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MULTI-GRID SOLUTION OF THE THREE-DIMENSIONAL THERMOCAPILLARY CONVECTION IN A CAVITY

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ABSTRACT

The thermocapillary convection of an incompressible Newtonian fluid in an open cubic container in the absence of gravitational forces is considered. Two opposite side walls of the container are differently heated and the other walls and the bottom are thermally insulated. Due to the temperature gradient the surface tension changes, which leads to a tangential flow from the hot to the cold side. The three-dimensional steady motion is computed with a FMG-FAS Multi-Grid method. Solutions for Reynolds numbers up to $5 \cdot 10^4$ are solved by a line coupled, block implicit relaxation method for the system of Navier-Stokes and continuity equation, and an alternating zebra-relaxa-tion method for the energy equation. The results of the three-dimensional model are compared with earlier solu-tions of a two-dimensional simulation of the thermo-capillary convection in a square cavity, Saß et al. (1992). The present Multi-Grid method is based on a finite difference-, primitive variable formulation with a staggered grid of $32 \times 32 \times 32$ points for each unknown.

NOTATION

$U = (u,v,w)^T$	velocity vector
p	pressure
T	temperature
T_0	temperature difference between the heated walls
d	distance between the heated walls
σ	surface tension
σ_0	average value for the surface tension
$\gamma = d\sigma/dT$	surface tension coefficient
μ	dynamic viscosity
ν	kinematic viscosity
κ	thermal diffusivity
$Pr = \nu/\kappa$	Prandtl number
$Ca = \gamma T_0/\sigma_0$	Capillary number
$Re = d\gamma T_0/\mu\nu$	Reynolds Zahl
$Ma = PrRe$	Prandtl Zahl

INTRODUCTION

Together with buoyant convection thermocapillary convection is an important natural transport mechanism. Thermocapillary convection occurs in all liquids with a free surface (liquid-liquid or liquid-gase), if the interfacial tension varies along the surface.

On the interface fluid particles move from the area of low surface tension to the area of high surface tension. Since the surface tension is generally a function of temperature, the particles move on the interface as a consequence of the temperature gradient. For most liquids $d\sigma/dT$ is negativ and the surface flow is from hot to cold. Due to viscosity and continuity fluid motion also occurs in the bulk.

Under the conditions of earth's gravity thermocapillary convection is superposed by buoyant convection. Only if the temperature gradient is high and if the aspect ratio is a large enough (length of free surface / depth) thermocapillary convection is predominant. Examples are the melt of a weld bath and the Czochralski crystal-growth configuration. In order to examine thermocapillary convection without the influence of buoyant convection, this study presumes conditions of weightlessness.

Thermocapillary flows in rectangular cavities have received much attention during the past years. Graziani et al. (1982) developed a two-dimensional stationary code for small Reynolds numbers (Re<200). It was improved by Strani et al. (1983), who could simulate, flows up to Re=3000. Both methods used a stream function formulation. Zebib et al. (1985) introduced a stationary two-dimensional finite difference method in primitiv variable formulation. As in most of the models the free surface is assumed not to be deformed. A subsequent evalutation confirms that this assumption is justified. The method is convergent for Reynold numbers up to 10000 with Pr=1. In a second part, Carpenter and Homsy (1990) reached Rey-

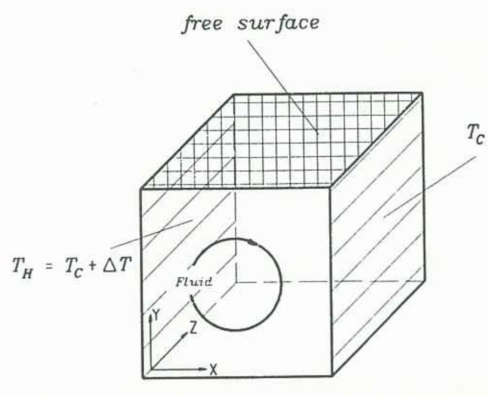

Fig. 1. Cubic container with two heated side walls

nolds numbers up to $1.9 \cdot 10^5$ with a multi-grid method and a stream-function formulation. In addition to this, the dependence of the convection on the Prandtl number was examined and an analogy to the driven cavity flow was found. The only three-dimensional simulation known to the authors was developed by Babu and Korpela (1990). This Calculations were carried out only for small Reynolds numbers. A weak three-dimensional flow characteristic was shown, i.e. the flow is nearly independent from z (fig.1). In this study a strong three-dimensional flow at high Reynolds numbers is observed.

ASSUMPTIONS

- Boussinesq liquid
- The surface tension is a linear function of the temperature
- the capillary number Ca is very small, so that the interface can be assumed not deformed
- the non-heated walls and the surface are insulated
- the heat flux from the fluid into the gas is negligible
- only stationary processes are examined

PHYSICAL MODEL

We consider an open-top cubic container shown in fig.1 with an edge length ***d*** containing an incompressible Newtonian liquid. The side areas of the container are heated isothermaly to different temperatures $\pm T_0/2$. The system is governed by the Navier-Stokes equation (1), the continuity equation (2) and the energy equation (3) inside the region $\Omega=(0,1)\times(0,1)\times(0,1)$

$$Re(U\cdot\nabla)U = \nabla p + \Delta U, \qquad (1)$$

$$\nabla\cdot U = 0, \qquad (2)$$

$$Ma(U\cdot\nabla)T = \Delta T, \qquad (3)$$

The length, velocity, temperature and pressure are dimensionless with respect to d, $\gamma T_0/\mu$, T_0, $\gamma T_0/d$. The Reynolds and Marangoni numbers are defined as

$$Re = \frac{d\gamma T_0}{\mu\nu},$$

$$Ma = Pr\, Re,$$

Boundary conditions for the velocity

Non-slip boundary conditions are imposed at the solid walls,

$$u = v = w = 0, \quad x = 0, 1, \qquad (4)$$
$$u = v = w = 0, \quad y = 0,$$
$$u = v = w = 0, \quad z = 0, 1.$$

For small capillary numbers Ca the surface deformation can be neglected, see Zebib et al. (1985),
thus

$$v = 0, \quad y = 1 \qquad (5)$$

The balance of forces on the free surface shows that the shear stresses are balanced by thermocapillary forces

$$\frac{\partial u}{\partial y} = -\frac{\partial T}{\partial x}, \quad y = 1, \qquad (6)$$

$$\frac{\partial w}{\partial y} = -\frac{\partial T}{\partial z}, \quad y = 1, \qquad (7)$$

Boundary conditions for the temperature

The side walls of the container are heated isothermally, all other walls are adiabatic and the heat flux between the fluid and the gas on the surface is negligible small,

$$T = 1, \quad x = 0, \qquad (8)$$

$$T = 0, \quad x = 1, \qquad (9)$$

$$\frac{\partial T}{\partial y} = 0, \quad y = 0, 1, \qquad (10)$$

$$\frac{\partial T}{\partial z} = 0, \quad z = 0, 1, \qquad (11)$$

NUMERICAL METHOD

A Multi-Grid method was used for an efficient iterative solver of the nonlinear system of equations (1)-(3). Good convergence properties and a stable algo-rithm were achieved with a simplified Full-Multi-Grid (FMG-FAS) scheme, see Brandt (1984). Equations (1)-(3) are numerical separated in two parts, the Navier-Stokes- and the continuity equation (NAS-C) and the energy equation (E). In each iteration step first the solution of the NAS-C system and second (E) is solved.

For the computation of the first part, a block implicit relaxation method (SCGS) is used. To get good relaxation properties the blocks are coupled line-wise. Best results have been obtained by an alternating zebra-relaxation method. The iteration for (E) is solved with the same relaxation method.

For stable solutions at high Reynolds numbers a special treatment of the convection terms is necessary. Instead of the hybrid scheme with first order accuracy, which is often used, a second order uncentered discretization was developed. Accurate solutions are obtained for Reynolds numbers up to $5 \cdot 10^4$ (Pr=1) on a staggered grid with $32\times32\times32$ points for each unknown.

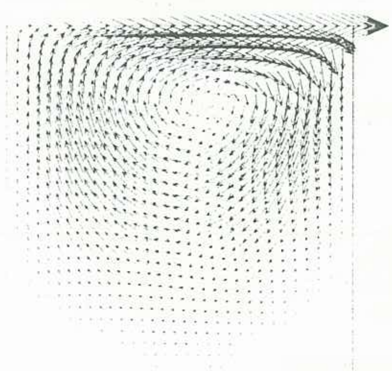

Fig.2. Velocity-field, x-y plane, Re=100, z=0.5

RESULTS

The numerical results of the three-dimensional simulation shown below have been calculated for Pr=1. A detailed analysis for Pr=1/100, 1, 10, 50 is given by Saß (1992). In order to show the three-dimensional-character of the flow field and the temperature field the results are discussed in the three projection planes.

<u>x-y plane</u>

In this projection plane the temperature gradient along the free surface causes a flow from hot to cold. As examples for the convection with low and high Reynolds numbers, the results for Re=100 and Re=50000 are explained.

Re=100: The temperature field is mainly determined by diffusive terms, so that the temperature function along the free surface in x-direction has only a small difference from the linearity. Therefore the thermocapillary convection on the free surface and also the flow fields inside are nearly symmetrical, see fig.2. The result is nearly two-dimensional, i.e. independent of z, see fig.5. Only in the boundary areas decreasing velocities are found due to the non-slip condition at the walls.

Re=50000: A large temperature gradient in the upper right and left corner and a nearly constant temperature in middle characterize the temperature field on the free surface, see fig.3 and fig.4. The temperature distribution inside the container is mainly determined by convection. In the flow field vortices have formed at the bottom left and right corners. The centre of the main vortex strongly varies in z-direction. The three-dimensional flow is shown in fig.5, where each function for Re=100,1000,10000,50000 describes the course of the main-vortex-centre from z=0 to z=0.5 in the direction of the arrow. As the Reynolds number increases the centre is displaced more and more. This show the increasing three-dimensional character of the flow with growing Reynolds number.

<u>x-z plane</u>

This projection plane is split into two symmetrical halves along z=0.5, so that the discussion of the results can be restricted one side of the container.

Re=100: The temperature field on the free surface shows slightly curved isotherms, see fig. 6. The temperature flux is largest for z=0.5 and decreases towards the boundary. This temperature behaviour is caused by the parabolic distribution of the velocity. Since the velocity is highest for z=0.5, more heat is transported here than anywhere else. Towards the boundary this effect decreases to its minimum value because of the adhesion.

Re=50000: The temperature field on the free surface shows strongly curved isotherms, see fig.7. The behaviour is contrary to the one for the Re=100 described above. The cause for the less intense temperature flux in the symmetry plane is the displacement of the vortex shown in fig. 5. Since the centre for z=0.5 is located more below and more towards the colder side than the centre for z=0.3, the fluid particles must cover a longer distance on their way from the surface along the colder side, following the streamline of the vortex, back to the free surface. Therefore the particles in the symmetry plane cool down more rapid than in the z=0.3 plane, and they get back to the surface within shorter distance from the hot side.

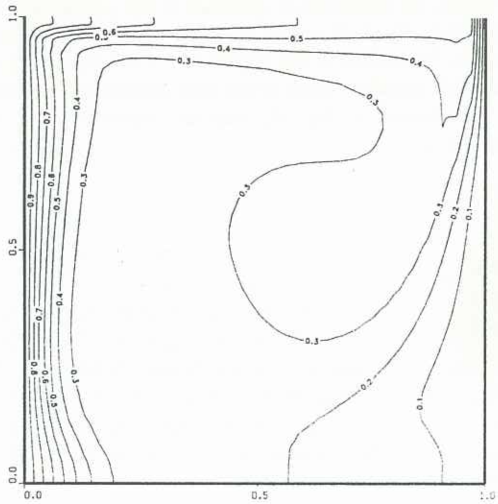

Fig.3. Temperature field, x-y plane, Re=50000, z=0.3

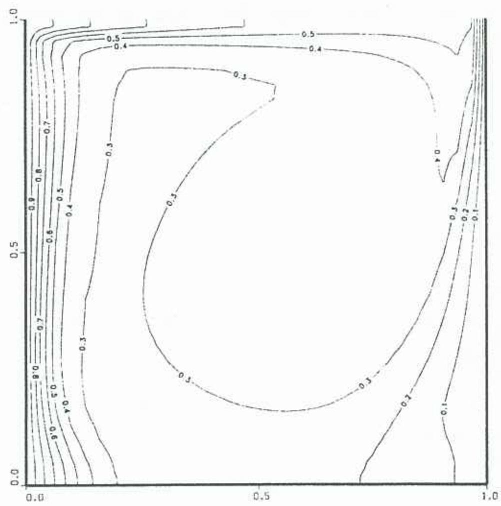

Fig.4. Temperature field, x-y plane, Re=50000, z=0.5

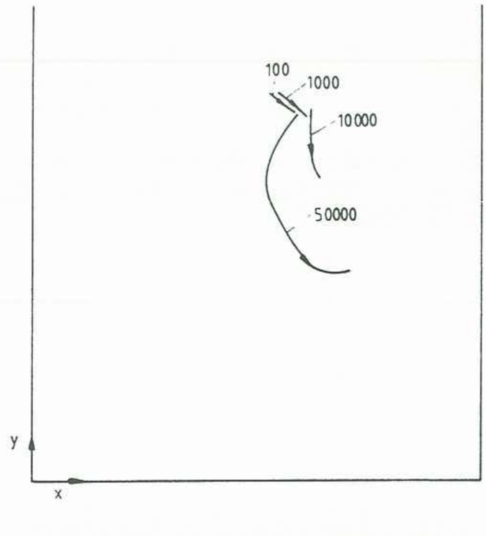

Fig.5. Displacement of the main-vortex-centre in the x-y plane from z=0 to z=0.5 in direction of the arrow for Re=100,1000,10000,50000

z-y plane

Just like the temperature gradient in the x-direction the one existing in z-direction induces a thermocapillary convection. Therefore in the region of x=0.5 depending on the Reynolds number, close to the free surface two vortices appear as shown in fig. 8, one on either side of the symmetry axis.

CONLUSION

On the one hand the results show that the solution for small Reynolds numbers is slightly three-dimensional, and the main vortex in the x-y plane can be approximated by a two-dimensional method. With growing Reynolds numbers on the other hand the three-dimensional character of the result increases. It remaines to be examined how the three-dimensional effects change when the aspect ratio is changed, especially when the distance to the wall is increased in direction of z.

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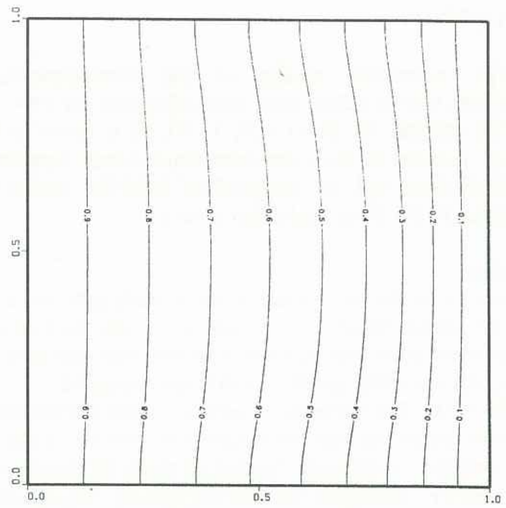

Fig.6. Temperature field, x-z plane, Re=100, y=1

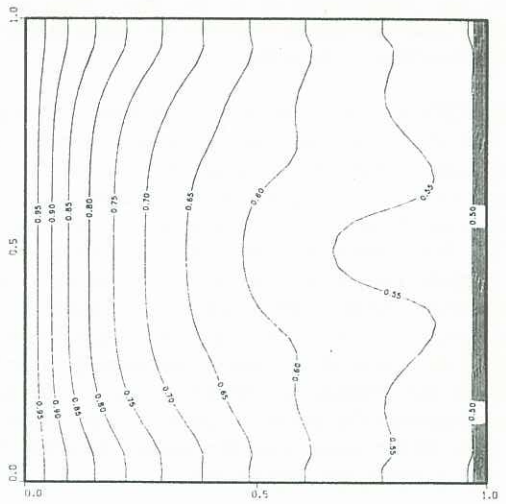

Fig.7. Temperature field, x-z plane, Re=50000, y=1

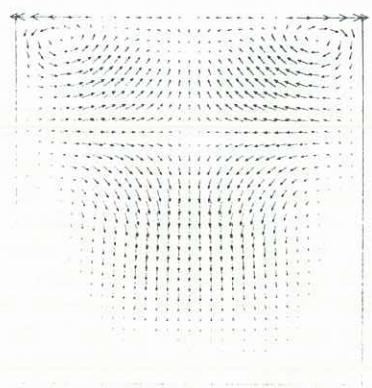
Fig.8. Velocity field, z-y plane, Re=1000, x=0.56