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MEASUREMENT OF CONCENTRATION STATISTICS DOWNSTREAM OF A LINE SOURCE IN GRID TURBULENCE

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ABSTRACT

Measurements of the temperature statistics downstream of a line source in grid turbulence are described. Data presented and discussed include the mean, variance, skewness and kurtosis of the temperature field.

INTRODUCTION

The measurement of concentration (actually temperature) statistics downstream of a line source (a heated wire) in grid turbulence is certainly not a new endeavour. Uberoi and Corrsin (1953) and Townsend (1954) made such measurements mainly in the far-field (several to many mesh lengths downstream) and they, and more recent workers (Stapountzis et al. (1986) and Warhaft (1984)), concentrated mainly on the mean field and the variance field.

There is presently increased interest in higher order statistics of the concentration field (variance, skewness and kurtosis, and the full probability density function) accompanying the need to treat problems such as transient odours, hazards due to flammable vapours, and chemical reactions in dispersing plumes, in both geophysical and engineering flows. There has been a similar interest in developing models describing and predicting these statistics of scalar concentration fields. Some of these models such as the Lagrangian two-particle models (Thomson, 1990) are complex, are in the early stages of development and are applicable at present to only the simplest turbulent flows. Other models are simpler and involve semi-empirical parameterisation of higher moments in terms of the mean field (e.g. Chatwin and Sullivan (1990), Kerstein (1991)). However, in both cases there is a need for more detailed measurements of concentration statistics with which to test the models.

This paper describes a new set of such measurements. In it we present data for the mean, variance, skewness and kurtosis for the temperature field downstream of a heated wire in grid turbulence. We discuss nondimensionalisation of the data and place some emphasis on the near-field where source effects are important.

EXPERIMENTAL DETAILS

The measurements were conducted in a suction wind tunnel with a rectangular test section 0.69m high, 1.07m wide and 3.3m long. The turbulence was generated with a planar 'punched plate' grid with circular holes of diameter 0.0208m in a hexagonal pattern, and 'mesh' length (i.e. hole spacing) $M = 2.54\times10^{-2}$m, giving a solidity ratio of 0.39. Temperature fluctuations were produced by a heated Nichrome wire of diameter $d_w = 0.213$mm placed a distance $x_0 = 12.2M$ downstream of the grid and spanwise across the tunnel. The experiments were carried out with a mean air speed $U = 5.0$m/s. The Reynolds number, $Re_M = UM/\nu$, was 8500 where ν is the kinematic viscosity of unheated air ($1.5\times10^{-5}m^2s^{-1}$ at 20°C). The power supplied to the wire was 200W/m, sufficient to eliminate vortex shedding.

Velocity fluctuations were measured with a crossed hot-wire probe with tungsten wires of diameter 5μm and length 1.25mm powered by a TSI-1053-A constant temperature anemometer circuit. Temperature fluctuations were measured with a 50Ω platinum cold wire 1.27μm in diameter and 0.4 mm long using an in-house designed temperature bridge.

Cross-stream traverses of the plume were carried out using screw-driven probe carriage. The cross-stream location of the probe was determined in relative terms to 0.01mm. Traverses were carried out at approximately logarithmically-spaced locations from 2mm to 3m downstream of the source.

The temperature signal was low-pass filtered at 2kHz and sampled at 4096Hz. Statistics were calculated from 20 separate 1s samples; i.e. from a total of 81920 points. At the beginning and end of each set of 20 samples, 'background' samples were taken with the source wire switched off and after approximately 30s stabilisation time. The trend in these background readings was used as a first estimate of the baseline relative to which the mean temperature is calculated. However, it was found that the true baseline temperature (estimated by inspection of the time series) is higher (probably due to the effect of heating of the probe stubs and supports) and a correction was made for this effect.

Properties of the turbulence are not reported here, but the usual checks on homogeneity and measurements of the decay rate downstream were made.

DEFINITIONS

We choose a coordinate system with the source wire as origin, and denote the streamwise coordinate by x and the coordinate transverse to the plume by z. Because we deal with a uniform line source, there is no variation in single-point statistics of the concentration in the y-direction.

We denote the scalar concentration (temperature) by $C(x,z,t)$; it has the mean value, $\overline{C}(x,z)$ and a fluctuating component $c(x,z,t) = C(x,z,t) - \overline{C}(x,z)$. Here we are concerned with the moments, $\overline{c^n}(x,z)$, which define the variance (n=2), third central moment (n=3) and fourth central moment (n=4). Experimentally, the overbar represents a time average as discussed in the previous Section, but theoretically is often equated to an ensemble average.

In addition to these central moments we also consider the following normalised quantities:

$$i(x,z) = \sigma_c(x,z)/\overline{C}(x,z) \qquad (1)$$

where $\sigma_c = \left(\overline{c^2}\right)^{1/2}$ is the standard deviation and i(x,z) is known as the intensity of the concentration fluctuations;

$$S(x,z) = \overline{c^3}(x,z)/\sigma_c^3(x,z) \qquad (2)$$

which is the skewness, and

$$K(x,z) = \overline{c^4}(x,z)/\sigma_c^4(x,z) \qquad (3)$$

which is the kurtosis.

We will also deal with the absolute moments, $\overline{C^n}(x,z)$.

MEASUREMENTS

Stream-wise Variation

Although the main purpose of this paper is to present data on higher moments of the concentration field, we first compare our results for the second moment with previous data. Figure 1 shows the intensity of fluctuations on the plume centreline, as a function of distance downwind. The nondimensionalisation is designed to collapse the data (at least well downstream from the source) and to identify any approach to self similarity in the scalar field. The data collapse reasonably well, although Warhaft's (1984) data generally are higher than the other data sets.

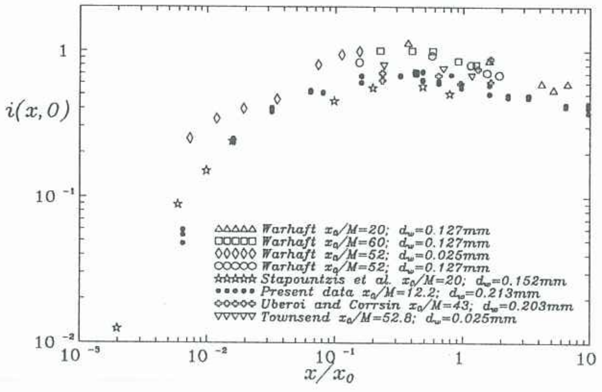

Fig. 1 Variation of the centreline value of the intensity of concentration fluctuations with distance downstream. Data referenced are from Warhaft (1984), Stapountzis et al. (1986), Uberoi and Corrsin (1953) and Townsend (1954).

Near the source there are fewer data but apparently larger differences between different data sets. In general, source (size and other) effects are expected to cause differences in the fluctuation intensity in the near field (see e.g. Sawford, 1983). However, for all of the near field data the source diameters were similar and were comparable with or smaller than the Kolmogorov microscale (typically ≅ 0.2-0.5mm for these flows) so that molecular effects ought to reduce any source size effects (Sawford and Hunt, 1986). The differences between different data sets in Figure 1 is therefore surprising and may be due to experimental difficulties such as errors in probe positioning or to vortex shedding.

In the far field the intensity is approximately constant, but there is a slight decrease with distance downstream indicating that the scalar field is at best only approximately self-similar there. This is not surprising since the velocity field is only self-similar on large scales, and the Reynolds number of the turbulence decreases slowly with distance downstream (Townsend, 1976).

Figure 2 shows the centreline value of the skewness and the kurtosis of the concentration as functions of distance downstream. Both vary strongly, particularly in the near-field. There the skewness and the excess, K-3, which represent the departure from a normal distribution, change sign. In the far field, the skewness decreases significantly with distance downstream, but the slight decrease in the kurtosis is probably not significant. For self-similarity, all the normalised moments (i.e. the intensity, skewness, kurtosis and higher moments) should be constant. This is clearly not the case for these data.

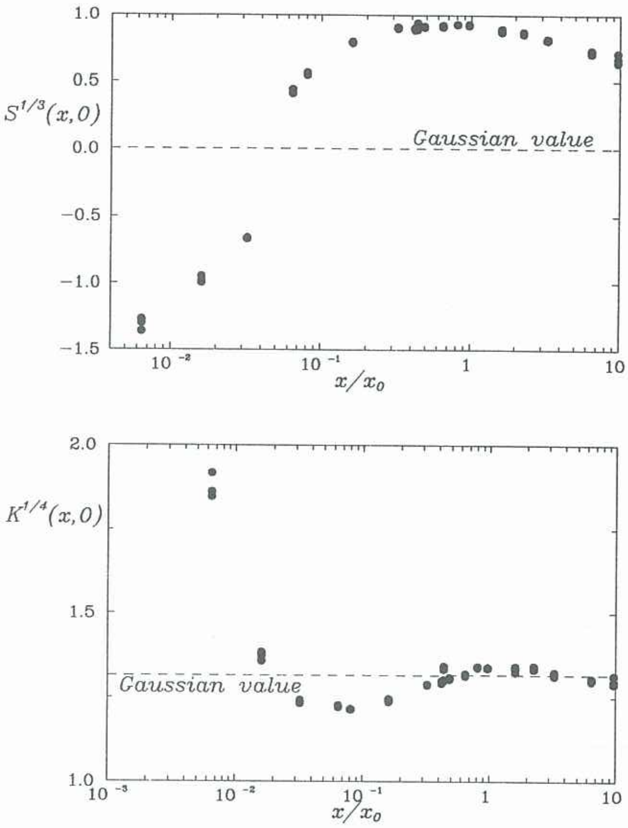

Fig. 2. Dependence of the centreline value of the skewness, S(x,0), and kurtosis, K(x,0), of the concentration on distance downstream.

Cross-stream Variation

It is usual to consider the central moments $\overline{c^n}$ which for localised sources in a variety of flows are known to exhibit complex structure in the cross-stream direction (Chatwin and Sullivan, 1990). This is certainly the case for the present data. Figure 3 shows cross-stream profiles of the variance and third central moment in the near field ($x/x_0 = 0.032$). Both have significant off-axis maxima, and the third moment is negative on the centreline. The mean profile is Gaussian and so is not shown, and the fourth moment is a similar shape to the second. Although we do not show the profiles here, for $x/x_0 \cong 0.5$ the off-axis maxima disappear from the second moment (which is still however not Gaussian in shape) but persists in the third and fourth moments. In the far field, the off-axis peaks re-appear in the second moment and all three higher moments for which we have data are similar to the variance in Figure 3. These features of the shape of concentration moments and their variation are consistent with the semi-empirical theory of Chatwin and Sullivan (1990). Detailed comparisons with their theory are in progress.

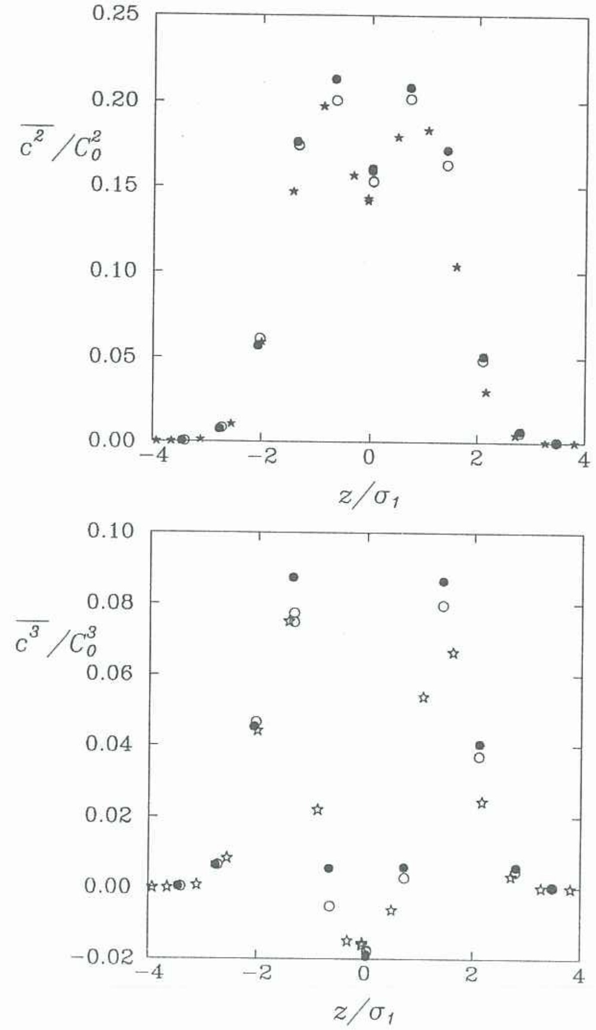

Fig. 3. Cross-stream profiles of the second and third central moments of the concentration field. The data are normalised using the centreline value of the mean concentration, C_0, and the width of the mean concentration profile, σ_1. The different symbols correspond to repeat experiments.

An alternative, and in some ways more natural, way to present these data is in terms of the absolute moments $\overline{C^n}$ which describe fluctuations relative to zero concentration. These are predicted directly by Lagrangian theory (Thomson, 1990) and are also most appropriate in describing the conservation properties of scalar fields in the absence of molecular diffusion (Chatwin and Sullivan, 1979).

The interesting feature of our data is that the absolute moments have a much simpler spatial structure than the corresponding central moments. In particular we find that cross-stream profiles for all the moments we have measured (i.e. up to the fourth) are Gaussian at all locations downstream of the source. In order to emphasise this point, and the precision with which it holds, we show in Figure 4 the profile of the third absolute moment at $x/x_0 = 0.032$; the contrast with Figure 3 is remarkable, and it is clear that the Gaussian shape is a very precise representation of the data.

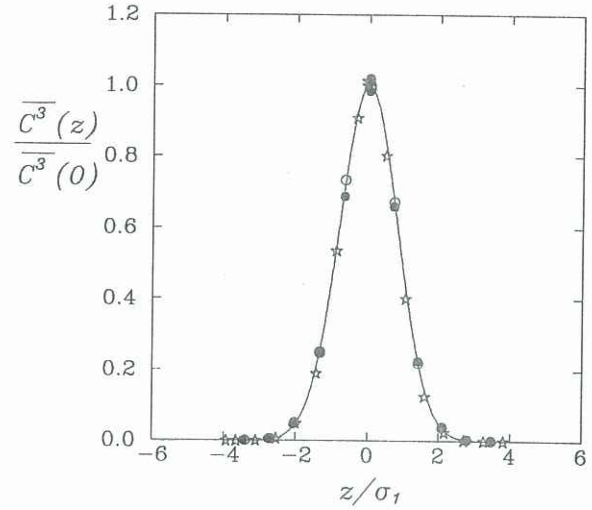

Fig. 4. Cross-stream profile of the third absolute moment of the concentration field corresponding to the data in Figure 3. The ordinate is normalised by the centre-line value.

Of course, the representations of the data in terms of central and absolute moments are entirely equivalent, and it is straight forward to transform from one to the other. The complex structure of the central moments arises from the much simpler structure of the absolute moments for two reasons. Firstly the binomial relationship between them is non-linear and nonhomogeneous

$$\overline{c^n} = \overline{(C-\overline{C})^n} = \sum_{j=0}^{n} (-1)^{n-j}\binom{n}{j}\overline{C^{n-j}}\;\overline{C}^{j} \qquad (4)$$

Secondly, the 'width' of the Gaussian profiles (defined here as the half-width at a height of $e^{-1/2}$ times the centreline value) for the absolute moments decreases systematically with the order of the moment; i.e. higher order moments have a narrower distribution across the plume than lower order moments.

Sawford (1991) has shown that the Gaussian form for profiles of the absolute moments is predicted by Lagrangian theory and furthermore that the dependence of the width of the profile on the order of the moment is predicted to within experimental uncertainty.

CONCLUSIONS

We have reported some new measurements of statistics of the concentration field downstream of a line source in grid turbulence. Although there are some differences between data sets, our measurements of the intensity of concentration fluctuations are consistent with previous measurements. The novel aspect of the present work is the detailed measurement of the downstream and cross-stream variation of third and fourth moments of the concentration. When displayed in the normal way as central moments, our data show a complex spatial dependence featuring changes of sign and off-axis maxima. However, when displayed as absolute moments, the spatial structure of our data is much simpler - all moments have a Gaussian cross-stream profile. There is no evidence of an approach to self-similar form far downstream of the source.

These measurements provide a data base suitable for testing recent theories of concentration statistics in plumes. Comparisons against the semi-empirical theory of Chatwin and Sullivan (1990), simple Lagrangian ideas (Sawford, 1991) and Lagrangian stochastic theory are proposed or under way.

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