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AN IMPROVED MODEL FOR WAVE EXCITATION IN LIQUID BATHS WITH GAS INJECTION

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ABSTRACT

When gas is injected into a tank of water through a nozzle in the base, a rotating wave can be set up on the surface of the bath. At the last Australasian Fluid Mechanics Conference, a mechanism was proposed to explain this wave excitation. According to this mechanism, the buoyancy force on bubbles that are displaced from the centre-line as a result of the oscillation is sufficient to sustain the oscillation under certain conditions. Interaction between the plume motion and wave motion was analyzed theoretically to determine an equation for the evolution of any particular mode of oscillation. In the present paper, improvements are made to the model by using more detailed expressions for the damping and bubble velocity. This improves predictions of wave amplitude as a function of bath depth when compared with data of Whalley and Davidson. In addition, the mode evolution equation is integrated numerically to obtain a more accurate solution.

NOTATION

b	Tank width
c_1, c_2, c_3	Damping coefficients defined by eqn(3)
$\mathcal{D}$	Damping term
$\mathcal{F}$	Forcing term
g	Acceleration due to gravity
h	Bath depth
k	Wavenumber
l	Tank length
n	Mode number ($= 1, 2, 3, \ldots$)
q	Function defined by eqn(5)
Q	Gas flow rate
t	Time
T	Period of wave
V_B	Vertical bubble velocity
z	Vertical coordinate
ω	Angular frequency
ν	Kinematic viscosity
ϕ	Mode amplitude
τ	Bubble transit time through the bath

INTRODUCTION

Injection of gas into a bath of liquid can cause a wave or oscillation to be set up in the body of liquid. The exact shape and amplitude of the wave motion depends on several factors: the gas flow rate, the tank geometry, the injection geometry and the bath depth. For example, in an upright cylindrical tank with gas injection through a nozzle centrally placed in the bottom, a rotating wave can be set up if the gas rate is sufficiently high (Leroy and Cohen de Lara, 1958; Schwarz et al., 1988). In all cases, wave excitation only occurs for certain ranges of bath depth, so that the phenomenon appears to be some sort of resonance.

The phenomenon is of importance in chemical and metallurgical engineering processes in which gas is injected in liquids, usually to increase mixing or reaction rates. Excessive wave motion can have detrimental effects as, for example, in a bottom-blown steel converter where large amplitude slopping can result in melt loss and vessel vibration.

At the last Australasian Fluid Mechanics Conference, a mechanism was proposed to explain this wave excitation, (Schwarz, 1989). According to this mechanism, the buoyancy force on bubbles that are displaced from the centre-line as a result of the oscillation is sufficient to sustain the oscillation under certain conditions. Interaction between the plume motion and wave motion was analyzed theoretically to determine an equation for the evolution of any particular mode of oscillation, and hence an expression for the amplitude of the wave excited.

The objective of the present work is to extend the model with the aim of improving the agreement between predictions and the measurements of Whalley and Davidson (1972).

MODEL OF THE MECHANISM

The Wave–Plume Interaction

Firstly, the physical mechanism for the self-excited sloshing proposed at the last Conference (Schwarz, 1989) is reviewed. Consider a long rectangular tank with injection through a line of nozzles bisecting the tank: this reduces the problem to two dimensions. Above the nozzles, a plume containing a mixture of gas bubbles and liquid will rise to the surface. If the plume is displaced to one side of the centre-line, the surface of the bath is raised on that side. There is then a restoring force (gravity) which moves liquid from the right hand side of the bath to the left hand side. In the absence of the plume, the bath, if initially disturbed to such a position, would oscillate back and forth, the amplitude decreasing with time because of damping. The presence of the plume leads to a self-sustaining oscillation.

Consider the disturbance shown in Figure 1(a) as an initial condition. The evolution of this disturbance is such that the liquid moves from the right hand side to the left as shown in Figure 1(b) everywhere in the bath. This moves the plume to the left, and importantly, it moves any bubbles released at the nozzle to the left hand side of the bath. All the bubbles released while the liquid is moving to the left end up in a plume on the left of centre, and this displaced plume tends to reinforce the deformation (the rise in the surface level) on the left hand side resulting from the free oscillation. The strength of this reinforcement depends on the number of bubbles released since the bath started moving to the left. The more bubbles, the greater the buoyancy force tending to raise the liquid on the left side of the tank.

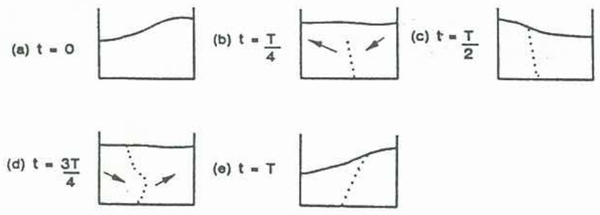

Figure 1: The evolution of an initial disturbance to the surface of a bath, and the effect of the resultant liquid velocity on the position of a bubble plume.

There is a second important effect to be considered. This effect is related to the length of plume released during one half cycle. If the plume reaches all the way to the surface at the end of the first half cycle, all those bubbles will leave the bath during the next half cycle, and a new line of bubbles reaching to the surface will be created. At the end of the second half cycle, each of these new bubbles will have been moving to the right all their life. Therefore the displacement of the plume to the right at the end of the second half cycle will be in some sense maximal.

On the other hand, if the line of bubbles created during the first half cycle only reaches half way to the surface, those bubbles are likely to remain in the bath during the next half cycle. Because they moved to the left during the first half cycle, their displacement to the right at the end of the second half cycle will not be maximal. In other words, the reinforcement of the wave by the buoyancy force will not be as great as it could be.

As a result, maximum reinforcement of the free oscillation is expected to occur when the transit time of bubbles through the bath is about one-half of the free oscillation period; for then a complete line of bubbles reaching all the way to the surface will be formed on the left side of the tank before the free oscillation moves the liquid back to the right-hand side.

Mathematical Model of the Mechanism

In the previous paper, the wave amplitude was described by a model involving a single degree of freedom:

$$\ddot{\phi} + \omega^2\phi = \mathcal{F} - \mathcal{D} \tag{1}$$

where ϕ is the instantaneous amplitude of the mode of the surface deformation being considered. The amplitude, ϕ, will be measured as the displacement of the free surface from equilibrium at an anti-node (i.e. at the vessel wall).

The two terms on the left hand side of eqn(1) represent the free oscillation terms. For the fundamental, the period, T, is given by (Lamb, 1932)

$$\omega^2 = \left(\frac{2\pi}{T}\right)^2 = \frac{\pi g}{l}\tanh\left(\frac{\pi h}{l}\right) \tag{2}$$

for a rectangular bath of length l.

The damping term, $\mathcal{D}$, was written as

$$\mathcal{D} = c_1\dot{\phi} + c_2\dot{\phi}|\dot{\phi}| + c_3\dot{\phi}^3 + \ldots \tag{3}$$

where the first two terms are likely to be sufficient if the amplitude is small. Wall friction is the main source of damping at low amplitudes, with other sources such as turbulence in the body of the bath and splashing becoming significant at high amplitudes.

The first term on the right hand side of eqn(1) represents the forcing due to the bubble displacement. The buoyancy force on bubbles displaced from a nodal position will allow energy to be pumped into the wave. A derivation of the forcing term was given in more detail in the previous paper (Schwarz,1990).

For the purposes of deriving a mode evolution equation, eqn(1), assume that the effect of the gas injection on the flow field is merely to increase (or decrease) the amplitude of the fundamental, and ignore any other influence of the injection on the flow. That is, the vertical plume motion and the fundamental oscillation are separate but coupled through eqn(1).

Assume that, Apart from the diffusion of bubbles within the plume, bubble lateral motion is a perfect response to the liquid sloshing motion. The vertical velocity of the bubbles, V_B, is taken to be independent of depth in the bath and includes a slip component relative to the liquid motion.

By considering the energy that is transferred to the wave, it is possible to show that the forcing term can be written

$$\mathcal{F} = q(Q,h,l)\int_{t-T}^{t} \sinh kV_B(t-t_o)\int_{t_o}^{t} \dot{\phi}(t')\cosh[kz(t')]\,dt'\,dt_o \tag{4}$$

where

$$q = \frac{2k^3Qg}{\pi b\cosh kh\sinh kh} \tag{5}$$

An analytic solution for wave amplitude can be found from eqn(4) in the usual way by assuming that ϕ is of the form

$$\phi = \phi_o\cos\omega t \tag{6}$$

An approximate expression for the amplitude is then

$$\phi_o = \frac{3\pi}{8c_2\omega}\left[\frac{q(\sinh 2kh - 2kh)}{4(\omega^2 + k^2V_B^2)} + \frac{q\omega(kV_B\cosh kh\sin\omega\tau - \omega\sinh kh\cos\omega\tau)}{(\omega^2 + k^2V_B^2)^2} - c_1\right] \tag{7}$$

whenever the expression is greater than zero, and zero otherwise. [The expression given in Schwarz (1989) neglected the first term in eqn(7) which is small for small h.] There is an apparent resonance (waves are most easily excited) when

$$T/2 \approx h/V_B. \tag{8}$$

The wall friction terms can be evaluated in a similar way to the forcing term, after matching the potential flow solution to a boundary layer solution. This gives estimates of c_1 (Schwarz, 1990):

$$c_1^b = \frac{k\sqrt{\omega\nu}}{2\cosh kh\sinh kh} \tag{9}$$

$$c_1^{sw} = \frac{\sqrt{\omega\nu}}{b} \tag{10}$$

and

$$c_1^{ew} = \frac{k\sqrt{\omega\nu}}{\pi}\left(1 - \frac{2kh}{\sinh 2kh}\right) \tag{11}$$

where c_1^b, c_1^{sw}, and c_1^{ew} are the contributions from the tank bottom, side walls, and end walls respectively. However, these expressions are only indicative of trends, since Case and Parkinson (1957) have shown that, even in the absence of gas injection, they do not predict the correct damping times if the wall surfaces are not highly polished.

In practice then, the damping coefficients should be determined empirically, for example by measuring the rate of decay of an oscillation.

COMPARISON WITH EXPERIMENT

Whalley and Davidson (1972) measured the amplitude of waves excited by gas injection in a rectangular tank, 10 cm wide and 40 cm long. Air was blown through five holes distributed along a line across the tank 20 cm from either end (i.e. halfway). In a second experiment, injection was through two lines of nozzles 10 cm from each end, thereby exciting the $n = 2$ mode. In a third experiment, the $n = 3$ mode was excited by three lines of nozzles at distances of 6.7 cm, 20 cm and 33.3 cm from one end. In each experiment there were five nozzles in each line, and the flowrate through each set of five nozzles was 700 cm^3/s. In each experiment the bath depth was varied and the wave amplitude measured as a function of depth.

Schwarz (1989) compared the amplitudes predicted by the analytic formula, eqn(7), with the measured values for each of the modes. The damping coefficients were fitted to obtain the correct maximum values. The agreement was in general good. However, the depth at which the amplitude maximum occurred was underestimated by about 10% for the n=2 mode and about 20% for the n=3 mode.

In the present study, the damping coefficients are allowed to be functions of depth. The coefficient, c_1, is taken to be proportional to the theoretical value, $c_1^b + c_1^{sw} + c_1^{cw}$. Surprisingly, over the range of depths of interest, c_1 actually decreases with decrease in depth. This is because the side wall friction component dominates. Only at smaller depths does c_1 increase with decrease in depth as the result of increasing bottom friction. The coefficient, c_2, is taken to be proportional to c_1. The proportionality constants are chosen so as to fit the maximum amplitudes.

The gas velocity is taken to be a function of bath depth:

$$V_B = V_\infty + V_o(h_o/h)^{0.2} \quad h \geq 0.1$$

$$V_B = 0.54 \qquad h < 0.1 \qquad (12)$$

with $V_\infty = 0.21$ m/s, $V_o = 0.31$ m/s and $h_o = 0.15$ m. There is much evidence that bubble velocity in a plume decreases with height (Castillejos and Brimacombe, 1987), and this would cause the average velocity, V_B, to decrease with increasing bath depth as in eqn(12).

Comparison of predicted and measured wave amplitude is given in Figure 2. The second and third modes are better predicted than previously, and the reason is essentially the decrease of V_B with depth.

The wave amplitude has also been obtained by numerically integrating eqns (1) and (4). This not only gives a more accurate result for finite amplitude waves, but also allows the use of an expression for bubble velocity which depends on height in the bath. As an example, consider

$$V_B = V_1/(z + z_1)^{1/3} \qquad (13)$$

with $V_1 = 0.19$ m/s and $z_1 = 0.21$ m. This is based on experimental and theoretical work which suggest an inverse 1/3 power (e.g. see Asai(1983)). The same expression is used for all bath depths, h. The agreement with the data of Whalley and Davidson (1972) is not significantly better than for the analytic theory (Figure 2), but eqn(13) has not been optimized to fit the data, and better fits can be obtained by modifying eqn(13). Nonetheless, the agreement obtained using eqn(13) is quite good, as shown in Figure 3 for the fundamental. The problem of determining the best $V_B(z)$ to fit the data is an inverse problem that could be tackled, but a better approach would be to

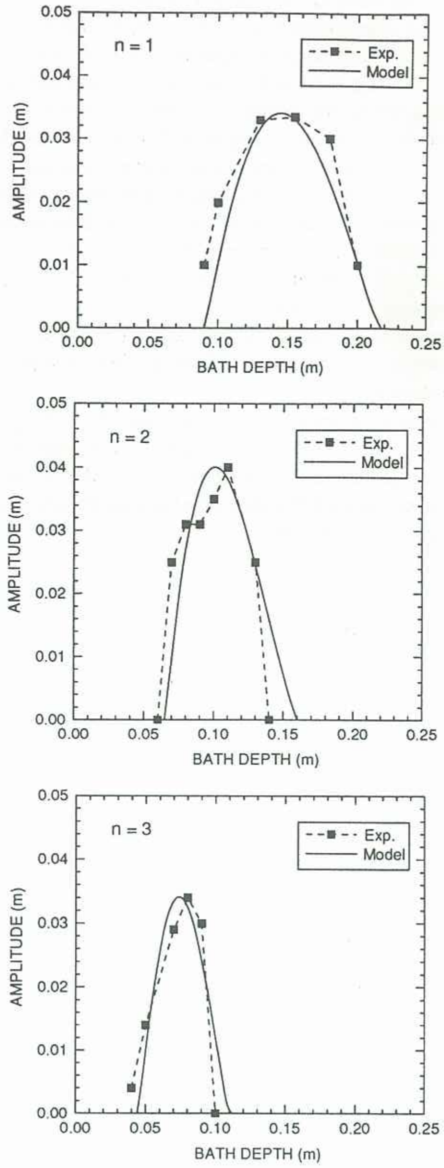

Figure 2: Predicted sloshing amplitude for the three lowest modes, compared with data of Whalley and Davidson (1972).

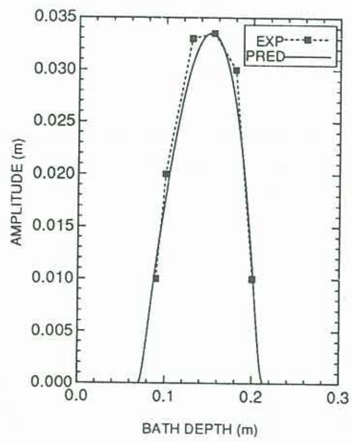

Figure 3: Sloshing amplitude computed by numerical integration of the evolution equation for the fundamental mode, compared with data of Whalley and Davidson (1972).

measure both wave amplitude and bubble velocity for the same experiment. The model could then be more rigorously tested.

Part of the numerical integration in eqn(4) is really equivalent to tracking the bubble motion. Insight can be gained by plotting the plume position determined in this way at a sequence of different times. Figure 4 shows four snapshots over one wave cycle showing the plume centre-line position and the bath surface. In this case, the experiment with a single row of nozzles is modelled, and $h = 0.15$ m. It should be observed that the plume motion appears to lead the wave. For example, when the surface is flat, the plume has already moved significantly to the rising side of the bath.

Figure 5 shows an equivalent plot for the experiment with three rows of nozzles, and $h = 0.08$ m. Only one phase of the cycle is shown.

A general principle following from the theory outlined above is that for maximum excitation of a particular standing wave, injection should be beneath the nodes of the wave. It is clear from Figure 5 that injection beneath an anti-node would not assist wave excitation.

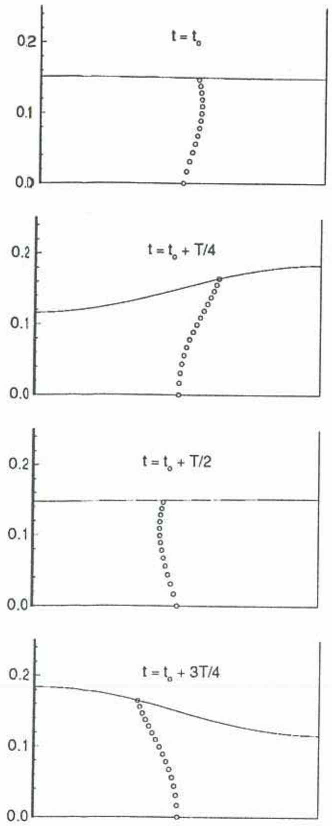

Figure 4: Four snapshots of the computed plume and surface shape over one wave period, for the n=1 experiment of Whalley and Davidson (1972).

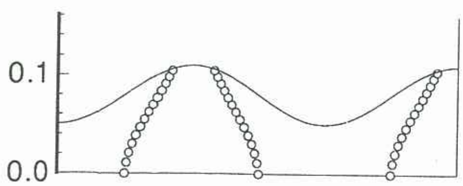

Figure 5: A single snapshot of the computed plume centre-line position and surface shape for the n=3 experiment of Whalley and Davidson (1972).

CONCLUSIONS

An improved model has been developed for predicting wave motion excited by gas injection into liquid baths. The model is based on a mechanism previously proposed to explain the formation of the waves. According to this mechanism, the buoyancy force on bubbles that are displaced from the centre-line as a result of the oscillation is sufficient to sustain the oscillation under certain conditions. This leads to a resonance-like phenomenon near

$$\tau = h/V_B = T/2 \tag{14}$$

where τ is the bubble travel time, and T is the wave period.

In the improved model, bubble velocity is taken to be a function of height in the bath and the equation for wave evolution is integrated numerically. This should be more accurate than the analytic solution (based on constant bubble velocity) given previously.

The improved theory gives a better prediction of the depths at which self-excited oscillation occurs, when comparison is made with data published in the literature. However the shape of the resonance depends strongly on the exact variation of bubble velocity with height, information that is not available for the experiment modelled.

The numerical approach also allows the phase relationship between the plume and wave to be investigated, and so provides insights into the excitation mechanism.

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