

$$2\pi\,\phi_i + \sum_{j=1}^{N} E_{ij}\,\phi_j = \sum_{j=1}^{N} G_{ij}\,\psi_j \tag{7}$$

Also by defining $H_{ij}=E_{ij}+2\pi\delta_{ij}$ ($\delta_{ij}=1$, if i=j & $\delta_{ij}=0.0$, if $i\neq j$) equation (7) is further simplified into a matrix expression:

$$[H]\,[\phi] = [G]\,[\psi] \tag{8}$$

From equation (8) the normal velocity on the bubble surface ψ can be calculated for every given distribution of velocity potential ϕ.

The tangential velocity, η, is approximated by the formula:

$$\eta = \frac{\partial\phi}{\partial s} = \frac{[\phi_{j-1} - \phi_j][d_j + d_{j+1}]^2 - [\phi_{j+1} - \phi_j][d_{j-1} + d_j]^2}{\frac{1}{2}[d_{j-1} + d_j][d_j + d_{j+1}][d_{j+1} - d_{j-1}]} \tag{9}$$

The term d_j is the length of element s_j . The discretized form of Bernoulli's equation for calculating the velocity potential, allows the velocity potential to be time-marched over a time increment of Δt:

$$[\phi_i]_{t+\Delta t} = [\phi_i]_t + \Delta t\left\{\frac{1}{2}[\psi_i^2 + \eta^2] + \frac{P_o - P_v}{\rho} + g z_i\right\} \tag{10}$$

The collocation points, p_i , is a vector in a cylindrical coordinates system (r_i, z_i) and are time-marched according to the Euler formula:

$$r_i(t + \Delta t) = r_i(t) + u_i \cdot\Delta t + O(\Delta t^2) \tag{11}$$

$$z_i(t + \Delta t) = z_i(t) + v_i \cdot\Delta t + O(\Delta t^2) \tag{12}$$

Here the velocity vectors (u_i, v_i) are derived from the velocity components which are normal to the bubble surface,ψ_i, and tangential to it,η_i.

The discretized form of equation (3) for calculating vapour pressure inside of the bubble is given by:

$$\frac{\Delta V}{V} = -\,\frac{\Delta\{\rho_v[u_g(P_v) - u_L(T)]\}}{P_v + \rho_v[u_g(P_v) - u_L(T)]} \tag{13}$$

Thus the change of volume, ΔV, will allow P_v to be evaluated from (13).

RESULTS AND DISCUSSION

The pulsation of a vapour cavity bubble near a horizontal rigid floor is carried out using the above computation technique. The aim here is to investigate the rebound of the bubble in relation to the penetrating water jet under various ambient temperature and pressure.

A sample of the calculations is shown in Figure 2. The calculation begins from a bubble at its maximum volume which is spherical in shape. The bubble is initially at a distance 1.5 times its maximum radius, R_m, and its initial vapour pressure, P_{VS}, is 3.75kPa. The ambient pressure (Po) and temperature are 5kPa and 30°C respectively. The computed results shows that the bubble under goes two cycles of collapse and rebound. The formation of liquid jet is a continuous process which is independent of the direction of growth of the bubble. It becomes very prominent at the end of the second pulsation. This gives more detail description on the evolution of the liquid jet in which many experiments have observed at the end of the second pulsation of the bubble.

The change in volume of the bubble is shown in Figure 3. Although the shape of the bubble continue to deform, the bubble has recovered its initial volume at the end of each rebound. This is expected in the framework of energy conservation. The increase of pressure in the bubble is synchronized with the reduction of volume as shown in Figure 4. It must be noted that in effect due to latent heat has resulted in a maximum pressure much higher than that predicted from the calculation which assumes polytropic gas law to be followed by the vapour.

Figure 5 illustrate the effect of varying the initial vaour pressure in the bubble. By reducing the initial vapour pressure, the times for the bubble to collapse lengthened while the sizes of the collapsed bubble increased. As shown in Figure 6, the increase in the fluid ambient temperature shorten the collapse times but has very small effect on the change of volume of the bubble. Qualitatively, the momentum exerted by the bubble on the rigid surface can be estimated from the rate of volume reduction during the collapse of a bubble. Rebound of the bubble is most likely to occur for a bubble which has a higher initial pressure. In other words, a bubble which over-expands (having a lower pressure at its maximum size) will deliver a stronger momentum onto the rigid surface.

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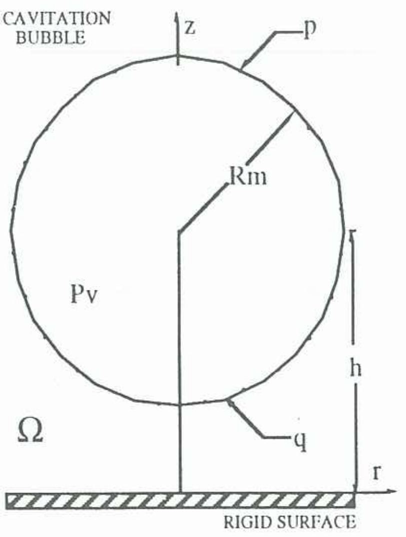


Figure 1 Schematic diagram of the computation domain

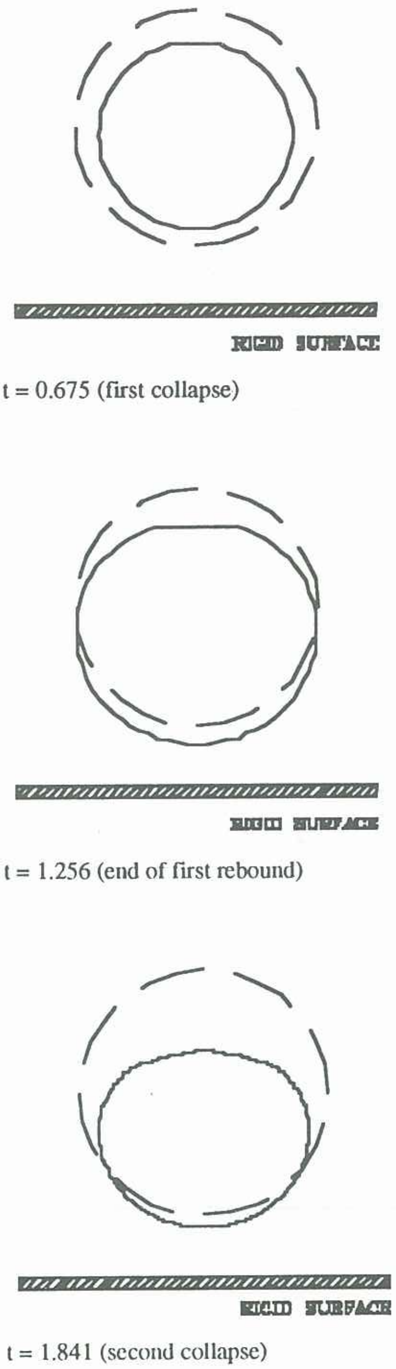
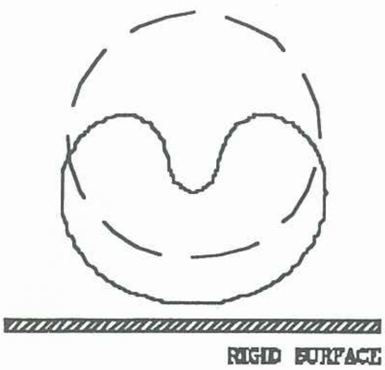
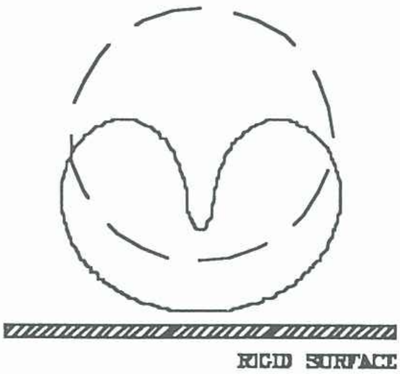


Figure 2 The pulsation of a vapour bubble. The broken line represents the position of the initial bubble.



t = 2.277 (growing state of second rebound)



t = 2.425 (end of second rebound showing well defined liquid jet)

Figure 2 (continued) The pulsation of a vapour bubble.

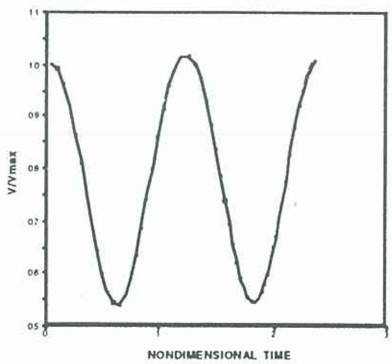


Figure 3 The change of bubble volume versus time; showing two pulsations.

$$\text{Nondimensional time} = \frac{\text{time}}{\text{Rm}}\sqrt{\frac{\text{Po-Pvs}}{\rho}}$$

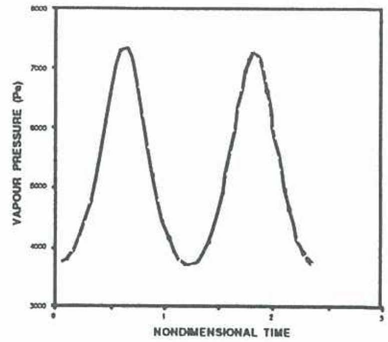


Figure 4 The variation of pressure in a bubble.

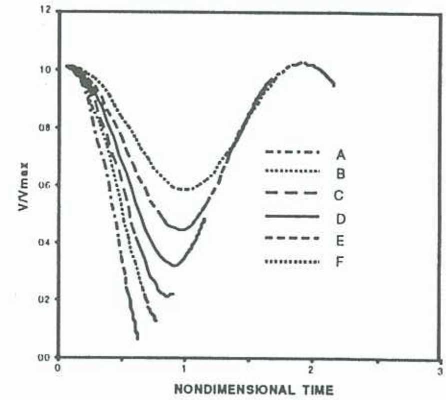


Figure 5 The pulsation of bubble as shown in volume changes versus time at an ambient condition of 25^{o}C, and 5kPa; the bubble is 1.5 times its maximum radius above a horizontal rigid surface. Initial vapour pressure: A=1kPa; B=2kPa; C=2.5kPa; D=3kPa; E=3.5kPa and F=4kPa.

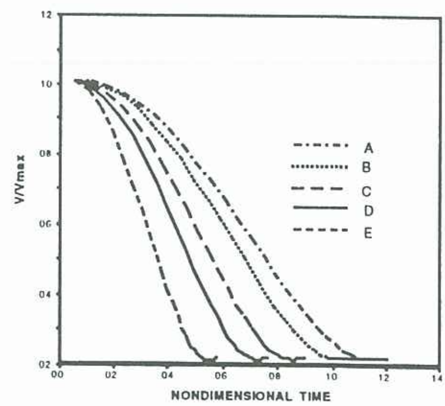


Figure 6 The pulsation of bubble as shown in volume change versus time at an ambient pressure of 5kPa and an initial vapour pressure of 2.5kPa and 1.5 times its maximum radius above a horizontal rigid surface. Ambient temperature: A=15^{o}C; B=20^{o}C; C=25^{o}C; D=27.5^{o}C and E=30^{o}C.