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AN INVESTIGATION OF VIBRATION-EXCITED BOUNDARY LAYERS USING THE METHODS OF CHAOS PHYSICS

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The dynamics involved in the transition from laminar to turbulence in flat plate boundary layers have been analysed utilising chaos physics techniques. The aim of this approach is to extract as much information as possible from a boundary layer flow whilst introducing as few variables as possible in the analysis. Measured boundary layer velocity fluctuations have been used to 'reconstruct system attractors' using the time-shifted coordinate technique. Providing certain conditions are met, these reconstructed experimental attractors have information on the original boundary layer system intwined in their own structure. Various available measures for quantifying both the static and dynamic properties of the attractors can then be related to the original boundary layer system. Experimental data obtained in an open circuit wind tunnel where the boundary layer transition process has been excited via global plate vibration and leading edge vibration are used to test these techniques. This paper summarises the results of the investigation.

INTRODUCTION

The chaos approach to the boundary layer transition problem, is motivated by the potential to provide a method which will complement traditional techniques in understanding this complex process.

The chaos techniques adopted in this paper consider the post transient oscillatory components of flow velocities in the boundary layer, and analyse the motion in state space. Figure 1 illustrates the classes of oscillatory motion that may exist under this state space representation. The possible classes of motion as depicted in Figure 1 are called 'attractors', as they represent the post transient long term structure that solution trajectories will eventually be 'attracted' to. Fixed point, limit cycle, tori and random attractors have long been known. Deterministic chaos is responsible for the discovery of the 'strange' attractor. Oscillatory motion governed by a strange attractor has apparent random behaviour in the time domain, but when viewed in the state space domain, exhibits complicated but nevertheless deterministic structure, unlike random motion.

The strange attractor is a relatively recent discovery in physics, which on a scale of complexity, fits between purely random motion and motion governed by a tori type attractor. Chaotic motion (i.e. motion governed by a strange attractor) is defined as exponentially divergent motion or motion which is sensitive to infinitesimal changes in initial conditions, a property which is not true for limit cycle and tori type attractors. This definition leads to the conclusion that it is impossible to develop long term predicability of a chaotic system. Chaotic motion has the added constraint that when the motion is plotted in state space, it remains confined for all time to an attractor of definite geometric structure, and doesn't simply fill the entire state space as random motion.

Experiments conducted to track the development of disturbances introduced into a laminar boundary layer are used to test the chaos techniques. The disturbances are introduced into the boundary layer via two methods, (i) leading edge vibration, and (ii) global plate vibration. The measured boundary layer velocity fluctuation time histories are used to reconstruct experimental attractors (Pruiti and Norton, 1991). A quantifying measure conceptually equivalent to the Lyapunov exponent of an attractor is applied to measure trajectory divergences of the attractors. This divergence estimate descriptor is assessed for its ability to identify strange or chaotic behaviour, quantify the increasing complexity of a boundary layer transition process and to distinguish between different reconstructed boundary layer attractors.

EXPERIMENTAL PROCEDURE

This study is concerned with the laminar turbulent transition process along a flat plate resulting from two different forcing conditions. The first forcing method is via local vibration of the leading edge only, and the second is via vibration of the entire test plate. A major objective in developing the experimental environment is the minimising of all other possible factors that could possibly affect the transition. In this way a direct qualification of the two forcing conditions can be made. A schematic diagram of the experimental set up used is shown in Figure 2.

Experimental Considerations

Factors that could affect transition in a boundary layer have been identified as (i) pressure gradients in the streamwise direction of the flow, (ii) surface curvature, (iii) free-stream turbulence, (iv) surface roughness, (v) acoustic disturbances, (vi) vortical disturbances, (vii) entropy disturbances and (viii) model vibrations.

To ensure experimental conditions that would satisfactorily examine the receptivity of the laminar boundary layer to model vibrations and reduce the effects of the other factors, the following considerations were accounted for in the set up of the wind tunnel:

(i) A 5 mm wide slot was cut into the reverse side of the test plate at 100mm from the tip of the leading edge. The slot was 21mm deep and was filled with silicone rubber to absorb the vibration energy from the vibrating leading edge. This configuration more effectively isolated leading edge vibrations from global plate vibrations.

(ii) The entire working side of the test plate and the leading edge were polished to an RMS roughness of approximately 0.6μm, making surface roughness effects secondary.

(iii) The wind flow generated by the centrifugal fan was passed through a wide angle diffuser, then through a honeycomb and a number of screens in the settling chamber and finally through a smooth three dimensional contraction before entering the working section. This arrangement was found to produce free stream turbulence levels in the working section which were under 0.05% for the experiments conducted.

(iv) The floor of the working section of the wind tunnel was adjustable. The position of the stagnation point of the flow could be located at a particular position on the leading edge by a combination of adjusting the floor and the

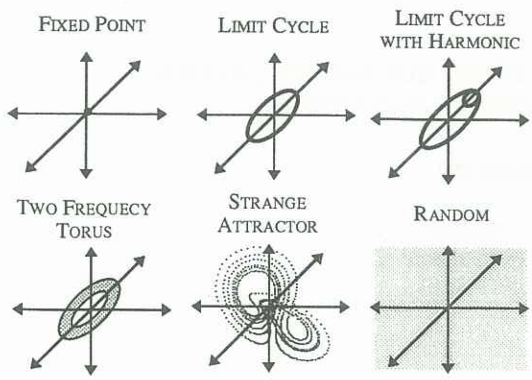

Figure 1: Types of attractors in state space coordinates.

adjustable splitter plate at the trailing edge of the test plate. This set up allowed also for the pressure gradient over the span of the test section to be varied.

(v) The adjustable splitter plate was also used to help destroy Karman vortex sheets from the otherwise blunt trailing edge of the test plate which would have been highly undesirable for the current receptivity experiments.

Work by Chiu and Norton (1990) and Soria (1989) at this facility found that the above considerations did in fact produce an environment suitable for boundary layer receptivity experiments to introduced model vibrations.

Experimental Program

Two experiments were conducted to document the laminar to turbulent transition process induced by leading edge vibration (experiment#A) and global plate vibration (experiment#B).

The experimental conditions for experiment (A) were: (i) The free stream velocity in the working section was U_∞ = 10 m/s; (ii) The stagnation point of the flow was set to the underside of the leading edge. This had the effect of moving the region of adverse pressure gradient upstream thereby accelerating the effects of any imposed disturbance to the flow. This was desirable as the full transition process from the laminar stage, to fully developed turbulent boundary layers was required to occur along the length of the test plate so that a chaos analysis at all stages of the process could be performed; (iii) The leading edge vibration was set to v_{le} = 5 mm/s peak @ 247 Hz; (iv) No global plate vibration was introduced.

The experimental conditions for experiment (B) were: (i) The free stream velocity in the working section was U_∞ = 10 m/s; (ii) The stagnation point of the flow was set to the underside of the leading edge for the same reasons as that of experiment (A); (iii) The global plate vibration was set to v_{le} = 5 mm/s peak @ 247 Hz; (iv) No leading edge vibration was introduced.

Without the presence of model vibrations, the boundary layer remained laminar and no flow separation occurred due to the locating of the stagnation point on the underside of the leading edge. In both experiments the desired transition to turbulence in the boundary layer took place within the length of the test plate and was found to have been caused by the introduction of the model vibrations to the system.

Velocity fluctuation time histories in the boundary layer were captured along the length of the test plate in both experiments. The measurement stations were situated at a height of 1mm from the test plate and were taken at x= 0.2 to x= 0.5m from the leading edge. These time histories provided a means to assess the development of instabilities introduced into a boundary layer via the experimental reconstruction of boundary layer state space attractors.

At each measurement station, 100,000 point time histories sampled at 40 kHz were acquired. The sampling rates and length of time histories captured at the boundary layer stations were determined from chaos considerations. Also taken at each station were 1024 point time histories sampled at 2kHz. These measurements were acquired for the purposes of attaining spectral information on the development of the boundary layer instabilities.

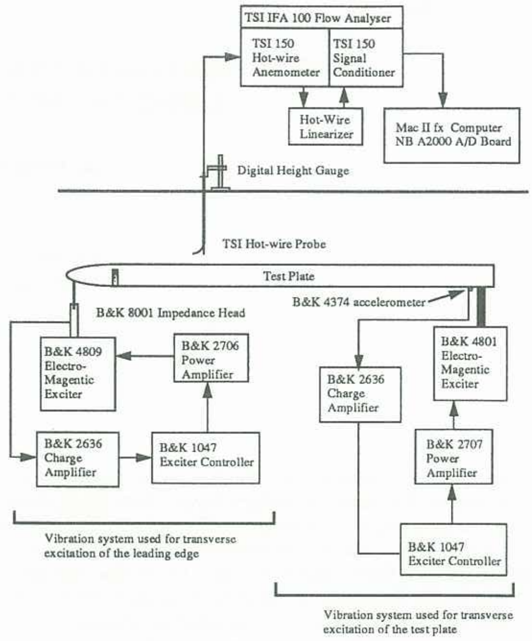

Figure 2: Schematic illustration of experimental set up.

The experiments conducted produced boundary layer flows which contained oscillations covering a wide range of flow categories and hence resulted in an ideal data bank from which to test the methods of chaos physics.

CHAOS ANALYSIS OF EXPERIMENTS

For a chaos analysis of the boundary layer transition process, the first requirement is a description of the dynamics in the state space domain. This approach leads to attractors of the classes described in Figure 1 which characterise the transition process. Chaos measures can then be applied to numerically quantify the attractors.

Reconstructing Experimental Boundary Layer Attractors

The generation of the exact boundary layer attractors can not be achieved as the equations of motion are not soluble. However it is possible to generate 'reconstructed boundary layer attractors' from experimental data which represent embeddings of the original attractors.

The method of time delay coordinates is adopted in this paper to transform boundary layer velocity fluctuations from the time domain to the state space domain. The method basically reconstructs an attractor from the time series of a single variable. More specifically the theory states, for almost every observable x(t), and for almost every time delay T_d, an attractor in m dimensional state space can be constructed from the vectors

$$\{ x(kt_s), x(kt_s + T_d), ..., x(kt_s + (m-1)T_d) \} \quad (k=1,2,...,\infty), \quad (1)$$

where t_s is the sampling time and m is called the embedding dimension. The reconstructed attractor will have the same properties as one constructed from measurements of N independent variables, if $m \geq 2N + 1$. The state space obtained in this manner gives an embedding of the original manifold.

State space reconstruction by delay coordinates is presently the most widely used choice for experimental attractor reconstruction, however it has the unpleasant property that in order to be applied to an application, the careful selection of values for the sampling time, time delay and embedding dimension must be made. For the current investigation, boundary layer attractors were generated with reconstruction parameters of t_s = 25 μs, T_d = 10*t_s and m = 3. The attractors generated from experiments#A and B are shown in Figures 3 and 4 respectively.

An embedding dimension of m=3 was chosen on the basis of being the minimum dimension required for a chaotic attractor to exist. Quite complex chaotic behaviour can be described by attractors in 3 dimensional state space, and 3D embeddings are sufficient to establish where and if chaotic oscillations first begin in the boundary layer transition process. It is expected that this embedding would not be sufficient to characterise the turbulent boundary layer and result in catastrophes in the reconstructed attractors. A catastrophe is defined as dissimilar points on the actual attractor being mapped to coincident points on the reconstructed attractor due to reduction in dimension of the reconstructed state space. However as the number of catastrophes should be proportional to the complexity of the flow, numerical measures can still feasibly detect the increasing complexity of the laminar turbulent transition via the detection of the increasing number of catastrophe points on the attractors.

Route to Turbulence

The analysis reported on in this paper is concentrated in the region of x=0.2m to x=0.5m from the leading edge. Spectral information presented in Figures 5 and 6 indicate that transition from a laminar to turbulent boundary layer occurs in this region for both experiments. Also in this region the boundary layer growth is reasonably well approximated by a parallel boundary layer. Therefore, taking boundary layer measurements at y = constant in this region can be used as an approximate description of the longitudinal fluctuation velocity development with x.

From Figures 3 and 4 it can be seen that leading edge induced boundary layer transition results in limit cycle attractors upto x=0.2m (Figure 3(a)) and global plate induced transition resulted in limit cycle attractors upto x = 0.25m from the leading edge (Figure 4(b)). The limit cycle attractors represent the initial stages of a 247 Hz Tollmien-Schlichting eigenfunction in the state space domain.

Chiu and Norton (1990) identified the 247 Hz TS eigenfunction as being the mechanism responsible for the transition to turbulence in the boundary layer. They studied the transition process described here in the initial linear stages of development, and found that the TS eigenfunction is an unstable mode and is introduced into the boundary layer flow via vibration of the leading edge. In fact, even for global plate vibration experiments conducted by Soria (1989), Chiu found that the offending TS eigenfunction responsible for transition was introduced into the flow via vibration of the leading edge, despite the fact that the overall leading edge vibration levels were much lower than the global plate levels.

In the state space domain, the development of the TS eigenfunction may be described by initial amplification of limit cycle attractors in the linear stages of the transition. Bifurcations then occur which transform the attractor to progressively higher levels of complexity and eventually culminating with the random attractor. From the frequency spectra collected, it is apparent that the bifurcations occurring in the transition process are of a period doubling nature. This is because harmonics of the fundamental 247 Hz disturbance develop progressively with increasing distance from the leading edge.

The boundary layer response observed in the state space domain reveals similar attractor development for both of the experiments conducted. However equivalent stages of the transition process induced by leading edge vibration occur approximately 0.05m earlier than global plate vibration induced transition.

The similar boundary layer responses observed in both experiments particularly in the early stages of development suggest that the excitation mechanism responsible for transition in the global plate vibration experiment is the same as that responsible for transition in the leading edge vibration experiment. This further suggests the excitation mechanism is caused by the vibration of the leading edge. This qualitative information extracted from the reconstructed attractors further supports the findings of Chiu.

Quantifying the Experimental Attractors

The qualitative information extracted from the reconstructed attractors provides significant information on the nature of the boundary layer transition process. However, the need to distinguish between different attractors and to assess for chaotic behaviour requires the application of quantifying measures.

The spectrum of Lyapunov exponents has proven to be the most popular dynamical diagnostic used by researchers of chaotic systems. Lyapunov exponents measure the average exponential rates of divergence or convergence of nearby orbits in state space. A positive exponent indicates exponential divergence while a negative exponent indicates exponential convergence. Any system containing at least one positive Lyapunov exponent is defined to be a chaotic system, with the magnitude of the exponent reflecting the degree of chaos. That is the larger the exponent, the larger the divergence rate of nearby trajectories and hence the shorter the time scale on which the system has predicability.

A numerical method proposed by Wolf et al. (1985) for estimating the largest Lyapunov exponents of reconstructed experimental attractors has been investigated. The method basically monitors the long term evolution of a single pair of nearby orbits on a reconstructed attractor and results in a trajcetory divergence estimate for the attractor. Providing certain conditions are met, the divergence estimate will correspond to the Lyapunov exponent of the actual dynamical system. The divergence estimate procedure has been applied on the reconstructed attractors of Figures 3 and 4. The results are presented in Figure 7 over the range of x=0.2 to x=0.5 metres from the leading edge. From Figure 7 it can be seen that the trajectory divergence estimates are approximately zero for the limit cycle attractors of both experiments. This zero value agrees with the theoretical Lyapunov value for limit cycles. The divergence estimates for attractors further from the leading edge are much higher than typical Lyapunov exponents of dynamical systems. This is attributed to the fact that the reconstructed attractors have been embedded to a dimension too small to characterise boundary layer motion completely. Hence catastrophe points present in these attractors are responsible for providing enormous contributions to the Lyapunov exponent estimate. As the flow becomes more complicated, more catastrophe points appear on the reconstructed attractors, resulting in larger and larger divergence estimates.

Although the divergence estimate fails to identify the true Lyapunov exponents of the boundary layer system, it provides a useful numerical measure of the increasing complexity of the flow. The point where positive divergence numbers commence also provides an indication where strange attractor behaviour begins. For experiment (A) this was x=0.25m and for experiment (B) it was x=0.3m from the leading edge. Figures 4 and 5 agree with this estimate. From Figure 7 it can be seen that the divergence estimate also detects earlier transition in experiment (A) and that the turbulent spectrum of experiment (A) is more complicated than that of experiment (B) at x=0.5m. It can be seen that this information is more easily arrived at from Figure 7 than from the measured frequency spectra of Figures 5 and 6.

CONCLUSIONS

This paper has utilised methods of chaos physics in an effort to complement traditional methods on interpreting the boundary layer transition problem. It has been found that via suitable reconstruction techniques, chaotic attractors which characterise experimental boundary layer flows can be generated. A numerical method based on the Lyapunov exponent algorithm has been used to quantify reconstructed experimental attractors. This divergence measure shows potential in characterising oscillations of increasing complexity and distinguishing between different oscillations.

The results of this study support the role of a chaos approach in analysing the boundary layer transition problem. Additional work is in progress on optimising the reconstruction process of experimental data and the calculation of attractor quantifying measures.

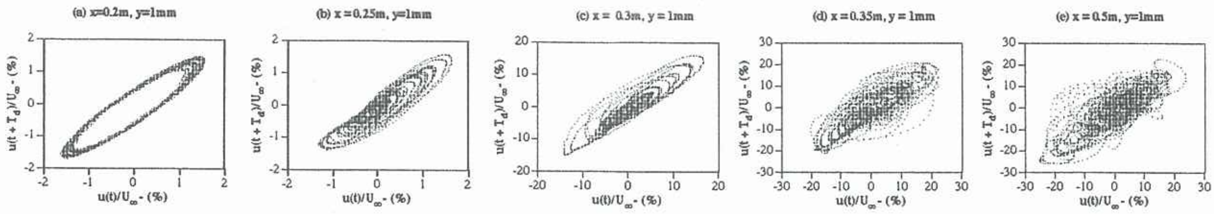

Figure 3: Reconstructed attractor development in the leading edge vibration excited boundary layer.

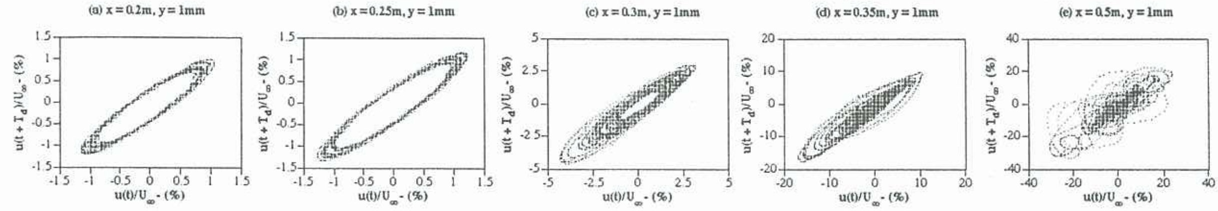

Figure 4: Reconstructed attractor development in the global plate vibration excited boundary layer.

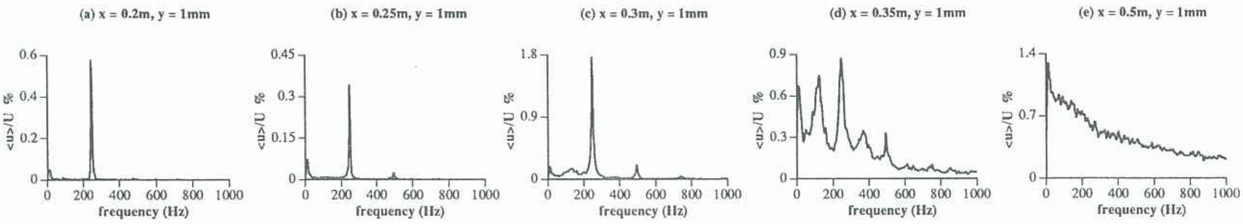

Figure 5: Spectral development of the leading edge vibration excited boundary layer.

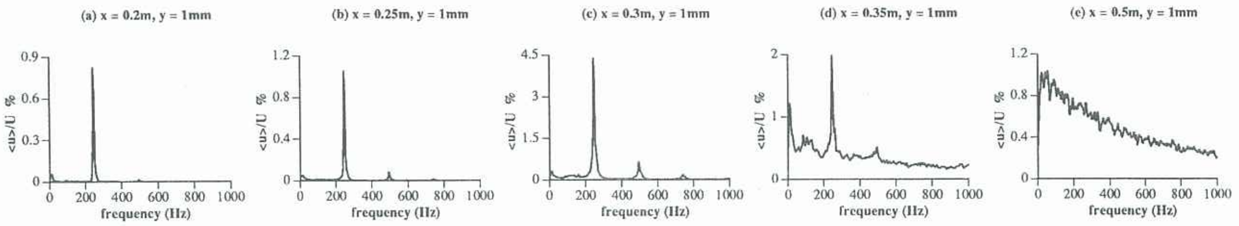

Figure 6: Spectral development of the global plate vibration excited boundary layer.

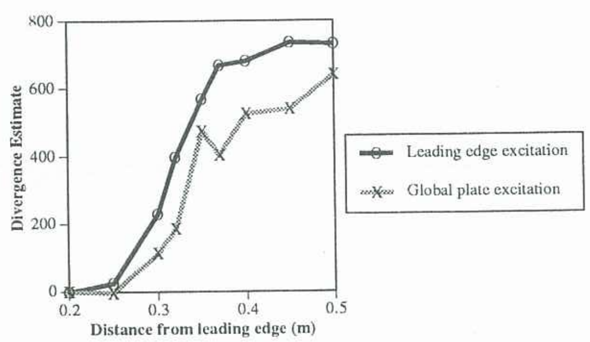

Figure 7: Divergence measure for the reconstructed boundary layer attractors.

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