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# INTERACTION OF ANTI-PARALLEL VORTEX FILAMENTS

**T.B. NICKELS and T.T. LIM**

Department of Mechanical and Manufacturing Engineering
University of Melbourne Parkville, VIC 3052, AUSTRALIA

## ABSTRACT

Laser cross-sections of the head-on collision of two vortex rings marked with dye are presented. The aim is to examine some features of the behaviour of anti-parallel vortex filaments brought into close contact. The experimental results are compared with the results of numerical studies of the same flow case - both inviscid and viscous.

## INTRODUCTION

The study of the interaction of anti-parallel vortex filaments has received much attention recently due to its importance in the understanding of vortex reconnection. Most of this work has been in the form of numerical studies because of the experimental difficulties involved. The head-on collision of vortex rings is one example of this interaction. This interaction has been chosen for study since numerical results exist for both the viscous and inviscid cases, so it is possible to compare the results of these studies with experiment. In this study fluorescin dye is used with a laser sheet to elucidate some features of the flow.

## APPARATUS

The apparatus used for the production of the vortex rings was the same as that given in Lim, Nickels & Chong (1991) (see also Lim & Nickels 1992). Briefly, it consisted of two 21 mm diameter horizontally-opposed tubes immersed in a glass tank of water. The two tubes were connected to 125 mm piston/cylinder arrangement driven by an electronically controlled stepping motor which provided a fixed impulse to the fluid. The two vortex rings thus generated then travelled toward the centre of the tank where they collided. Fine adjustment of one of the nozzles meant that the rings could be made to collide head-on. To visualise the flow Fluorescein dye was injected around the circumference of one of the nozzles and therefore only one of the colliding rings can be seen in the photographs. In order to obtain cross-sections of the flow a 5 watt argon-ion laser was spread into a sheet by a cylindrical lens and used to illuminate a plane through the axis of symmetry of the flow. A Nikon f501 camera with a motor-drive was used so that several photographs could be taken in succession at equal time-intervals.

## EXPERIMENTAL RESULTS

Figures 1 to 4 show the comparison of the experimental results with the direct numerical simulation of Stanaway et al. (1988). While flow cases at six different Reynolds numbers were studied in the experiment only two are shown here. They were chosen to be approximately the same as those used in the simulation. Unfortunately, it was difficult to ensure that both the Reynolds number and the core-to-ring diameter ratio ($\alpha$) were the same as in the simulation. Here $\alpha$ for both Reynolds numbers is 0.1 whereas it is 0.35 in both the simulation cases.

The non-dimensional time scale, $\bar{t}$ used is the same as that used by Stanaway i.e.

$$\bar{t} = \frac{\nu^2}{I/\rho}t \qquad (1)$$

where $I$ is the fluid impulse.

For ease of presentation the time of the first illustration in each sequence has been arbitrarily assigned to be $\bar{t} = 0.0$;the times shown for the results of Stanaway et al. (1988) are measured from the results presented in that paper. The times are approximate and are presented only as a guide.

Even though the comparison is between dye signatures and vorticity contours the similarity between the simulation of Stanaway et al. (1988) and the photographs is quite marked for the higher Reynolds number case. This would suggest then that **for this flow case** the behaviour of the dye is a reasonably good indicator of the behaviour of the vorticity. This is due to the fact that in this case the time-scale of the interaction is reasonably short in comparison with the diffusion timescale. There is, however, no one-to-one correspondence between the presence of dye and vorticity in this case, or in general. In the low Reynolds number case the interaction time is longer and the results must, therefore, be interpreted with more caution. The similarity of the dye and vorticity patterns would also suggest that the simulation gives a good indication of the behaviour of the real flow. The times appear to indicate that the interactions proceed more slowly in the experimental case than in the simulation, however, given the error involved in estimating the impulse this difference may not be significant.

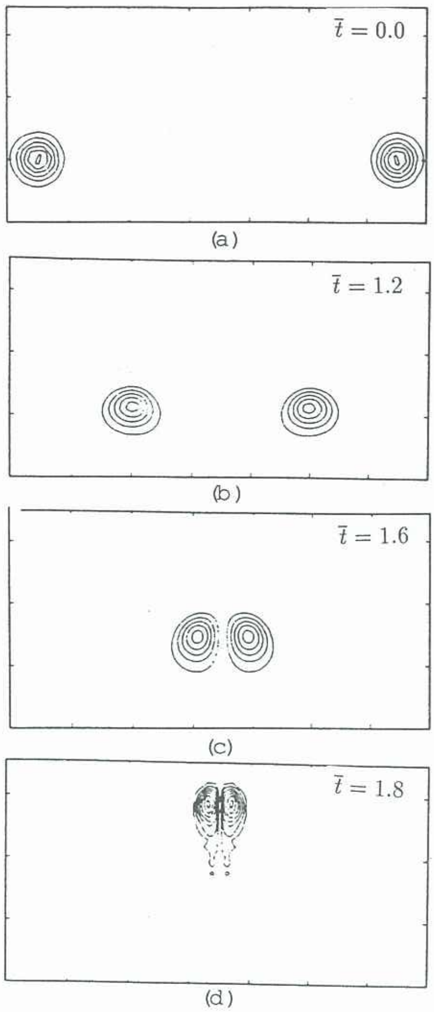


Figure 1: Vorticity contours of Stanaway et al. for $\Gamma/\nu$=1000

The most surprising observation, however, is the behaviour of the tail at large times. It may be seen in figure 4(e) and (f) that shortly after the head-tail structure has formed, the tail is drawn back into, and wrapped around the head. This behaviour is more obvious in the video of the interaction and consistently occurs. At the higher Reynolds number, however, this unusual behaviour is not observed (even at larger times than are shown here). One possible explanation for this phenomenon is that the dye tail no longer contains significant vorticity since the vorticity has diffused much faster than the dye. Thus the dye is convected by the local velocity field of the cores.

Video sequences of the laser cross-sections also provide a means for obtaining more quantitative measurements of the interactions. One measurement of particular interest is the rate of increase of the diameter of the rings. It is possible to measure this quantity reasonably accurately from the video sequences and it may be compared to predictions from inviscid models. Inviscid models (see Shariff et al. (1989)) suggest that at large times, if the shape of the cores do not change significantly as they are stretched, the rate of growth scales with $\overline{\sigma}^{\frac{1}{2}}$ where $\overline{\sigma}$ is the radius of the rings. Shariff et al. (1989) found that the core shape in their simulation was very close to the core shape calcu-

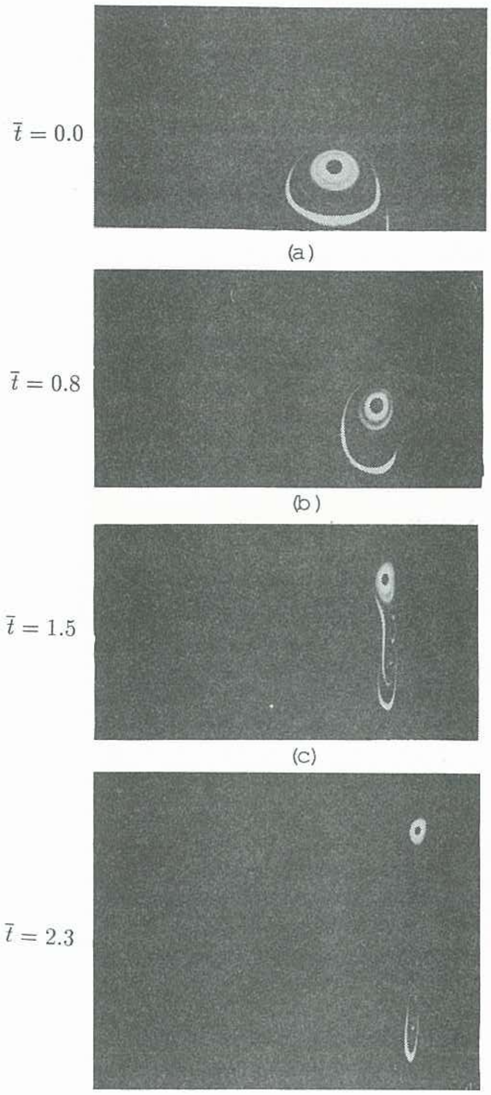


Figure 2: Laser cross-sections for $\Gamma/\nu$=1075

lated by Sadovskii(1971) for a translating vortex pair. It may be shown that if these conditions are met,

$$\frac{(d\overline{\sigma}/dt)}{(\overline{\sigma}\Gamma A)^{\frac{1}{2}}} = constant \qquad (2)$$

where $\Gamma$ is the circulation of the core and $A$ relates to the increase in vorticity as the rings are stretched(i.e. $\omega_\phi = A\overline{\sigma}$). In the inviscid case, however, there is no cancellation of vorticity and thus the circulation of the cores remains constant. In the real case initially the growth rate should be similar to that of the inviscid case but will become less as viscous diffusion cancels the vorticity. The difference between the growth-rate in the inviscid and viscous cases then gives some indication of the rate at which the vorticity is annihilated. Figure 5 shows the growth rate scaled with $\overline{\sigma}^{\frac{1}{2}}$. It should be noted that the circulation used to non-dimensionalise this graph is the initial circulation of the rings and is thus constant and $A$ is calcu-

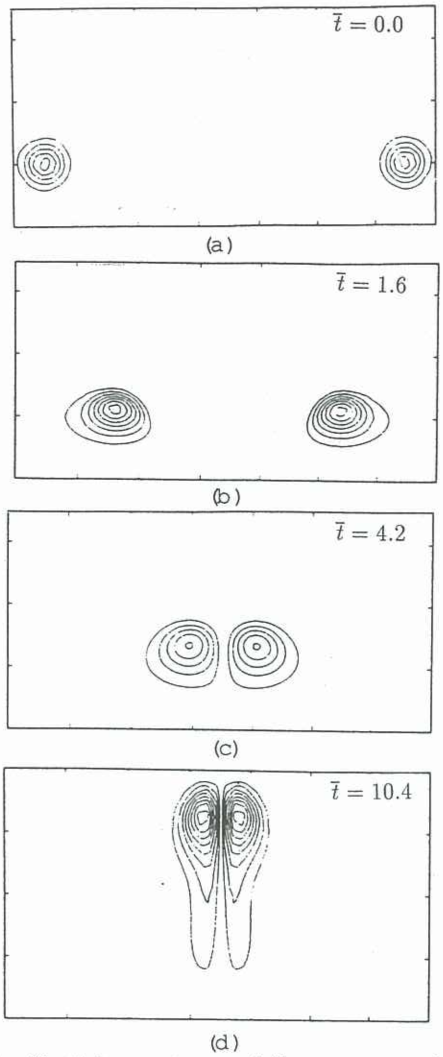


Figure 3: Vorticity contours of Stanaway et al. for $\Gamma/\nu$=350

lated assuming a circular cross-section. 5(b) is taken from Shariff et al. (1989). The solid line shows the growth rate calculated from the inviscid simulation of the interaction. Although this plot is for $\alpha = 0.5$ the behaviour up to the formation of a tail ($U_o t/L_o = 5$) should be similar to that for smaller values of $\alpha$ and should reach the same non-dimensional value (for details see Shariff et al. (1989)). The dashed line shows the growth rate predicted from the analysis of Dyson(1893) which is also inviscid but in which the vortex cores are constrained to be circular. The dotted line shows the expected behaviour if the cores have a self-similar shape which is the same as that calculated by Sadovskii(1971) for a two-dimensional vortex pair. It may be seen that the growth rate in the experimental case is much less than for the inviscid simulation of Shariff et al. (1989) and drops off sharply during the experimental collision . Shariff et al. (1989) also found a drop in the growth rate at $U_o t/L_o \approx 5$ as shown in figure 5(b). Since the simulation is inviscid this drop cannot be due to the cancellation of vorticity. Shariff et al. (1989) attribute it to the formation of a tail which reduces the circulation of the head (which otherwise maintains its shape). Interestingly, the experimental results for both Reynolds numbers reach a maximum at approximately the same non-dimensional time as the inviscid simulation. Vorticity cancellation and

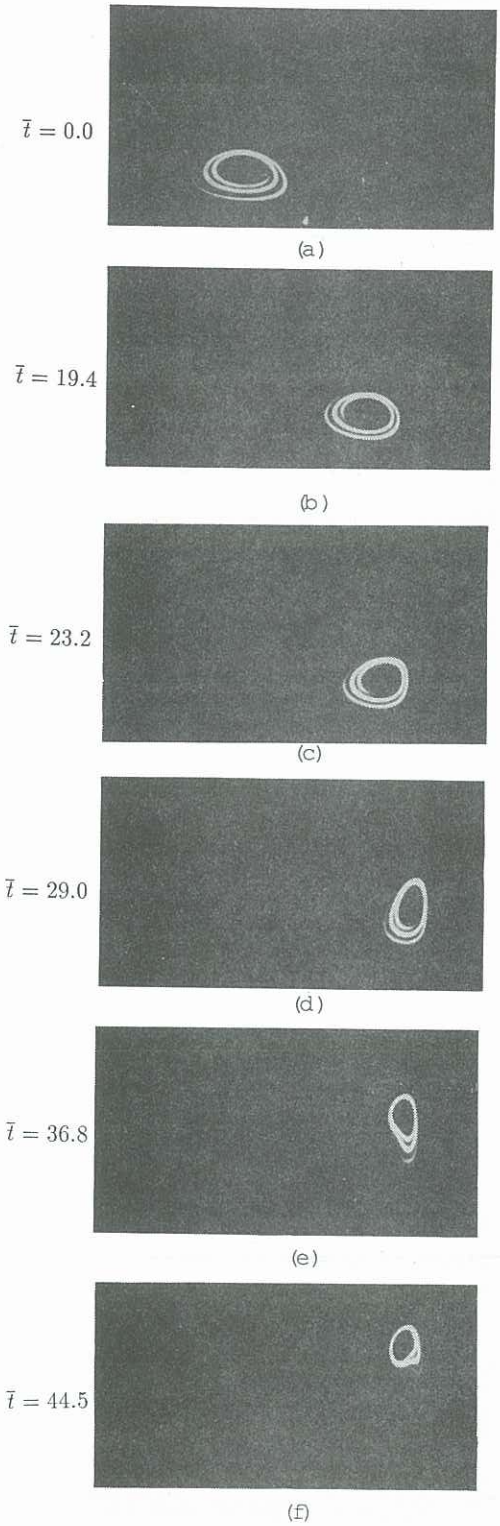


Figure 4: Laser cross-sections for $\Gamma/\nu$=350

tail formation would both be expected to contribute to the drop in growth rate, however it is not possible from these results to establish their individual contributions. However, the fact that the growth rate is overall much less than the inviscid simulation would seem to be due to the cancellation and diffusion of the vorticity.

This plot is also used to make some estimate of the "annihilation time" described by Stanaway et al. (1988). This time is the time during which most of the annihilation of the vorticity occurs. In the plot this may be interpreted as the time from when the real growth first departs significantly from the inviscid growth rate to the point where they cease to grow significantly. From the plot this time is approximately $\bar{t}_{ann} = 0.17 \times 10^{-4}$. For the same Reynolds number Stanaway et al. (1988) found $\bar{t}_{ann} = 0.2 \times 10^{-4}$. This estimate is approximate and takes no account of tail formation. The closeness of the agreement may be fortuitous since the estimation involves several approximations. It is included here since the value of $\bar{t}_{ann}$ is of some interest,since the annihilation time is an important factor in attempts to understand vortex reconnection. Stanaway et al. (1988) found that for this flow case the annihilation time scale is less than the circulation time-scale but larger than the viscous time scale, indicating that both local and non-local effects are important in the annihilation of the vorticity. Estimates from the experiments lead to the same conclusion.

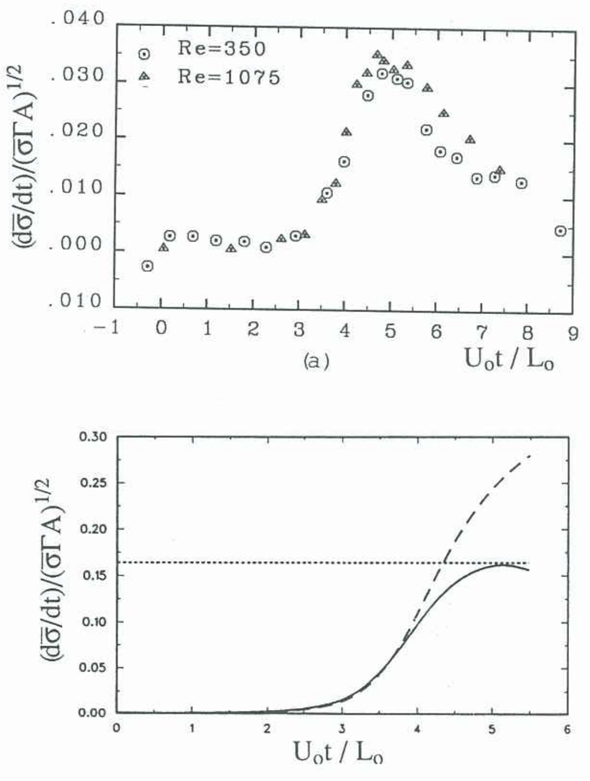


Figure 5: Growth rate of rings - $\bar{\sigma}$ is ring radius, $L_o$ is initial ring radius, $\Gamma$ is circulation, A is a constant relating to the vorticity (see Shariff et al. (1989)).

## CONCLUSIONS

The experimental results at $\Gamma/\nu = 1075$ agree well with the direct numerical simulation of Stanaway et al. (1988). This result lends support to the contention that in this particular flow case the behaviour of the dye is a good indicator of the behaviour of the vorticity.

It has also been shown that the annihilation of vorticity leads to a significant departure from the behaviour of inviscid models particularly reducing the growth rate and leading rapidly to the final "death" of the rings.

The "annihilation time" has been estimated and also agrees approximately with the numerical simulation of Stanaway et al. (1988) its value falling between the values of the viscous and circulation time scales.

A further interesting observation is that the dye signatures of the rings consistently exhibit an unusual behaviour at low Reynolds number. To the authors' knowledge this behaviour has not been observed before and is not, as yet, fully explained.

## ACKNOWLEDGEMENTS

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