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EFFECTS OF TURBULENCE ON THE SETTLING OR RISE VELOCITY OF ISOLATED SUSPENDED PARTICLES

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ABSTRACT

The effect of turbulence on the settling or rising velocity of single particles is discussed in terms of theory and the available experimental data. The coherent eddy structures of turbulence have a strong delaying effect on bubbles which have a tendency to be trapped inside steady vortices. The vortex trapping mechanism is less efficient for heavy particles because they tend to escape due to the centrifugal effect. Furthermore, heavy particles in arrays of steady vortices tend to get fast tracked along certain paths (Maxey & Corrsin 1986). Grid turbulence experiments indicate that heavy particles are delayed by relatively weak turbulence while strong turbulence tends to increase the settling velocity.

INTRODUCTION

The behaviour of particles suspended in a turbulent fluid is of concern to scientists and engineers in a variety of fields such as meteorology, oceanography, chemical reactions, sewage treatment and sediment transport. The two main questions for all of them are: For how long will the particles stay in suspension, and how fast will a cloud of suspended particles spread? The first question which concerns the average settling/rising velocity $\overline{w_p}$ has received far less attention than the question about the dispersion rate.

There are at least three existing theories which predict an effect of turbulence on the average settling/rise velocity of non-neutral, isolated suspended particles. The trouble is that while two of these predict a reduction, the third predicts an increase, and their relative importance in real turbulence of various types is incompletely defined by the sparse experimental data.

Settling velocity reduction may occur as a result of non-linear drag for fairly large particles and as a result of vortex trapping for particles which are small in the sense that their still water settling velocity w_o is smaller than the typical turbulent velocities. An increase of the settling velocity was described by Maxey & Corrsin (1986) for particles settling through a periodic array of steady vortices. The heavy particles tend to congregate along fast tracs along the vortex boundaries, and this results in an increase of the average settling velocity. The same effect was also found by Maxey (1987) in a less special, simulated flow field.

EXPERIMENTAL DATA

Few experiments have been reported on settling or rise of single particles through turbulence. Murray (1970) monitored the settling of particles with still water settling velocities *0.2, 1.0, 2.0, 3.0* and *4.0cm/s* through turbulence generated by horizontal bar grids spaced *30cm* apart and oscillating in their own plane. His results in terms of the relative settling velocity $\overline{w_p}/w_o$ versus the relative turbulence intensity σ/w_o are shown in Figure 1 together with results from the present study. For Murray's experiments, the turbulence intensity σ was taken as the standard deviation of the vertical velocity of a particle along its path.

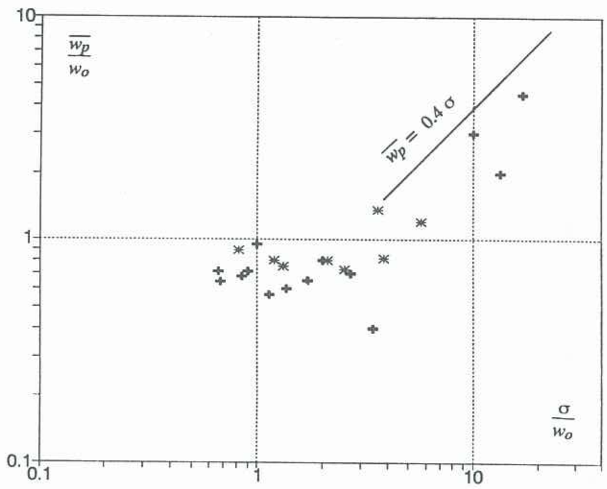

Figure 1: Relative settling velocity versus relative turbulence intensity. Legend, + : Murray (1970), * : present study.

The data indicate that heavy particles tend to be slowed down by 20 to 40 percent in relatively weak grid turbulence while the settling velocity may be considerably increased in stronger turbulence.

In the present study the settling of two types of particles was monitored in a tank with *50cm* sqare base and a height of *70cm*. The turbulence was generated by two connected horizontal grids spaced *50cm* apart, see Figure 2.

The grids were oscillating vertically at frequencies between *0.34* and *1.64Hz* with a fixed excursion of *5cm*. The settling velocity in each drop was determined from still camera photographs taken with a known time difference of the order four seconds. Two syncronised cameras, looking from perpendicular directions, were used in order to obtain a complete determination of the particle positions in three dimensions. For these experiments the nominal turbulence intensity used in Figure 1 was taken as the grid velocity amplitude. A summary of the experimental results are given in Table 1.

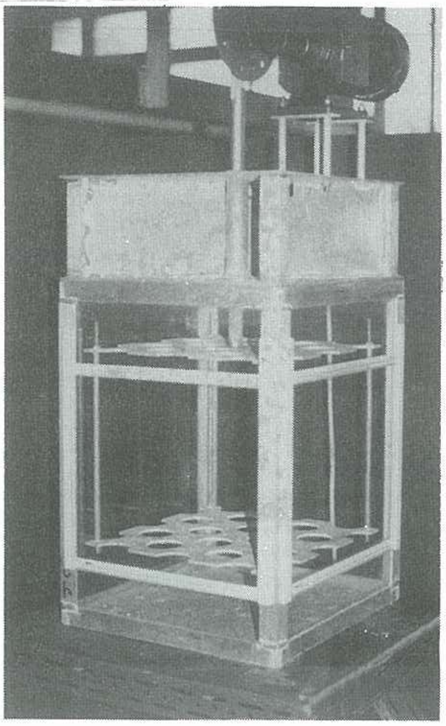

Figure 2: The turbulence tank used in the experiments of the present study.

Grid freq [Hz]	Turb int, σ [cm/s]	Particle diam [mm]	Number of drops	w_o [cm/s]	Average $\overline{w_p}$ [cm/s]	St dev of $\overline{w_p}$ [cm/s]
0.365	5.7	2.0	17	7.0	6.2	0.7
0.582	9.1	2.0	14	7.0	5.3	1.4
1.13	17.7	2.0	8	7.0	5.1	1.1
1.62	25.4	2.0	10	7.0	9.5	2.3
0.342	5.3	6.1	28	4.5	3.6	0.7
0.586	9.2	6.1	20	4.5	3.6	1.2
1.10	17.3	6.1	23	4.5	3.7	2.2
1.64	25.8	6.1	29	4.5	5.4	4.0

Table 1: Summary of experimental conditions in the present study. In all experiments the grid excursion was *5.0cm* (vertically) and the nominal turbulence intensity σ is taken as the grid velocity amplitude.

Some less direct experimental evidence was provided by Jobson & Sayre (1970). They observed the average horizontal distance travelled along a flume by particles released at the water surface. Their data indicates a 4-5% increase of the settling velocity due to the stream turbulence. Volkart (1985) studied the behaviour of bubbles entrained in spillway flows and found evidence of delayed rise of the bubbles due to trapping in votices. He noted (p 6) "the bubbles actually remained in the flow for a period that exceeds the theoretical value by a factor of about 10" and the bubbles were observed to move in a spiralling motion.

NON-LINEAR DRAG

Non-linearity of the drag force can cause a delay of particle settling through turbulent water. Ho (1964) studied this phenomenon experimentally by measuring the settling velocity of heavy particles through a body of fluid which was shaken vertically as a whole. He also provided a numerical solution to this problem. Nielsen (1984) derived an analytical solution which showed that the settling velocity reduction due to this mechanism is approximately

$$|w_o - \overline{w_p}| \approx \frac{|w_o|}{16}\left(\frac{A_{max}}{g}\right)^2 \qquad (1)$$

where A_{max} stands for the maximum fluid acceleration.

For most practical situations, including the experiments reported above, the typical fluid accelerations are of the order $10^{-2}g$ or less. Under such conditions, the effect of non-linear drag is without practical importance.

VORTEX TRAPPING

It was shown experimentally by Tooby et al (1977) that forced vortices with horizontal axes can trap bubbles and heavy particles and thus eliminate their settling velocity completely.

This effect was subsequently studied in detail by Nielsen (1984) who showed that it is unrelated to non-linearity of the drag force but can be easily derived under the simple assumption of

$$u_p(x,z) = u(x,z) + w_o \qquad (2)$$

i e, that the particle velocity is equal to the local fluid velocity plus the still water settling velocity. Under this assumption, sediment particles in a forced vortex with angular velocity ω, can move indefinitely along any circle with centre at $(-w_o/\omega, 0)$. See Figure 3.

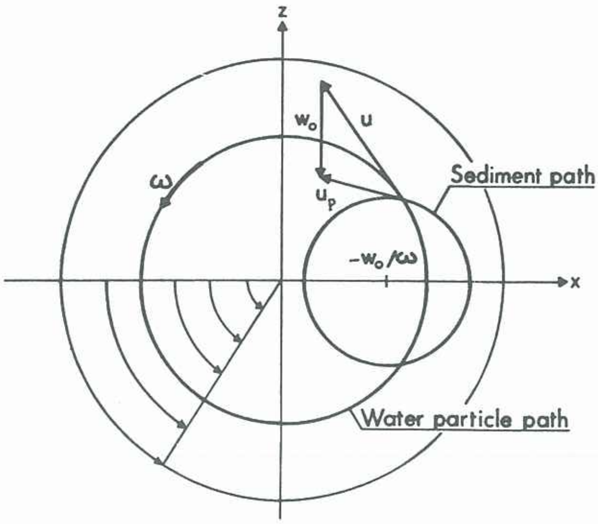

Figure 3: In a forced vortex with velocity field $\begin{bmatrix} u \\ w \end{bmatrix} = \omega \begin{bmatrix} -z \\ x \end{bmatrix}$ a non-neutral particle with settling velocity $\mathbf{w_o} = (0, w_o)$ can be trapped indefinitely on any circle around the point $(-w_o/\omega, 0)$ if its instantaneous velocity is given by Equation (2). The angular velocity of the orbiting particles or bubbles is ω.

The trapping effect is not peculiar to the somewhat artificial forced vortex. It is possible in most vortices including the Rankine vortex and the irrotational vortex see Nielsen (1984, 1992).

FAST TRACKING BETWEEN VORTICES

Inertia (or density) differences will cause particles to deviate from the closed orbits described above. Heavy particles will essentially spiral outwards and light particles to spiral inwards. The time scale of this spiralling process is $g/(w_o\,\omega^2)$, see Tooby et al (1977) and Nielsen (1984, 1992).

One surprising consequence of these inertial effects is the fast tracking which was discovered by Maxey & Corrsin (1986). They simulated the motion of heavy particles which were initially uniformly distributed in an array of steady vortices with the velociy field

$$\begin{pmatrix} u \\ w \end{pmatrix} = \begin{pmatrix} U_o \sin kx \cos kz \\ -U_o \cos kx \sin kz \end{pmatrix} \tag{3}$$

($k = \pi/D$). They found that small particles ($w_o \lesssim 0.5\, U_o$) tended to become concentrated along the s-shaped curves which follow the righthand edges of clockwise vortices and the lefthand edges of anticlockwise vortices see Figure 4.

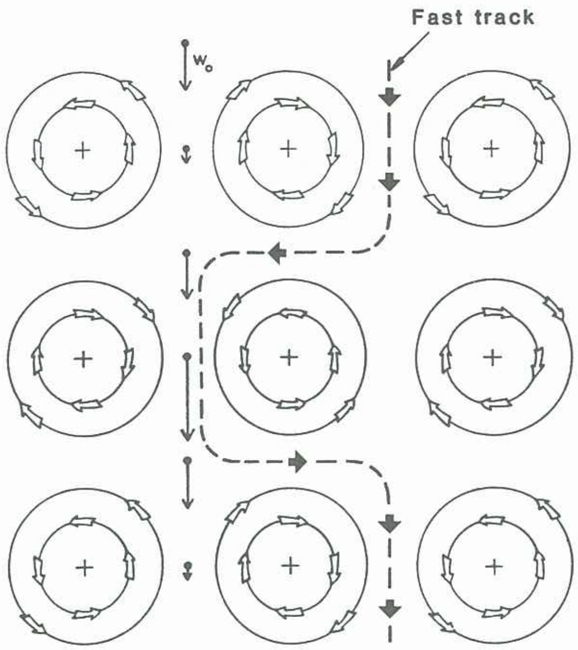

Figure 4: Small particles which settle through a velocity field of the form (3) will tend to be fast tracked. Larger particles will tend to be delayed like the particle which settles along the vertical line of symmetry. Bouyant particles will drift towards the vortex centres and may remain trapped for ever.

This fast tracking effect can lead to very large settling velocity increases for small ($w_o << U_o$) particles because the asymptotic mean velocity along a fast track is proportional to U_o . For example, consider a particle which follows the s-shaped curve past a pair of vortices with diameter D with average speed $0.8U_o$. It will be travelling horizontally for half the time and vertically for half the time. Hence its average vertical velocity is approximately $0.4U_o$. For turbulence with intensity σ it may thus be expected that the line $\overline{w_p} = 0.4\sigma$ is an upper bound for the asymptotic behaviour of small, heavy particles. This agrees reasonably with the experimental results in figure 1.

THE LOITERING EFFECT

If the partiles are too fast to be effectively guided along the s-shaped fast tracs in Figure 4 or if the vortices are too short lived, the effect of the vortices will tend to be a decrease rather than an increase of the settling velocity. Correspondingly, the effect of "structureless" turbulence will be a settling delay due to the loitering effect.

The essence of the loitering effect is that a particle which is settling or rising through a non-uniform velocity field will spend more time with fluid which moves opposite to its natural settling or rise velocity. It may be illustrated by the particle which settles along the vertical symmetry line in Figure 4.

To quantify the effect, assume for simplicity that the vertical fluid velocity along the symmetry line varies as

$$w(z) = w_o A \cos\frac{\pi z}{D} \tag{4}$$

which under the simplifying assumption (2) gives a particle velocity of

$$w_p = w_o \left(A \cos\frac{\pi z}{D} + 1\right) \tag{5}$$

Then, the time-averaged particle velocity over an integral number of vortices is reduced to

$$\overline{w_p(t)} = \begin{cases} w_o\sqrt{1-A^2} & \text{for } A < 1 \\ 0 & \text{for } A \geq 1 \end{cases} \tag{6}$$

and the variance is

$$Var\{w_p(t)\} = w_o^2\left[\sqrt{1-A^2} - 1 + A^2\right] \tag{7}$$

The particle velocity variance is thus always less than the spatial variance ($= \frac{1}{2}A^2 w_o^2$) of the fluid velocity along the path, and it vanishes for $A \to 1$, which corresponds to stagnation.

Some of the simulations by Maxey & Corrsin as well as the experimental data in Figure 1 indicate that the loitering effect is effective for fairly large particles $\sigma/w_o \lesssim 3$ while it becomes overshadowed by the fast tracking effect for $\sigma/w_o \gtrsim 5$. The loitering effect will affect bubbles as well as heavy particles while fast tracking will not.

The strength of the loitering effect will depend on various aspects of the turbulence structure as well as on the relative turbulence intensity σ/w_o. It is suggested however that a reasonable, initial quantification may be given in the form

$$\frac{\overline{w_p}}{w_o} = F\left(\frac{\sigma}{w_o}, A_E\right) \tag{8}$$

where $A_E = \sigma T_E/L_E$ measures the persistense of the coherent eddy structures. T_E is the Eulerian time scale and L_E is the Eulerian length scale of the turbulence.

RANDOM WALK WITH LOITERING EFFECT

At the simplistic, initial level, where no consideration is given to the structure of the eddies, it is possible to model the settling delay, due to the loitering effect, with a simple random walk model. An example of this is given in the following, and the model is very similar to the one used by Taylor (1921) for analysing the dispersion rate of a cloud of particles.

Taylor considered an Ornstein Uhlenbeck process for say the vertical component w_p of the particle velocity $u_p = (u_p, v_p, w_p)$, i e

$$w_{p,i+1} = \rho\, w_{p,i} + \sigma\sqrt{1-\rho^2}\,\chi \tag{9}$$

where χ is a random variate with zero mean and unit variance. The variance of $w_p(t)$ resulting from this process is σ^2 irrespective of the constant ρ. The value of ρ is the correlation between successive values of w_p, and in Taylor's model it is related to the Lagrangian integral scale and the time step by

$$\rho = e^{-\delta_t/T_L} \quad (10)$$

The following model is designed to predict the loitering effect for non-neutral particles. One central aspect of the loitering effect (Figure 4) is that small velocities lead to small changes of velocity, or in other words, the correlation $\rho(w_{p,i}, w_{p,i+1})$, is low if $w_{p,i}$ is large, high if $w_{p,i}$ is small.

In the random walk model, this may be obtained by replacing Taylor's correlation formula (10) with

$$\rho_i = e^{-\sqrt{[\frac{\delta_t}{T_E}]^2 + [\frac{u_i \delta_t}{L_E}]^2 + [\frac{v_i \delta_t}{L_E}]^2 + [\frac{(w_i+w_o)\delta_t}{L_E}]^2}}$$

$$= e^{-\frac{\delta_t}{T_E}\sqrt{1 + A_E^2\{[\frac{u_i}{\sigma}]^2 + [\frac{v_i}{\sigma}]^2 + [\frac{w_i+w_o}{\sigma}]^2\}}} \quad (11)$$

The expression (11) is equivalent to (10) under certain assumptions about the correlations between velocity components and spatial velocity derivatives (Nielsen, 1992). It is presently unknown however, to what extent these assumptions are valid.

The fluid velocity vector $u_{i+1} = (u_{i+1}, v_{i+1}, w_{i+1})$, in step $i+1$, is then determined by

$$u_{i+1} = \begin{pmatrix} u_{i+1} \\ v_{i+1} \\ w_{i+1} \end{pmatrix} = \begin{pmatrix} \rho_i u_i + \sigma\sqrt{1-\rho_i^2}\,\chi_u \\ \rho_i v_i + \sigma\sqrt{1-\rho_i^2}\,\chi_v \\ \rho_i w_i + \sigma\sqrt{1-\rho_i^2}\,\chi_w \end{pmatrix} \quad (12)$$

where χ_u, χ_v, χ_w are independent, normalised normal variates. In accordance with Equation (2), the particle velocity is subsequently found by simple superposition of the still water settling velocity

$$u_{p,i+1} = \begin{pmatrix} u_{p,i+1} \\ v_{p,i+1} \\ w_{p,i+1} \end{pmatrix} = \begin{pmatrix} u_{i+1} \\ v_{i+1} \\ w_{i+1} + w_o \end{pmatrix} \quad (13)$$

Results of simulations with this model are shown in Figure 5.

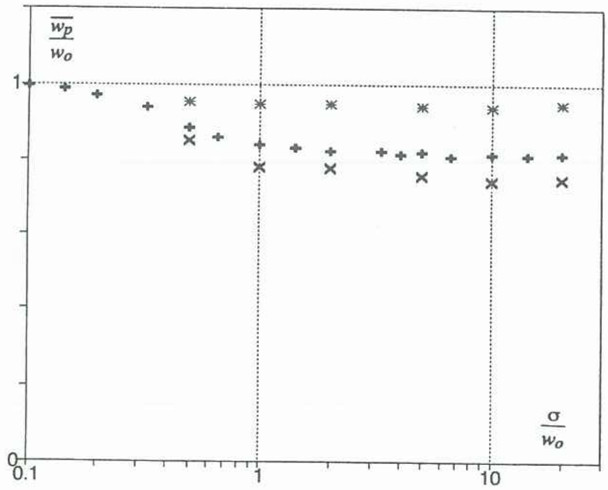

Figure 5: Simulated settling velocity reductions based on the model above, each point corresponds to several runs with 10^5 time steps. Legend, *: $A_E = 0.3$, +: $A_E = 1.0$, x: $A_E = 3.0$.

We see that the random walk model predicts a settling delay which increases from zero with increasing relative turbulence intensity towards a maximum delay of the order *19%* for $A_E = 1.0$. For $A_E = 0$ there is no settling delay. For A_E values in the realistic range $0.3 < A_E < 3$ the asymptotic maximum delay varies between 5% and 25%.

DISCUSSION

There are at least three specific mechanisms by which a turbulent velocity field with zero mean (e g homogeneous, isotropic turbulence) may influence the settling or rise velocity of single, non-neutral particles. They are the delaying effect of non-linear drag on large particles, the likewise delaying loitering effect and the fast tracking of small, heavy particles.

From the available theory, simulation results and experiments, it seems that both bubbles and heavy particles tend to be delayed by relatively weak turbulence ($\sigma/w_o \lesssim 3$). Settling velocity reductions of up to 40% have been observed in grid turbulence. The corresponding reductions for bubbles lack experimental verification but are expected to be greater because bubbles can be trapped indefinitely in steady vortices.

Strong and persistent eddies are able to fast track small heavy particles leading to substantial settling velocity increases. An increase by a factor 4 has been observed in grid turbulence. Fast tracking will only occur with heavy particles. Bubbles will spiral towards the centre of persistent eddies and remain trapped.

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