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SOME FUNDAMENTAL ASPECTS OF CENTRIFUGAL PUMP DESIGN

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SUMMARY

Centrifugal pumps have an optimum (best efficiency) specific speed at which maximum energy transfer takes place. From this it is possible to determine impeller efficiency and optimum values of design coefficients which may be used to determine impeller dimensions at the design specific speed. These theoretically determined coefficients agree with with values generated by Stepanoff and others.

When the optimum specific speed is the same as the reference specific speed the impeller is designed for maximum impeller efficiency. There is an efficiency penalty associated with selection of an optimal specfic speed other than the reference specific speed. This takes place if a design criterion such as pump rangeability is used instead of maximum efficiency.

NOTATION AND DEFINITIONS

A	area (m^2)
D	diameter (m)
H	pump head (m)
N	rotational speed (rev/s)
P	fluid power (kW) = ρQgH
Q	volume flow rate (m^3/s)
z	number of impeller blades
U	tangential velocity of impeller
V	absolute fluid velocity
Vr	fluid velocity relative to impeller
Vu	tangential component of fluid absolute velocity
Vm	meridional velocity; normal to area
b	impeller blade height (m)
k	constant
β	blade angle measured from tangent to impeller
α	inlet fluid swirl angle measured radially
ρ	fluid density (kg/m^3)
η	efficiency

Subscripts

1	pump impeller inlet
2	pump impeller outlet
d	design
dr	design based on recirculation
e	euler
h	hydraulic
i	input or ideal
ip	inlet pipe
l	leakage
m	mechanical
Ns	specific speed
o	optimum for maximum efficiency
r	relative or ratio
rc	recirculation
s	shaft
u	tangential direction or uncoupled

Superscripts

'	optimum energy transfer
*	optimum for turbo-machine type
"	effective

Definitions

η_b	blade efficiency = H_i/H_e
η_o	overall efficiency = $\eta_h\, \eta_m\, \eta_{vol}$ = P/Ps
η_u	uncoupling efficiency = P'/P*
η_{vol}	volumetric efficiency = $Q/(Q + Q_l)$
η_{visc}	viscous efficiency = $1 - 0.075/Q^{.25}$
η_∞	efficiency of infinite capacity pump = P_i/P'
ϕ	flow parameter = V_m/U
ψ	head parameter = gH/U^2
ξ	fluid power ratio = P/P_i
N_s	specific speed = $2\pi N$ [rad/s] $\sqrt{Q[m^3/s]}/(g\,H)^{3/4}$
S	stodola slip factor = $k\pi \sin \beta_2/z$
μ	= 1 – S

Following Stepanoff $u_2 = k_u \sqrt{2gH}$
$Vm_2 = k_{m2} \sqrt{2gH}$

1. INTRODUCTION

Historically a body of design theory and practice has been developed to facilitate centrifugal pump design. This paper provides a theoretical insight into the parameters employed by Stepanoff and others, and develops an equation relating the maximum pump efficiency to the best efficiency specific speed.

The paper commences with a brief resume of pump impeller theory and design practice. This is followed by an outline of the theory and derivation of pump design coefficients based on maximum impeller efficiency, which until now, have been essentially empirically based. The paper concludes with a study of the effects of selection of design criterion other than maximum efficiency. Such requirements produce an uncoupling of reference and optimum specific speeds.

2. FUNDAMENTALS

If the flow through a centrifugal pump impeller conformed exactly to the geometry the head produced across the pump could be calculated from the Euler equation

$$H_e = (U_2\, V_{u2} - U_1\, V_{u1})/g \qquad (1)$$

When there is no inlet swirl, as will be assumed here, then $V_{u1} = 0$.

Because the flow does not conform to impeller geometry then the theoretical input head (H_i) is much less than the Euler head. There is an effective V''_{u2} which differs from the ideal by a slip component such that

$$H_i = U_2\, V''_{u_2}/g = U_2\,(V_{u_2} - SU_2)/g. \qquad (2)$$

or

$$\psi = \eta_h (\sigma - \phi_2 / \text{Tan } \beta_2) \quad (2a)$$

or

$$1 = 2\eta_h k_u^2 (\sigma - k_{m_2}/k_{m_1} \text{ Tan } \beta_2). \quad (2b)$$

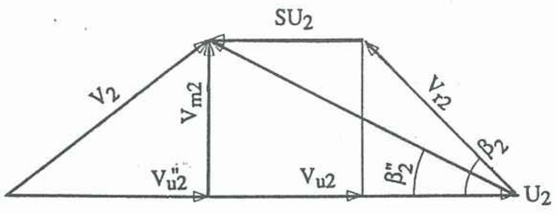

Figure 1. Ideal velocity triangle at impeller outlet.

There are various estimates of the slip velocity (S), but the one used here, for simplicity, is due to Stodola. From these results it is possible to construct a non-dimensional pump characteristic curve (Fig. 2). Note that it is assumed that $s \neq f(\phi)$.

Stepanoff provides guidelines for values of the constants k_{m_2}, k_u, k_{m_1} and the Pump Handbook gives values for ϕ_2. These design values are only guidelines to provide efficient design and must be adjusted to achieve the actual design specification.

Estimates of overall efficiency have been obtained from tests on a range of pumps to provide a correlation between specific speed at the best efficiency point and the overall efficiency (Fig. 3).

The values of the design constants have been obtained from experience. The objective of this paper is to give theoretical insight into the reasons behind that experience.

3. DESIGN COEFFICIENTS AND EFFICIENCY VERSUS Ns

The function of the impeller is to transfer energy. An optimal design would transfer power as efficiently as possible.

From Fig. 3 it is evident that energy transfer is most efficient, at least for pumps where viscous effects are minimal, at $N_s = 1$. This is here defined as the reference specific speed N_s^*, and it indicates another aspect of specific speed.

Meaning of Specific Speed

From the definition of Ns it follows that

$$N_s = k_{Ns} P_r \quad (3)$$

where

$$P_r = U^2 k/2g H \quad (4)$$

and

$$k_{Ns} = f(\eta_h, k, \sigma, \beta_2)(D_{ip}/D_2)\sqrt{V_{ip}}/U_2. \quad (5)$$

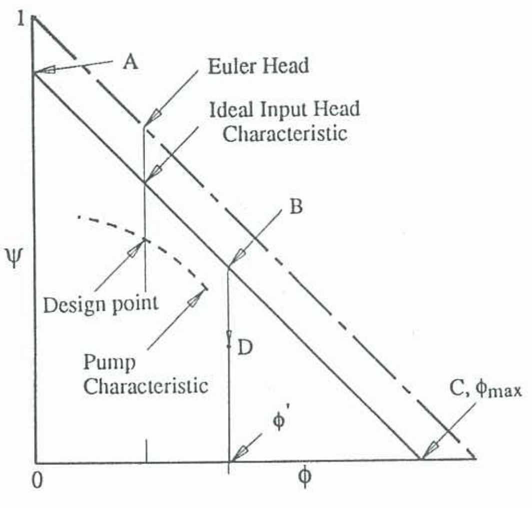

Figure 2. Non-dimensional pump characteristic curve.

In this partition, which has to be validated, P_r is an energy ratio, and k_{Ns} relates geometry and dynamics. The correlation between specific speed at the best efficiency point and the pump geometry is well known.

Anticipating the results of the ensuing discussion, it appears that three specific speeds can be identified; namely the reference, optimum (the N_S selected to optimise the design) and the design specific speed. An essential condition, established below, to delineate these three speeds is the requirement that

$$P_r' = P_r^* = 1. \quad (6)$$

The following conditions apply at each of the three specific speeds.

Reference specific speed N_s^*. $P_r^* = P_r' = 1$ and $k_{Ns}^* = N_s^* = 1$ since $N_s^* = N_s' = 1$. This is unique condition which, if satisfied, maximises impeller efficiency.

Optimum specific speed N_s'. $P_r' = 1$ and $k_{Ns}' = N_s' \neq 1$. This condition arises when a design criterion other than maximum efficiency is selected. It results in an efficiency penalty, as will be discussed later.

Design specific speed. $P_r \neq 1$ while $N_s = k_{Ns} P_r$.
If a pump is to be designed for maximum efficiency then the optimum specific speed is set equal to the reference specific speed. If another criterion is used then N_s' is selected for that, but there is an efficiency penalty by virtue of the uncoupling from N_s*.

Design Coefficients at N_s'

At the optimum specific speed the energy transfer is maximum. Combining the equation for fluid power and equation (2), and assuming that $\eta_h \neq f(\phi_2)$, then it follows that

$$k_u' = 1/\sqrt{d\eta_h} \quad (7)$$

$$k_{m_2}' = \sqrt{\sigma/\eta_h}\,(\text{Tan } \beta_2)/2 \quad (8)$$

$$\phi_2' = \sigma\,(\text{Tan } \beta_2)/2 \quad (9)$$

$$\psi' = \sigma\,\eta_h/2 \quad (10)$$

These are optimal design coefficients which will give maximum energy transfer and maximum impeller efficiency.

Reference to previously cited Stepanoff literature shows that coefficient values derived from these equations for typical values of β_2, σ and η_h agree with values used in practice at N_s^* (Fig. 4).

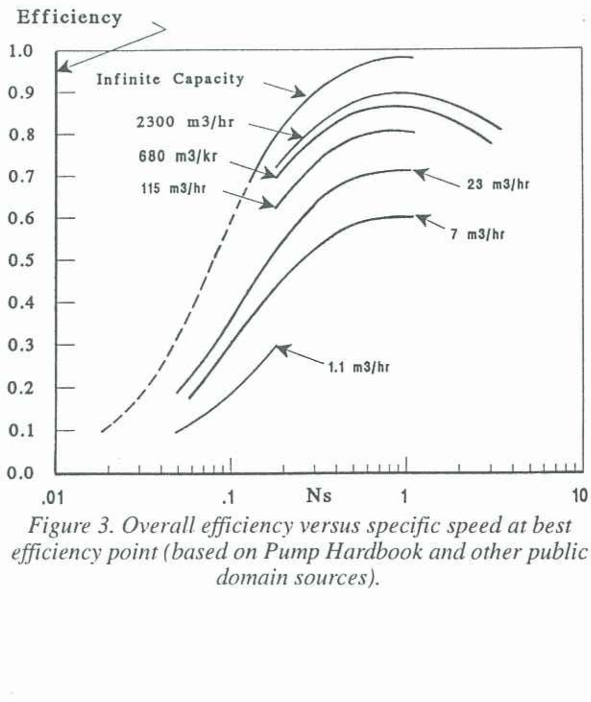

Figure 3. Overall efficiency versus specific speed at best efficiency point (based on Pump Hardbook and other public domain sources).

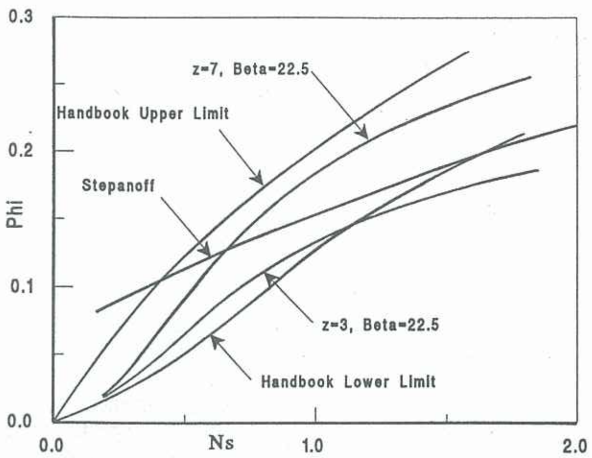

Figure 4. Comparison of theoretically derived ratios with those quoted by Stepanoff and Pump Handbook based on Ns=1.0.*

For all values of H' along the line $\phi = \phi'$ it follows, from equation (1), that

$$P'_r = \frac{U_2^2}{2\ gH'_e} = \frac{U_2^2\ k}{2\ gH'_i} = 1. \tag{11}$$

Thus the condition stated in equation (6) is established. Note that, from the definition of blade efficiency, when $H'_i = H^*_i$ then $k = \eta_b$.

Variation of design coefficients with N_s. In practice the optimum values for the design coefficients cannot be used at all N_s.

With reference to the non-dimensional pump characteristic curve (Fig. 2), it is possible to determine a relationship between N_s and ϕ along the ideal input head line. Along that line N and U_2 are constant, so that

$$u_2 = k_u \sqrt{2g\ H} = k'_u \sqrt{2g\ H}$$

it follows that

$$\frac{\phi_2}{\phi'} = \left(\frac{N_s}{N'_s}\right)^2 \left(2 - \frac{\phi_2}{\phi'}\right)^{3/2}. \tag{12}$$

This equation predicts values of ϕ_2 which agree, for realistic values of blade angle, hydraulic efficiency and slip, with ranges and values derived from the Pump Handbook and Stepanoff (Fig. 4).

Variation of efficiency with N_s. With the above result it is possible to obtain the variation of efficiency for an infinite capacity pump, in which the viscous effects are negligible. The efficiency will be zero at points A and C (Fig. 2), and will be 100% at B.

From the ratio of power at the actual design point to that of the optimum design point,

$$\xi = P/P' = QH/(Q'H')$$

and introducing N_s again, it can be shown that

$$\xi = \left(\frac{N_s}{N^2}\right)^2 \left(\frac{\phi_2}{\phi'}\right)^{5/3} \tag{13}$$

Dimensional analysis shows that

$$P = \rho\ N^3\ D^5\ f\ (\eta_\infty). \tag{14}$$

Changing the frame of reference from a particular pump to a general pump in which rotational speed is allowed to vary, then, while there is no a priori justification for so doing, assume that for large pumps that N and D are functions of η_∞ then

$$\xi = \eta_\infty^8. \tag{15}$$

Thus from equations (13) and (15) the efficiency of an infinite capacity pump is

$$\eta_\infty = \left[\left(\frac{N'_s}{N_s}\right)^{4/3} \left(\frac{\phi_2}{\phi'}\right)^{5/3}\right]^{1/8}. \tag{16}$$

When this result is compared with the generalised pump data for infinite capacity pumps (Fig. 5) there is close (± 1%) agreement in the range $0.2 < N_s/N'_s < 0.6$. For $N_s/N'_s < 0.2$ actual pumps are influenced by flow recirculation at impeller inlet and outlet and the maximum efficiency design criterion no longer applies. Above 0.6 there is also a deviation but this is possibly due to assumptions about the data.

4. CONSEQUENCES OF CHANGES TO DESIGN CRITERIA

Under certain conditions an impeller, which is designed for maximum efficiency, may exhibit poor performance according to other criteria. For example, a pump may exhibit significant impeller inlet or discharge recirculatory back flows at larger than normal partial flow rates. This may reduce the rangeability of the pump because such back flows increase the magnitude of pump discharge fluctuations.

Effect on dfficiency of uncoupling N'_s and N^*_s. Uncoupling of N'_s and N^*_s causes a decrease in overall efficiency. This can be demonstrated as follows. N and U_2 remain constant, but there is also the constraint that $\phi' = \phi^*$ so that ψ' moves along line BD (Fig. 2), and also $Q_{rd} = Q$. From the definitions of uncoupling efficiency and specific speed then

$$\eta_u = \left(\frac{N'_s}{N^*_s}\right)^2 \left(\frac{2 - \frac{\phi_o}{\phi'}}{2 - \frac{\phi_d}{\phi'}}\right)^{5/2}. \tag{17}$$

This equation yields $\eta_u < 1$ for $N'_s < N^*_s$ and $\phi_{rd} < \phi'$. This indicates a lack of generality, since η_u must always be less than unity, but the constraints may be a mathematical rather than physical difficulty.

The adoption of another design criterion, such as discharge recirculation, will require a uncoupling of N^*_s and N'_s Consider, an example, discharge recirculation. Fraser proposed a model which predicts that recirculation occurs when

$$\phi_{rc} = 0.26\ \beta''_{2(\text{radians})} - .01. \tag{18}$$

When a design margin is required so that recirculation commences at less than a specified percentage of design flow, then if

$$\phi_{rd} = \frac{\phi_{rc}}{x} > \phi_2 \text{ for maximum efficiency}$$

recirculation becomes a design consideration. ϕ_{rd} at design N_s will give, from equation (12), the required value of N'_s, which, in all cases, is less than N^*_s. From that the efficiency penalty can be calculated.

The effect of the Fraser model is to produce a fixed ϕ_{rd} for particular z, β_2 and x with a corresponding critical Ns_{rd}

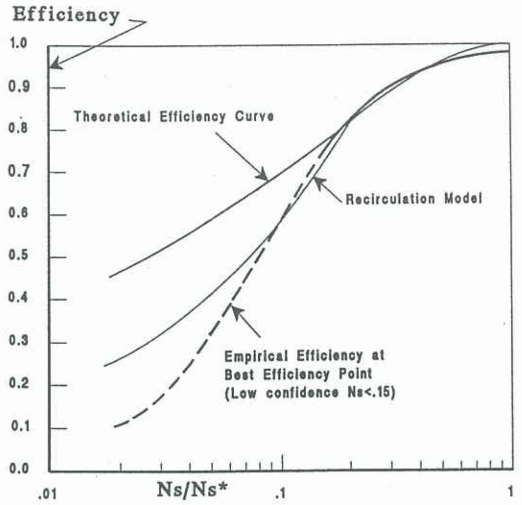

Figure 5. Comparison of theoretically derived efficiency verses N_s/N_s^* curve to empirically derived result for infinite capacity pump.

(typically 0.5 to 0.8) which leads to unrealistically large efficiency penalties for low Ns pumps. This may result from the interaction of two simple models; the Stodola model which produces poor estimates of slip under many circumstances, and the Fraser model in which ϕ_{rc} is not, as may be expected, an explicit function of Ns. In an arbitrary step, if the prediced ϕ_{rd} from the Fraser model is retained, with the consequence that critical Ns_{rd} is constant, and ϕ_{2rd} is referenced to that value then it follows that

$$\phi_{2rd}/\phi_{rd} = (Ns/Ns_{rd})^{4/3}. \tag{19}$$

When this model is used and Ns_{rd} is set equal to 0.16 the result is close to the limiting efficiency for an infinite capacity pump (Fig. 5). This correspondence should not be taken as giving credence to the recirculation model for obvious reasons.

Hydraulic efficiency. As a consequence of this analysis, hydraulic efficiency can be seen to have three components such that

$$\eta_h = \eta_{visc}\, \eta_u\, \eta_\infty. \tag{20}$$

5. CONCLUSION

This paper has extended the classical pump impeller design approach and has provided a systematic basis for existing practice.

When a pump is designed for maximum impeller efficiency the optimum specific speed and the reference specific speed are required to be the same. The impeller design coefficients for the design specific speed can be calculated on the basis of the theory outlined here rather than employing coefficients based on empirical results.

An efficiency penalty is incurred if the reference and optimum specific speeds are uncoupled. This may be required if a design criterion other than maximum efficiency is dominant. These theoretical results are based on significant assumptions which indicate directions for further research and which must not be forgotten.

6. REFERENCES

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