11th Australasian Fluid Mechanics Conference
University of Tasmania, Hobart, Australia
14-18 December 1992

ANALYSIS OF SINGLE-COMPONENT TWO-PHASE CRITICAL FLOW THROUGH A CONVERGING NOZZLE : APPLICATION OF A THREE LAYER MODEL

Junji OCHI, Kyozo AYUKAWA, Genta KAWAHARA and Takahiro UMEZAKI

Department of Mechanical Engineering
Ehime University
Matsuyama 790, JAPAN

ABSTRACT

The two-phase critical flow with phase change through a converging nozzle is investigated and discussed on the applications of three layer model analysis reported by Ochi and Ayukawa(1991) in a case of two component two-phase flow.

In the theoretical analysis on single component two-phase flow in rapid phase change, there arose some difficulties from the modeling of a complicated flow pattern with an entrainment of liquid droplets and submerged gas bubbles at the interface, caused by the large acceleration of fluids and larger velocity difference between gas and liquid phases towards the downstream direction.

To overcome these difficulties, three layer model based on the assumption that a flow consists of three layers with a mixing region between gas and liquid phase layers, is applied to the single component mixtures through the converging nozzle in critical conditions.

The calculated results using the model are compared with the experiments for the steam-water flows on the critical flow rates and the pressures, and are discussed in comparison with the complete separated flow or the homogeneous flow model.

NOMENCLATURES

A	:	cross sectional area
h	:	enthalpy
p	:	pressure
T	:	temperature
u	:	velocity
W	:	mass flow rate
x	:	quality
x_m	:	mixing ratio
ρ	:	density

subscript

f	:	liquid
g	:	gas
m	:	mixture
mf	:	liquid in mixing layer
mg	:	gas in mixing layer
o	:	nozzle inlet
t	:	nozzle throat

INTRODUCTION

Gas and liquid two-phase flows are usually seen in many industrial applications. The two-phase flows through channels take various flow patterns under the flow conditions of the fluids such as physical fluid properties, flow rate of the gas or the liquid and the flow direction. In particular, the single-component two-phase flow in high speed have very complicated flow pattern because of its phase change and acceleration.

The accurate estimation of flow characteristics through a channel is required in the analysis of the two-phase critical flow for a flashing of pressurized water, a loss of coolant from nuclear reactors and transport of refrigerants or cryogens. However, it is difficult to execute the theoretical analysis exactly. In a two-phase critical flow with rapid phase change through a converging nozzle, the velocity of each phase accelerates towards down stream direction, and, since the phases have different densities, the pressure gradient tends to accelerate the lighter vapor phase more than the liquid and cause large velocity differences between the gas and the liquid, and it eventually results in the flow of a complicated two-phase mixture near the interface by the entrainment of droplets and bubbles.

Many previous analyses on two-phase flow have been based on a homogeneous or a complete separated flow model in the basic equations. But these models are not sufficient for the analysis of actual two-phase flows. Especially for the estimation of the flow rate under critical conditions, the analytical results from the above models revealed some deviations from the experimental data.

The three layer model was developed with the supposition that the entrainment of droplets caused by shearing force at the interface plays an important role in the flow pattern through a converging nozzle. The mixing layer composed of the vapor bubbles and the liquid droplets is assumed instead of the interface between a vapor and a liquid layer.

BASIC EQUATIONS IN THREE LAYER MODEL

The flow of three layer model through a converging nozzle is illustrated in Fig.1. The mixing layer between the vapor and the liquid phase layers as seen in this figure is composed of liquid droplets, the vapor and submerged

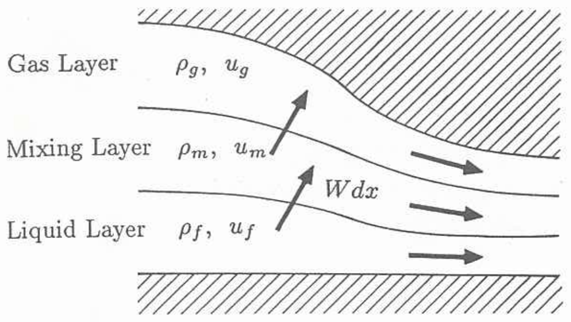

Fig.1 Three Layer Flow Model

bubbles. The assumptions on a single-component two-phase flow in this analysis are

1. The fluid of each layer is assumed to be in a state of thermodynamical equilibrium and saturated.
2. The fluid of each layer flows through the channel without shear stress or heat exchange with the wall.
3. The gravitational effect can be neglected.
4. The velocity of the fluid in each layer is uniform respectively.
5. In the mixing layer, the vapor and the liquid phases are assumed to be in equilibrium and homogeneous with equal velocities and temperatures.
6. The density of liquid is invariant through the channels.

The continuity equation is written from the volumetric change of each layer.

$$\left(\frac{1-x_{mg}}{\rho_g u_g}+\frac{x_{mg}}{\rho_g u_m}-\frac{1}{\rho_f u_f}\right)W dx-\frac{A_g+A_{mg}}{\rho_g}\frac{\partial \rho_g}{\partial p}dp$$
$$-\frac{A_g}{u_g}du_g-\frac{A_m}{u_m}du_m-\frac{A_f}{u_f}du_f=dA \qquad (1)$$

where the whole of the mass flow rate is shown that $W = W_g + W_m + W_f$, for the mixing layer $W_m = W_{mg} + W_{mf}$, and the quality x is usually defined as follows

$$x=\frac{W_g+W_{mg}}{W} \qquad (2)$$

and x_{mg} or x_{mf} is mixing ratio defined for the flow rate of gas and liquid in this layer divided by the whole gas or the liquid phase flow rate in a channel. That is

$$x_{mg}=\frac{W_{mg}}{W_g+W_{mg}} \qquad (3)$$

$$x_{mf}=\frac{W_{mf}}{W_f+W_{mf}} \qquad (4)$$

The momentum equation is expressed for a whole of the flow through a channel.

$$\{(1-x_{mg})u_g+x_{mg}u_m-u_f\}\,W dx$$
$$+W_g\,du_g+W_m\,du_m+W_f\,du_f+A\,dp=0 \qquad (5)$$

For the vapor or the mixing layer, the momentum equation is written respectively,

$$E(1-x_{mg})(u_g-u_m)W dx+A_g dp+W_g du_g=0 \qquad (6)$$

$$[(1-E)(1-x_{mg})u_g+\{E-(1-E)(1-x_{mg})\}\,u_m$$
$$-Eu_f]\,W dx+W_m du_m+A_m dp=0 \qquad (7)$$

where E is the coefficient which takes unity in evaporating flow $(dx > 0)$ or takes zero in condensating flow $(dx < 0)$.

The energy equation is shown as follows.

$$\{(1-x_{mg})(h_g+\frac{u_g^2}{2})+x_{mg}(h_m+\frac{u_m^2}{2})-(h_f+\frac{u_f^2}{2})\}W dx$$
$$+\left\{(W_g+W_{mg})\frac{\partial h_g}{\partial p}+(W_{mf}+W_f)\frac{\partial h_f}{\partial p}\right\}dp$$
$$+W_g u_g du_g+W_m u_m du_m+W_f u_f du_f=0 \qquad (8)$$

The critical flow is achieved as a local mathematical singularity, that is, the critical condition for three layer model is occurs when the determinant of the coefficients of the variables dx, dp, du_g, du_m, du_f from the above simultaneous equations is zero at the nozzle throat($dA = 0$), where the numerator that is divided by this determinant can also be arranged to be zero. It is the same method as Katto's principle(1968,1969) for two-phase critical flow.

The critical condition for this proposed model is written as

$$\begin{vmatrix}
\left(\frac{1-x_{mg}}{\rho_g u_g}+\frac{x_{mg}}{\rho_g u_m}-\frac{1}{\rho_f u_f}\right)W & -\left(\frac{A_g+A_{mg}}{\rho_g}\right)\frac{\partial \rho_g}{\partial p} & -\frac{A_g}{u_g} & -\frac{A_m}{u_m} & -\frac{A_f}{u_f} \\
\{(1-x_{mg})u_g+x_{mg}u_m-u_f\}\,W & A & W_g & W_m & W_f \\
E(1-x_{mg})(u_g-u_m)W & A_g & W_g & 0 & 0 \\
[(1-E)(1-x_{mg})u_g+\{E-(1-E)(1-x_{mg})\}\,u_m-Eu_f]\,W & A_m & 0 & W_m & 0 \\
\left\{h_g-h_f+(1-x_{mg})\frac{u_g^2}{2}+x_{mg}\frac{u_m^2}{2}-\frac{u_f^2}{2}\right\}W & (W_g+W_{mg})\frac{\partial h_g}{\partial p}+(W_f+W_{mf})\frac{\partial h_f}{\partial p} & W_g u_g & W_m u_m & W_f u_f
\end{vmatrix}=0 \qquad (9)$$

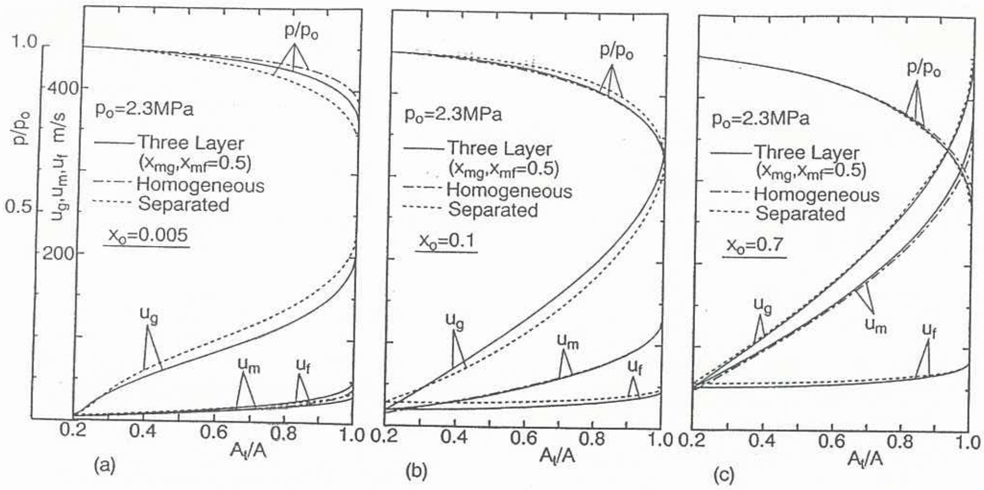

Fig.2 Typical Profiles of Pressure and Velocities in a Converging Nozzle

RESULTS AND DISCUSSIONS

The characteristics on two-phase flow approximated three layers was investigated for a steam-water flow through a converging nozzle. The initial values for pressure or temperature, flow rate, quality and mixing ratios should be given to solve these equations. At the critical condition, the flow rate must be chosen as equation(9) is satisfied for the quantities at the throat. In the numerical calculation from above equations, the mixing ratio x_{mg} and the flow rate of liquid in mixing layer is assumed to be constant in flowing through the nozzle. The properties such as thermodynamic quality can be obtained from Steam Tables.

Critical Flow through a Converging Nozzle

The change of pressure and velocity in each layer are illustrated for steam-water critical flows at typical qualities in Fig.2-a,b,c. The lateral coordinate is presented by A_t/A instead of the position of the flow direction in the channel as example that the location at the inlet is shown $A_t/A = 0.2$ and 1.0 at the throat. While flowing through the converging nozzle, the velocity of fluid in each layer is increased and the pressure is decreased. Their changes tend to be steeper closer to the throat. This tendency is remarkable in higher qualities.

In single component two-phase flow, the pressure change varies the quality of the flow due to the vaporization or the condensation in the channel. The rate of the change of the quality from inlet to throat are shown in Fig.3 for some mixing ratios over wide range of inlet qualities. From the figure it is found that the phase changes are greater at lower quality and lower mixing ratio.

The velocities at the throat in each layer are presented in Fig.4. It is found that each of these velocities is increased at higher quality. In a two component two-phase flow without phase change, the velocity in gas layer was conversely increased at low quality region. However, it is considered that the tendency towards the acceleration of vapor phase in the one component mixture is suppressed by the force due to momentum exchange by intensive vaporization in lower quality flows as shown in Fig.3.

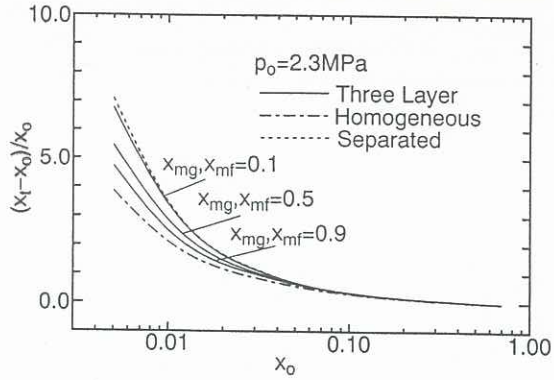

Fig.3 The Rate of Change of the Quality from Inlet to Throat

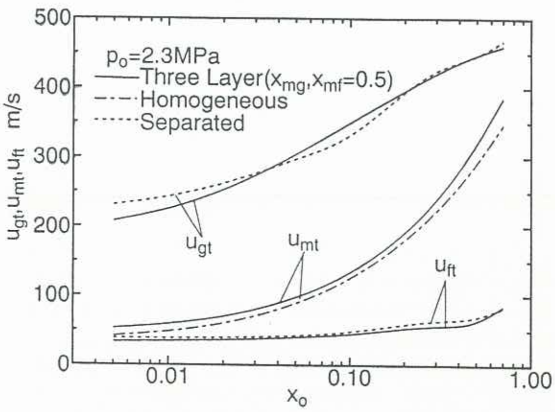

Fig.4 Velocities at the Throat under Critical Conditions

The effect of mixing ratios x_{mg} and x_{mf} on the critical flow rate is shown in Fig.5. Reducing the value of either x_{mg} or x_{mf}, that is, narrowing the width of mixing layer in the channel, the critical flow rate increases and approaches the results of completely separated flow model. The upper limit of critical flow rate for the three layer model is equivalent to a completely separated flow; that is, $x_{mg} = 0, x_{mf} = 0$ and the lower limit is equivalent to a homogeneous flow for $x_{mg} = 1, x_{mf} = 1$.

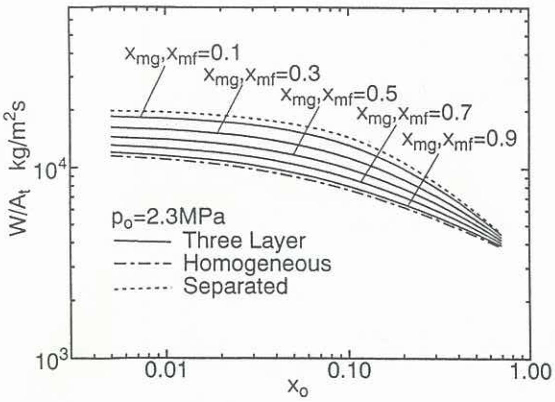

Fig.5 Effect of Mixing Ratio on Critical Flow Rate

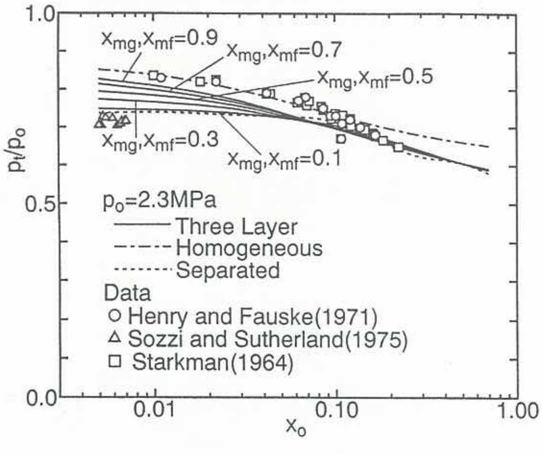

Fig.6 Calculated Critical Pressure versus Experimental Data

Comparison with Experimental Results

The critical pressures and critical flow rates obtained from the proposed model is compared with the experimental data of Henry and Fauske (1971), Starkman et al.(1964), Sozzi and Sutherland(1975) and Hutcherson (in Wallis, 1980) in Fig.6 and 7 for inlet pressures $p_o = 1.38-3.45Mpa$. The effects of the value of mixing ratio on critical pressure are slightly found in this calculation at low quality region. In a higher quality than about $x_o = 0.1$, the critical pressures are independent of the mixing ratios and gradually approach a complete separated flow.

The calculations of the critical flow rate presented in a case of $x_{mg} = 0.5, x_{mf} = 0.5$ have a good agreement with the experimental data. The mixing ratios for these flows with phase change was estimated by considering the extent of spreading of mixing phases based upon our experiments conducted with air-water two-phase flows. It is observed in the low quality region that the experimental results are larger than the calculated results from this model. The reason for the increase in this region is considered that the interfacial phase change in actual flow is not sufficient between the phases and results in a state of thermodynamic nonequilibrium.

CONCLUSIONS

The three layer model is developed for the two-phase critical flow of single-component mixtures through converging nozzle. The main results obtained here are summarized as follows.

(1) It is found that the three layer model is to be considerably an useful method for single-component two-phase flow with rapid change of the state in variable area channels.

(2) The flow rate under critical conditions calculated from three layer model are shown in good agreement with the experimental data of steam-water flow over a wide range of the quality except the flow in nonequilibrium state at low quality.

(3) The vapor velocities at the throat rise with increasing quality and the critical pressures correspondingly decrease in contrast to the tendency for two component two-phase flow.

(4) Intensive phase change appears in lower quality and lower mixing ratio.

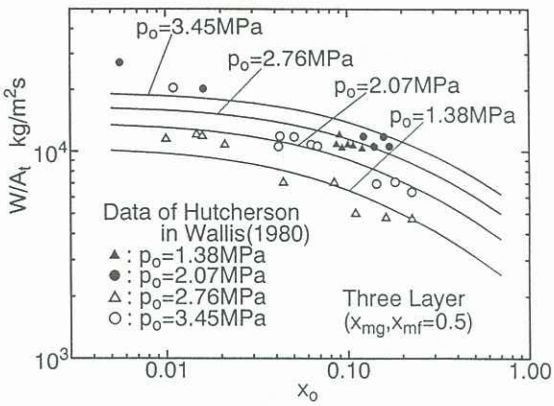

Fig.7 Comparison of the Calculated Critical Flow Rate with Experimental Data

REFERENCES

HENRY,R.J., FAUSKE,H.K., (1971), "The Two-Phase Critical Flow of One-Component Mixtures in Nozzles, Orifice, and Short Tubes," Trans. ASME, J.Heat Transfer 95, 179-187

KATTO,Y., (1968), "Dynamics of Compressible Saturated Two-Phase Flow (Critical Flow)," Bull. JSME, 11-48, 1135

KATTO,Y., (1969), "Dynamics of Compressible Saturated Two-Phase Flow (Critical Flow-sequel, and Flow in a Pipe)," Bull. JSME, 12-54, 1417

OCHI,J., AYUKAWA,K., (1991), "Three Layer Model Analysis on Two-Phase Critical Flow through a Converging Nozzle: In a Case of Two-Component Two-Phase Flow" Proc. The ASME/JSME Thermal Engineering Joint Conference, 2, 249-256.

SOZZI,G.L., SUTHERLAND,W.A., (1975), "Critical Flow of Saturated and Subcooled Water at High Pressure," ASME Non-Equilibrium Two-Phase Flow Symp., 47-54

STARKMAN,E.S., SCHROCK,V.E., NEUSEN,K.F., MANEELY, D.J., (1964), "Expansion of a Very Low Quality Two-Phase Fluid through a Convergent-Divergent Nozzle", Trans. ASME J.Basic Engineering 83, 247-256.

WALLIS,G.B., (1980), "Critical Two-Phase Flow," Int. J. Multiphase Flow, 6, 97-112.