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THE EFFECT OF INITIAL CONDITIONS ON THE DEVELOPMENT OF TWO-STREAM MIXING LAYERS SUBJECTED TO LONGITUDINAL CURVATURE

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ABSTRACT

Curved two-stream mixing layers originating from untripped and tripped initial boundary layers, with a velocity ratio of 0.6 and Re_δ of about 27,000 have been investigated experimentally. Destabilizing curvature retarded the decay of spatially stationary streamwise vorticity naturally present in the untripped case, resulting in a higher growth rate than exhibited in a comparable straight mixing layer. On the other hand, stabilizing curvature accelerated the streamwise vorticity decay and resulted in a lower growth rate. Spatially-stationary streamwise vortical structures were *not* evident in the tripped cases. Therefore, due to reduced entrainment, the tripped unstable mixing layer exhibited a lower growth rate than that of the corresponding untripped case. However, the growth rate of the tripped unstable case was still higher than the tripped straight case, while that of the stable case was not affected by the change in initial conditions. The Reynolds stresses in the unstable mixing layer were elevated with respect to those in the straight layer, and by approximately the same amount in both tripped and untripped cases. On the other hand, Reynolds stresses in the stable case were comparable to, or slightly lower than, those for the straight case for both initial conditions. There is some evidence that the mixing layer turbulence structure in all cases continues to change beyond the measurement domain ($Re_x > 10^6$).

INTRODUCTION

It is now well established that a mixing layer developing from laminar initial boundary layers develops a complex three-dimensional structure in the form of streamwise vorticity (see Bell & Mehta 1992, for a review). Bell & Mehta investigated, quantitatively, the role of these streamwise structures in a mixing layer at high Reynolds numbers ($Re_\delta \sim 2.5 \times 10^4$). They found that small spanwise disturbances in the flow were initially amplified just downstream of the first spanwise roll-up (in the braid region), leading to the formation of streamwise vortices. The vortices first appeared in clusters containing streamwise vorticity of both signs, but further downstream, they re-organized to form counter-rotating pairs. The vortex structure was then found to grow in size, scaling approximately with the mixing layer vorticity thickness, and to weaken, the maximum mean streamwise vorticity diffusing as approximately $1/X^{1.5}$. The mixing layer appeared to be nominally two-dimensional in the far-field, "self-similar" region.

It is also well-known that an inviscid instability occurs in any curved flow where the angular momentum decreases away from the center of curvature (Bradshaw 1973). A mixing layer with curvature in the streamwise direction is subject to a destabilizing effect if the low-speed stream is on the outside of the curve (at larger radius of curvature) and a stabilizing effect if it is on the inside of the curve. The subject of streamwise curvature effects on mixing layer structure has received limited attention (see Plesniak & Johnston 1989, for a review). Plesniak & Johnston's results suggested that the Taylor-Görtler instability mechanism, present in the unstable case, acts to strengthen the streamwise vortical structures. Dramatic increases in the primary Reynolds shear stress and enhanced transport of turbulence were documented in the unstable case. Thus, the streamwise structures are believed to play an integral role in the generation and evolution of turbulence in two-stream mixing layers. In a recent study, Plesniak *et al.* (1991) found that although well-organized streamwise vorticity was generated in both the stable and unstable cases, the rate of decay for the unstable case was slower. As a consequence of that, the unstable layer exhibited noticeable spanwise variations in the mean velocity and Reynolds stresses. Both cases achieved linear growth, but the rate of growth for the unstable case was higher. The far-field spanwise-averaged peak Reynolds stresses were significantly higher for the destabilized case compared to the stabilized case, which exhibited levels comparable to those of a straight case.

The main objective of the present study was to establish the effects of changes in initial conditions (laminar or turbulent state of the boundary layers) on the three-dimensional structure of curved two-stream mixing layers. These effects were investigated through measurements of the mean and turbulence properties on large cross-plane grids at several streamwise locations.

EXPERIMENTAL PROCEDURE

The experiments were conducted in a *Mixing Layer Wind Tunnel* consisting of two separate independently driven legs. Two air streams merge at the sharp trailing edge of a slowly tapering splitter plate. The curved test section (Fig. 1) has a fixed radius of 305 cm, giving δ/R of less than 5%, and measures 36 cm in the cross-

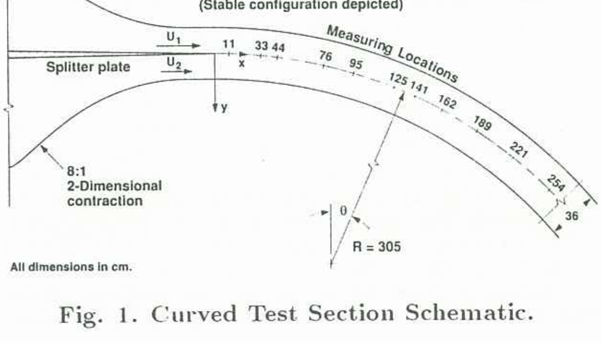

Fig. 1. Curved Test Section Schematic.

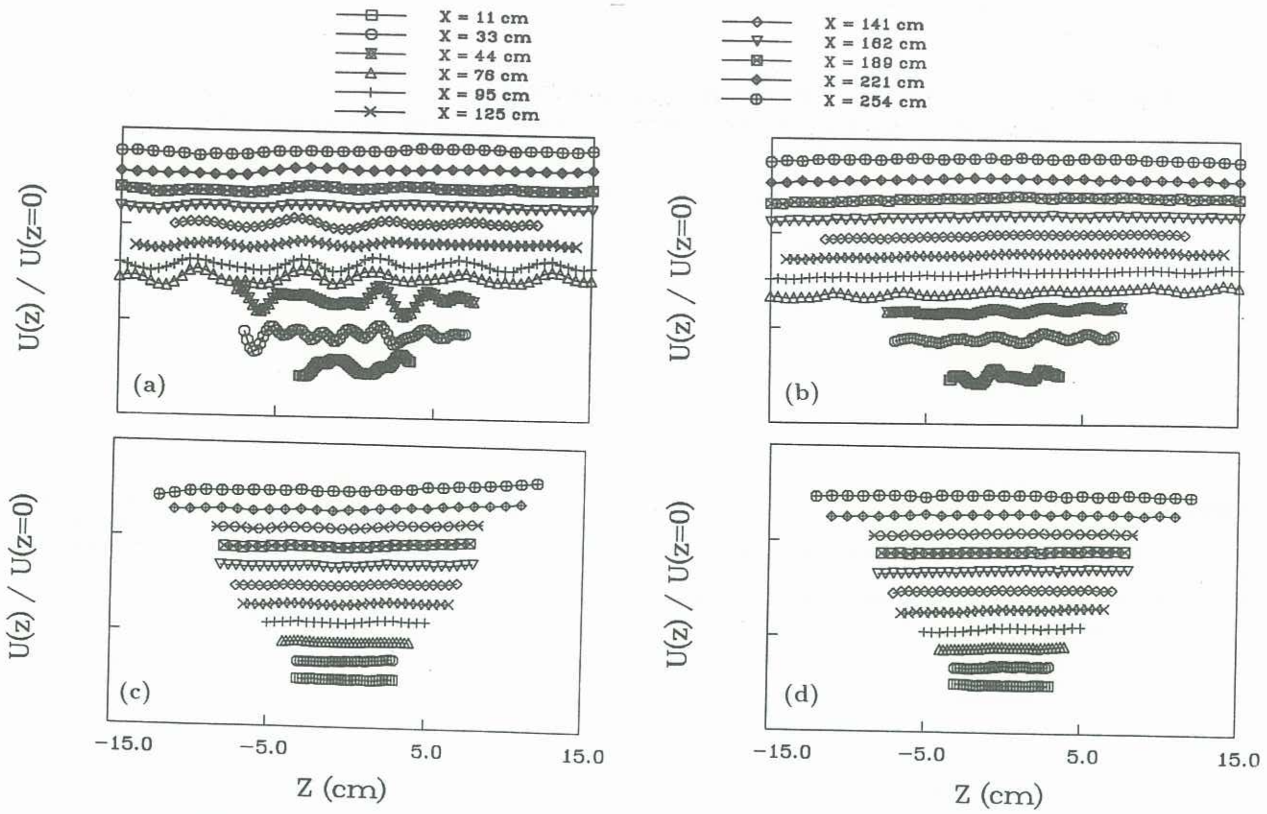

Fig. 2. Spanwise Variation of Streamwise Velocity. (a) Untripped Unstable, (b) Untripped Stable, (c) Tripped Unstable, (d) Tripped Stable.

stream (Y) direction, 91 cm in the spanwise (Z) direction and 366 cm in length (X, measured along the centerline arc). One side-wall is adjustable, and it was set to give a nominally zero streamwise pressure gradient.

The two sides of the wind tunnel were run at free-stream velocities of 9 and 15 m/s, thus giving a mixing layer with initial velocity ratio, $U_2/U_1 = 0.6$. Note that in these curved mixing layers, the local "velocity ratio" is given by the moment of momentum ratio, $(RU)_2/(RU)_1$, and varies slightly from case to case. The high-and low-speed sides were interchanged to produce the stabilizing and destabilizing cases. To generate the turbulent initial boundary layers, 1.5 mm diameter rods (trips) were glued across the span of the splitter plate, 15 cm upstream of the trailing edge.

Measurements were made using a cross-wire probe mounted on a 3-D traverse and linked to a fully automated data acquisition and reduction system controlled by a MicroVax II computer. Individual statistics were averaged over 5,000 samples obtained at a rate of 1,500 samples per second. Data were obtained in cross-sectional (Y-Z) planes with the probe oriented in the uv - and uw-planes, at eleven streamwise locations.

RESULTS AND DISCUSSION

The spanwise variation of the streamwise component of mean velocity (U) is plotted in Figs. 2a-b for the untripped curved mixing layers. The distance between tick marks on the vertical scale is equivalent to a variation of 25%. The "wiggles" in the streamwise velocity are due to the presence of spatially-stationary streamwise vortices (Bell & Mehta 1992). In the unstable mixing layer, the spanwise variation appears only quasi-periodic at first, but by $X = 76$ cm, a more regular and periodic variation is exhibited. The spanwise variation in U (peak-to-peak) is 13% at the first station, increasing to about 15% at X = 33-44 cm, after which it decreases, but still exhibits a variation of about 5% at the last two stations. At the first station ($X = 11$ cm), the stable mixing layer exhibits somewhat lower ($\sim 8\%$) spanwise variation in U compared to the unstable mixing layer. However, the wiggle magnitude decreases almost monotonically further downstream, giving a smaller variation in U. All of the stations downstream of $X = 125$ cm exhibit nearly constant distributions of U across the stable mixing layer span. In both untripped cases, the spanwise wavelength of the wiggles increases with increasing streamwise distance, although, at a given station, the wavelength is higher for the stable case implying that the streamwise vortical structures for this case are larger. The increasing wavelength is consistent with the notion that the spacing between pairs of counter-rotating streamwise vortices scales approximately with the mixing layer thickness (Bernal & Roshko 1986; Bell & Mehta 1992). The spanwise variation of U for both tripped cases is presented in Figs. 2c-d. It is clearly evident that the variation for both cases, at all streamwise locations, is minimal. This implies that *spatially-stationary* vorticity is *not* produced for the tripped cases, and furthermore that, the imposition of curvature does not change the mixing layer structure, at least in terms of

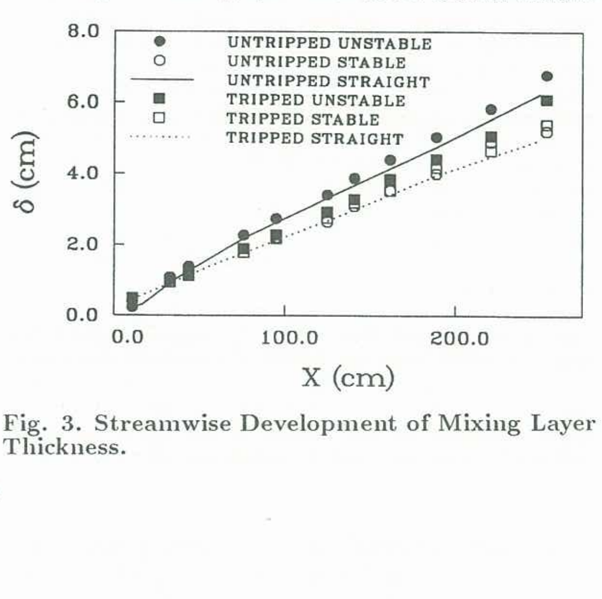

Fig. 3. Streamwise Development of Mixing Layer Thickness.

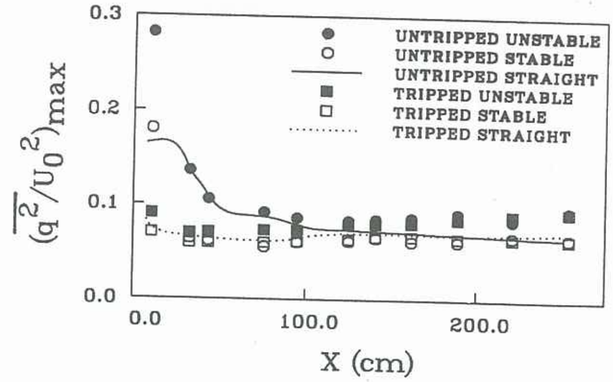

Fig. 4. **Streamwise Development of Maximum Turbulent Kinetic Energy.**

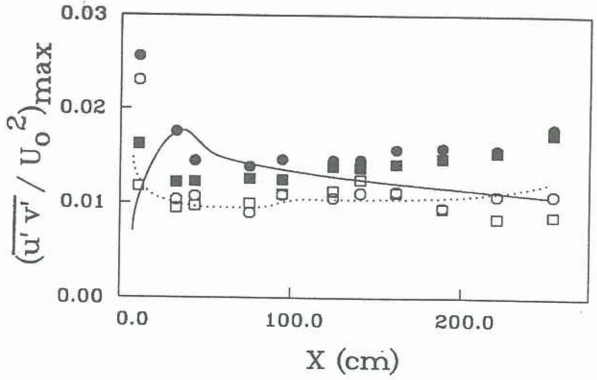

Fig. 5. **Streamwise Development of Maximum Primary Shear Stress.**

producing Taylor-Görtler type streamwise vorticity. Although this is true for the present study of mild curvature, it does not necessarily mean that large-scale roll cells will not be generated for stronger curvature.

Figure 3 shows the streamwise development of the mixing layer thickness (determined by an error function fit to the U data) for the stable, unstable and straight cases with untripped and tripped initial conditions. In all the streamwise development plots, the quantities are averaged across the measurement span. All the mixing layers exhibit linear growth downstream of $X = 76$ cm. A linear least-squares fit to the data downstream of $X = 76$ cm yields a growth rate, $d\delta/dx$ of 0.020 for the untripped stable layer, 0.023 for the untripped straight layer and 0.025 for the untripped unstable layer. Thus, the stable layer growth rate is 13% lower than the straight layer value, while the unstable layer grows at a rate 9% higher than the straight layer, and 25% higher than the stable mixing layer. Note that the stable untripped mixing layer is initially thickest. However, at all locations downstream of $X = 44$ cm, the unstable layer becomes significantly thicker than both the stable and straight layers. The growth rates of δ for the tripped mixing layers are 0.020 for the stable case, 0.019 for the straight case, and 0.023 for the unstable case. Hence, the tripped unstable and straight layers, which do not possess organized streamwise vorticity to provide additional entrainment, do not grow as rapidly as their untripped counterparts. However, the curvature effects cause the unstable tripped mixing layer to grow at a rate about 15% greater than that of the stable tripped mixing layer.

The streamwise evolution of the peak Reynolds normal stress components is shown in Fig. 4 in terms of twice the turbulent kinetic energy ($\overline{q^2} = \overline{u'^2} + \overline{v'^2} + \overline{w'^2}$). All the stresses presented here are normalized by the velocity difference across the layer, U_0. Not surprisingly, the trends for the two different initial conditions are very different, with the untripped cases showing the characteristic overshoot in the very near-field which is associated with transition in the mixing layer. Downstream of approximately 100 cm, the tripped and untripped data pairs agree well, for both, the straight and curved cases. The stable and straight cases asymptote to approximately the same value of approximately 0.70. In contrast, the turbulent kinetic energy levels in the unstable cases, which consistently lie above the straight and stable curves, continue to grow monotonically with downstream distance. At the most downstream stations, where the streamwise vorticity is quite weak (even in the unstable cases), the continued evolution of the turbulent kinetic energy is due to the destabilizing curvature, and indicates that an asymptotic "self-similar" state is not established.

Figure 5 illustrates the streamwise evolution of the maximum primary Reynolds shear stress ($\overline{u'v'}$). At the first station, the maximum shear stress levels for both untripped curved cases are higher than those for the straight case. Furthermore, the peak stress levels in the curved cases decrease monotonically with streamwise distance until an apparent asymptote is reached, whereas the straight case has a distinct maximum at $X \sim 40$ cm. Beyond the first station, the shear stress levels for the (tripped and untripped) unstable cases are consistently higher than those for the stable cases. Again, not much effect of tripping is apparent downstream of $X \sim 50$ cm. Also, a definite trend is exhibited in the primary shear stress plots beyond $X = 150$ cm. In the unstable case, these levels increase monotonically, and decrease in the stable case, especially for the tripped case. Again, this is an effect due to curvature, and not due to the streamwise vorticity, present only weakly at this downstream location in the untripped cases, and not at all in the tripped cases.

The effects of curvature and initial conditions are clearly apparent relatively early in the mixing layer development. This is true for both the mean flow, in terms of the streamwise vorticity, and also for the Reynolds stress distributions. In order to further investigate the curved mixing layer structure, velocity spectra for all three components were measured at several locations. The streamwise velocity (u) component spectra, measured on the mixing layer centerline and at $X = 11$ cm, for all four curved cases are presented in Fig. 6a. Only the untripped unstable case exhibits the characteristic fundamental peak associated with the Kelvin-Helmholtz vortex roll-up (at $f \sim 600$ Hz) and some higher harmonics. Similar trends are exhibited in the v- and w- velocity spectra, which are not presented here for brevity. A more broad band distribution is observed in the stable untripped case, quite similar to the spectral distributions for the two tripped cases, which are almost identical. Thus, for the tripped cases, there does not seem to be much effect of the curvature (stabilizing versus destabilizing) on the spectral content of the mixing layer, at least in the very near-field. However, the untripped cases are affected significantly, even at this early stage of their development. Note that the differences are not due to changes in initial conditions caused by interchanging the high- and low-sped sides the spectra for the two straight cases (with initial conditions corresponding to the stable and unstable cases) both exhibited the fundamental and higher-order peaks (Plesniak *et al.* 1992). The implication of the spectral results is that the stable case generates more small scales in the near-field, making it comparable to the tripped cases where the smaller scales are injected from the turbulent boundary layers. This is despite the fact that the streamwise vortical structures in the stable case are *initially* of a larger scale than those in the unstable case.

In the far-field region ($X = 221$ cm), the spectra for the four curved cases appear to have the same overall shapes (Fig. 6b). However, a closer examination reveals that in the region, $f > 100$ Hz, the stable and unstable pairs collapse very well and that the unstable cases have more energy, implying that they have developed more fine scales, in contrast to the near-field behavior. It is clear

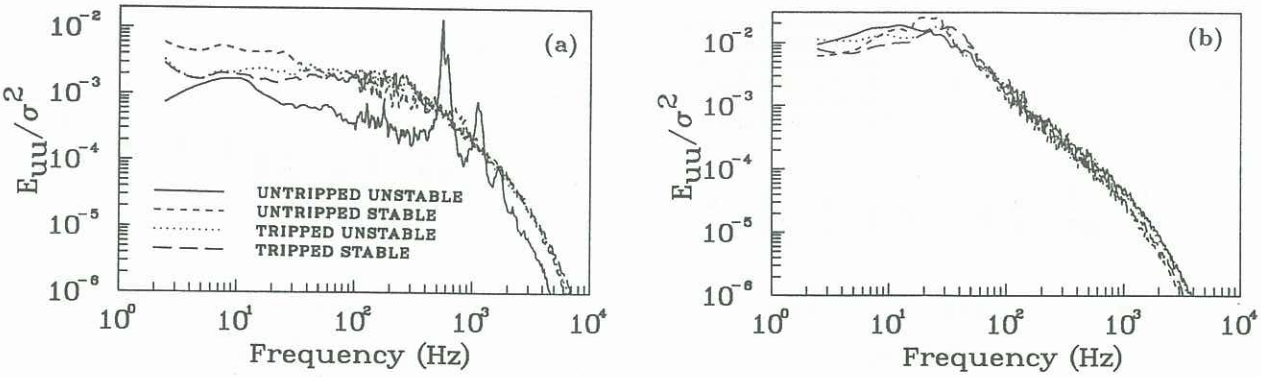

Fig. 6. Spectral Measurements on Mixing Layer Centerline. (a) X = 11 cm, (b) X = 221 cm

that the turbulence structure for the two curved cases is quite different in the far-field region, both in terms of the Reynolds stress levels and spectral content, and that the differences are independent of initial conditions.

CONCLUSIONS

The effects of the state of the initial boundary layers on the development of curved two-stream mixing layers were investigated. Destabilizing curvature retarded the decay of the secondary streamwise vorticity (naturally occurring in mixing layers originating from laminar initial conditions), resulting in a higher growth rate than exhibited by a straight mixing layer. Stabilizing curvature accelerated the decay of the naturally-occurring streamwise vorticity, and resulted in a lower growth rate than a straight mixing layer. The growth rate of the unstable mixing layer was about 9% higher than that of the straight layer, while that of the stable layer was about 13% lower. In the curved mixing layers originating from turbulent initial boundary layers, spatially stationary streamwise vortical structures were not evident. Therefore the tripped unstable mixing layer exhibited a lower growth rate than that in the corresponding untripped case. However, the tripped unstable case growth rate was still about 15% higher than the corresponding straight case while that of the stable case was not affected by the change in initial conditions. Both stable case growth rates were comparable to that of the straight tripped layer.

The Reynolds stresses in the unstable mixing layer were elevated with respect to those in the straight layer, and by about the same amount for both initial conditions. For the stable case, the Reynolds stress levels were comparable to, or slightly lower than, those for the straight mixing layers. There is some evidence that the stress levels continue to change towards the end of the measurement domain, especially for the unstable cases. The spectral data show that, in the near-field, only the untripped cases are affected by curvature, presumably through effects on the streamwise vortical structure behavior. In the far-field, the spectral data show that the effects of initial conditions are minimal, but that the turbulence structure of the unstable cases contains more fine scales.

The primary effects of unstable curvature are to increase the layer growth rate, Reynolds stress levels and spectral energy content at the higher frequencies. For the untripped cases, the secondary effect of curvature on the streamwise vorticity decay rate also plays a role, at least in the near-field region. In the far-field region, where the vorticity has decayed, the behavior is essentially independent of the initial conditions. Therefore, the implication is that the observed differences between the two curved cases in the far-field region are due to the direct effects of curvature, rather than through the secondary effects on the behavior of the streamwise vortical structures. Since the curvature is applied continuously, the curved mixing layers should *not* be expected to reach an equilibrium state, and that certainly appears to be the case in the present study.

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