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PANEL METHODS IN MARINE HYDRODYNAMICS

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ABSTRACT

A variety of hydrodynamics problems concerning ships and offshore platforms can be addressed within the context of potential theory. Most numerical solutions are based on panel methods. The basic methodology is described, and illustrated by various special topics including wave effects on offshore platforms, ship motions in the time domain, ship interactions in a channel, higher-order panel methods, and other related works.

1. INTRODUCTION

The field of marine hydrodynamics includes applications of fluid mechanics which are pertinent to ships, offshore platforms, and other vessels. Traditionally these problems have been addressed experimentally in towing tanks and water tunnels. Advances in theoretical knowledge and computational ability have made it possible to transfer much of this work from physical experiments to numerical solutions.

Many problems of practical importance can be analysed as potential flows, neglecting viscous effects. These include the wave resistance of ships, motions of ships and platforms in waves, propeller performance, and the interactions between adjacent ships manoeuvring in close proximity. In these cases the Reynolds number is large, and separation is avoided either because the geometry is streamlined, or because the Keulegan-Carpenter number is small. Boundary-layer corrections can be applied when it is appropriate to do so.

Within the assumptions of potential theory, we seek numerical solutions of Laplace's equation in the fluid domain, subject to appropriate boundary conditions. In the simplest case the normal velocity is specified on the body, the fluid extends to infinity in all directions, and the solution is specified at infinity. Since the fluid domain is unbounded, an effective numerical approach is to distribute sources and (optionally) normal dipoles on the body surface. Justification for these representations follows from Lamb (1932, §§57-8). In the 'potential formulation' Green's theorem is used directly, with the source strength equal to the known normal velocity and the dipole moment equal to the unknown potential. A second-kind Fredholm integral equation can then be solved for the velocity potential on the body. In the alternative 'source formulation' the potential is represented by sources alone, with unknown strength. Evaluating the normal derivative on the body leads to a similar Fredholm equation for the source strength.

The first use of this approach, for three-dimensional bodies of arbitrary shape, was by Hess and Smith (1964). In their numerical technique the source formulation is used, the body surface is approximated by a large number N of small flat quadrilateral 'panels', and the source strength is assumed constant on each panel. A set of N linear algebraic equations follows by imposing the body boundary condition at one collocation point on each panel. In this linear system the N^2 influence coefficients represent the normal velocity induced on each panel by a unit-density source distribution on the other panels. The right-hand-side vector is the normal velocity at each collocation point, determined from the body boundary condition. The solution can be obtained by standard matrix algebra, yielding the appropriate source strength on each panel. It is straightforward to solve for the velocity potential, velocity vector, and pressure on the body or in the fluid domain.

A vital element in the original work of Hess and Smith was their derivation of analytic expressions for the potential and velocity induced by a unit-density source distribution on a flat quadrilateral panel. Without these algorithms it would be necessary to use numerical integration, a slow and potentially inaccurate approach when the field point is near or on the panel.

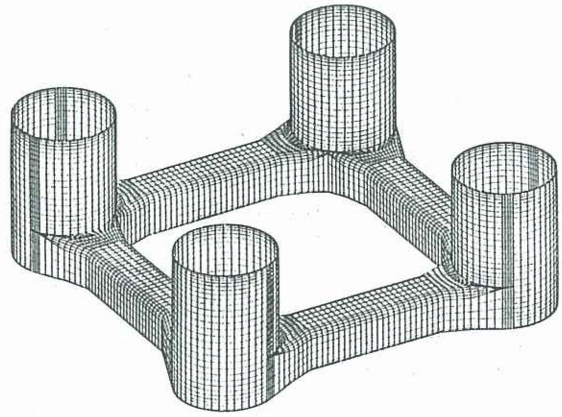

Figure 1 – Discretization of the submerged surface of the Snorre Tension Leg Platform with 13952 panels.

Numerous extensions and modifications have followed the seminal work of Hess and Smith. Some have used the source formulation, and others have adopted the potential formulation. This distinction is less important than one might suppose. The kernels of the respective integral equations are related by transposition, a consequence of the reciprocity between the normal velocity induced by a source and the potential of a normal dipole. Thus the computational effort involved in setting up and solving the linear system is practically the same in both cases. The following differences vary in importance depending on the application:

a. For thin bodies (or appendages), normal dipoles are more stable than sources in representing cross-flow components. The equivalence of normal-dipole and vortex distributions is another hint of the utility of dipoles, particularly in lifting problems.

b. In the source formulation the fluid velocity can be evaluated from first derivatives of the Green function, but in the potential formulation second derivatives are required. The latter is not robust when a curved surface is approximated by flat panels, since the local velocity field induced by the dipole distribution varies rapidly over distances comparable to the panel dimensions.

c. For bodies which intersect the free surface, the solution of the integral equation breaks down at a discrete set of 'irregular frequencies'. This difficulty arises in both formulations, but it is more serious in the source formulation (Yeung, 1982).

d. If the only physical parameter of interest is the linearized pressure on the body, proportional to the velocity potential, the potential formulation is advantageous in terms of programming, storage, and computational cost.

The number N of panels required to achieve the desired computational accuracy, and the associated computational burden, are important practical issues. From geometric and hydrodynamic considerations we can expect that simple body shapes with continuous curvature, such as spheres, spheroids, and ellipsoids, will require relatively few panels, and conversely for more complicated body shapes. In the early days of this field pragmatic considerations limited N to be on the order of 100. Many current applications involve 1000-10,000 panels.

Four distinct computational tasks may be identified in estimating the cost of increasing N:

a. Pre-processing to derive the panel representation and auxiliary parameters (unit vectors, areas, moments, cross products) is directly proportional to N.

b. Set-up of the linear system requires the evaluation of N^2 influence coefficients.

c. Solution of the linear system requires $O(N^3)$ computations if direct solution algorithms are used, or $O(N^2)$ computations per iteration, if a suitable iterative algorithm can be utilized.

d. Post-processing to evaluate integrated parameters such as the pressure force and moment requires $O(N)$ computations.

Clearly the set-up and solution are dominant. Their relative importance depends on the algorithms used and the magnitude of N.

In the era when N was relatively small, direct solution of the linear system was appropriate, based on Gauss elimination. This permitted the use of standard algorithms. The computational burden was not a concern, particularly for free-surface applications where the cost of evaluating the influence functions was dominant. However the development of special algorithms for the influence functions, and advances in hardware, made it feasible to increase N to the level O(1000), where the $O(N^3)$ cost of direct solution became dominant. With the implementation of effective iterative solutions, the cost of both the set-up and solution increase in proportion to N^2.

A survey of the mathematical basis for panel methods is given by Atkinson (1990). A recent review of applications and numerical techniques is given by Hess (1990).

It is beyond the scope of this paper to survey the extensive field of panel methods, even within the context of marine hydrodynamics. Instead an account is given of several developments in which I have participated, supplemented by a brief summary of parallel work by my colleagues in the Department of Ocean Engineering at MIT. In addition to wave problems these include the complementary field of propellers and lifting surfaces, a higher-order panel method based on B-splines, and a method where the computational cost is reduced to $O(N)$.

2. WAVE LOADS ON OFFSHORE PLATFORMS

From the standpoint of the relevant fluid mechanics, wave loads on offshore platforms fall into two categories. For platforms with cross-sections which are small relative to the wave trajectories, free surface effects are not as significant as the oscillatory drag due to separation. This leads to the approximation known as 'Morison's formula,' where a quasi-steady viscous drag is combined with an inertial force proportional to acceleration. While it is a source of continuing debate between practicing engineers and the research community, Morison's formula has been used extensively for platforms consisting of multiple cylindrical elements with relatively small diameter.

Wave diffraction and radiation are more significant for platforms with larger cross-sections. These are of increasing importance, especially in deep water. The tension leg platform illustrated in Figure 1 is an example of particular current interest. Viscous effects are less significant due to the reduced Keulegan-Carpenter number. In these circumstances it is appropriate to assume a potential flow, and to account for free-surface effects.

Starting about twenty years ago, the panel methodology was extended to linear three-dimensional water-wave radiation and diffraction problems. This extension is conceptually straightforward, since the Green function is known for a source in the presence of a linearized free surface. With the oscillatory time dependence represented by the complex factor $\exp(i\omega t)$, both the Green function and the unknown source strength or velocity potential are complex. A more significant complication is that the free-surface Green functions are difficult to evaluate, particularly in their original forms expressed as integrals in wavenumber space. For this reason early programs required substantial computer time, restricting the number of panels which could be utilized, and the results were of uncertain accuracy in some cases. Eatock Taylor and Jefferys (1986) focussed attention on these limitations by comparing the results from different codes.

We were attracted by the challenge of this problem about ten years ago, and extensive research has followed. Algorithms have been developed to evaluate the Green functions using Chebyshev approximations in place of direct evaluation of the underlying Fourier integrals, as described by Newman (1992). A special iterative solver was developed by Lee (1988) to overcome the $O(N^3)$ barrier of Gauss elimination. For most practical applications convergence is achieved in 10-20 iterations. On rare occasions in special problems convergence is not readily achieved, and it is necessary to revert to the direct solution. (The most frequent cause of poor convergence is defects in the panel representation of the body. This can be excused on the basis of 'user error', but the practical difficulties of preparing effective discretizations for complicated geometries cannot be ignored.)

The program WAMIT, which incorporates these algorithms, is now used extensively in the offshore industry. An example of computations is presented in Figure 2, which shows the longitudinal (surge) exciting force due to incident plane waves acting upon the platform shown in Figure 1. Wave interference between the vertical columns is evident from the oscillatory character of the force. The peaks and minima coincide closely with values of kL equal to $n\pi$ or $(n+\frac{1}{2})\pi$, respectively.

In these results a modified integral equation is solved in the manner described by Lee (1988), to provide a robust solution at the irregular frequencies. For this platform the first two irregular frequencies occur near kL=14.6 and kL=23.3. The solution of the original integral equation displays small fluctuations near these points.

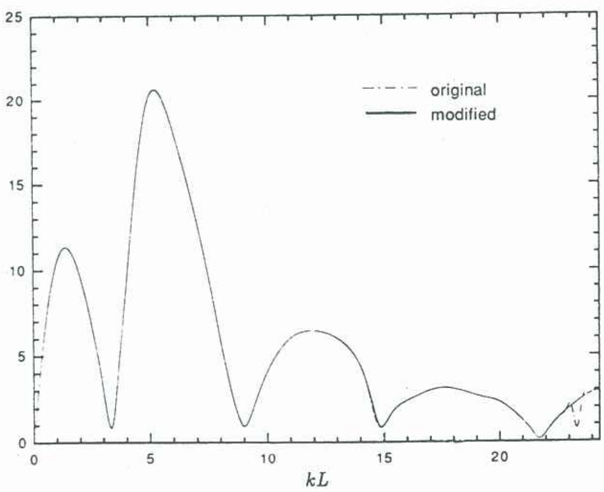

Figure 2 – The surge exciting force for the platform in Figure 1. The ordinate is the force amplitude normalized by the water density, wave amplitude, gravity, and the square of the column radius $a = 12.5m$. The normalized wavenumber kL is based on the spacing between the column axes L=76m.

To indicate the convergence of these results, Table I compares the peak values of the exciting force in Figure 2 for a sequence of three discretizations with N=872, 3488, and 13952 panels. Assuming the discretization error is of order $1/N$, Richardson extrapolation can be used as shown in the last two columns of Table I. This appears to be effective only at the middle three wavenumbers, where the first extrapolant based on the coarse discretizations improves the agreement with the finest discretization. The proximity of the second irregular frequency may explain the inefficacy of Richardson extrapolation for the largest wavenumber. There is no obvious explanation why the first wavenumber also is an exception.

The same approach can be extended to multiple interacting bodies. Such problems are important for marine operations, *e.g.* the installation of offshore platforms with floating cranes and other vessels in close proximity. It is convenient to consider the union of all bodies as one extended boundary surface, defined by the ensemble of each separate set of panels. The total number of degrees of freedom is the sum of the number of independent motions for each body. In each mode one body oscillates and the others are fixed. In this way the principal extension required is simply to extend the definitions of the vectors defining the degrees of freedom, and corresponding forces and moments.

A more profound extension is to evaluate second-order wave effects including the time-average and second-harmonic components associated with a monochromatic first-order input, and the corresponding difference- and sum-frequency components in a bi-spectrum. The most difficult task is to satisfy the inhomogeneous second-order free-surface boundary condition, which requires distributions of singularities over the entire free surface. The resulting surface integrals are oscillatory, and converge slowly with increasing radial distance from the body. Further details and computational results are given by Newman and Lee (1992).

kL			
1.46	11.2743		
	11.3101	11.3220	
	11.3127	11.3135	11.3129
5.35	20.9011		
	20.6509	20.5676	
	20.5652	20.5366	20.5346
11.92	6.6882		
	6.5178	6.4611	
	6.4699	6.4539	6.4534
17.51	3.0743		
	3.1032	3.1128	
	3.1136	3.1171	3.1173
24.32	2.7459		
	2.9136	2.9695	
	2.8948	2.8885	2.8831

Table I – Peak-values of the surge exciting force coefficient in Figure 2. The second column shows the computed results for N=872, 3488, and 13952 panels. The third and fourth columns are the Richardson extrapolants.

3. TIME-DOMAIN SHIP MOTIONS

For the 'seakeeping' analysis of a ship, the unsteady motions are superposed with the steady-state flow due to the ship's mean forward velocity. In the frequency domain the appropriate Green function is the potential of an oscillatory translating source, which is unusually difficult to evaluate. This problem can be circumvented by solving an initial-value problem in the time domain, using the transient Green function and advancing its position in space. Green's theorem then is used to solve for the velocity potential, with a convolution in time over the history of the motion. Algorithms for the transient free-surface Green function are described by Newman (1992).

The convolution integral is evaluated by the trapezoidal rule, in a sequence of equal time steps. Only the current value of the potential is unknown at each time step. One simplification, associated with the initial conditions, is the absence of wave effects in the influence coefficients at the upper limit of the convolution integral corresponding to the current value of the unknown. Thus the left-hand-side coefficient matrix is the same for all time steps, and can be inverted at the outset. The solutions for subsequent time steps involve simple matrix products with $O(N^2)$ computations.

The evaluation of the convolution integral requires either the storage or recomputation of the Green function at earlier time steps. With current workstations disk-access is faster than recomputation, but the storage requirement is substantial. For a typical problem with O(1000) panels and 200 time steps, 1-2 gigabytes of storage is required to avoid recomputation. With expected improvements in CPU speed relative to the access time of disk drives, it soon may be faster to recompute the Green functions and storage requirements then will be greatly reduced.

Figure 3 shows a typical computation, the time history of the vertical force on a ship hull due to an impulsive incident wave. Here the incident wave is a superposition of regular waves of all frequencies, advancing toward the ship's bow with suitable amplitude and phase to produce a delta-function wave height at $t = 0$. For $t > 0$ the delta function disperses into the same regular wave components, moving on in the same direction in a manner analogous to the waves in a Cauchy-Poisson disturbance. The resulting force is peaked about $t = 0$ as one would expect, with a small number of precursor oscillations and more persistent oscillations after the waves are past. For increasing forward velocity (Froude number) of the ship, moving in the direction opposite to the waves, the peak force is increased but its duration is reduced. Transforming to the frequency domain by Fourier integration, and converting from the frequency of encounter to the incident wavelength, gives the results shown in Figure 4. A more detailed account of this work is given by Bingham *et al* (1993).

4. SHIP INTERACTIONS IN A CHANNEL

Computations in the time domain also can be used to analyse the interactions between ships moving with unequal velocities. Such problems are particularly important in narrow channels. If the ships' velocities are small the free surface can be replaced by a homogeneous Neumann condition. This simplification, equivalent to supplementing each ship's submerged hull by its image above the 'rigid' free surface and reflecting the fluid domain above the same plane, is referred to as the 'double-body' flow.

We have applied panel methods to the case where one ship is moored, while a second ship moves past with constant velocity. The principal concern is the time-history of the horizontal force and vertical moment acting on the moored ship. Two complementary approaches are described and compared by Korsmeyer *et al* (1992). In the first, a uniform rectangular channel section is assumed and the Green function is suitably modified to satisfy the boundary conditions of zero normal velocity on the channel sides and bottom. The algorithms for this procedure are described by Newman (1992). In the second approach, panels are used not only to represent the two ship hulls, but also the bottom and sides of the channel. In this manner the hydrodynamic effects of irregular channel topographies can be reproduced. The example in Figure 5 shows the submerged portions of two ships and the surrounding channel.

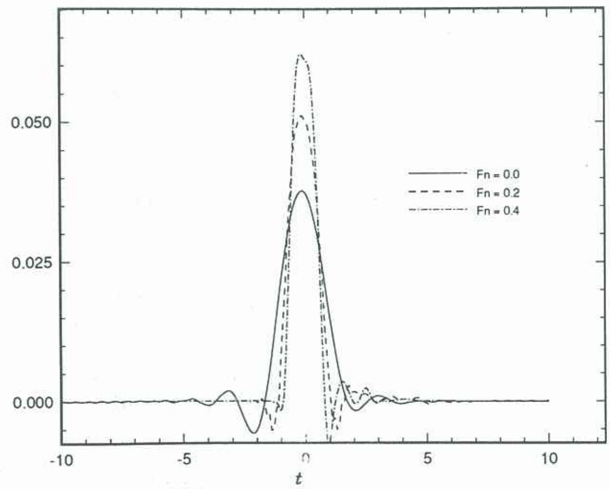

Figure 3 – Heave exciting force on a Wigley ship hull moving with Froude number Fn into impulsive head waves. The hull is discretized with 264 panels.

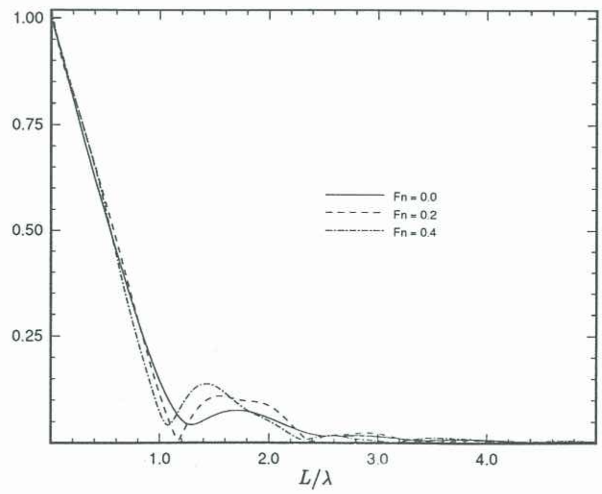

Figure 4 – Heave force in Figure 3, transformed to the frequency domain. The abscissa is the ratio L/λ between the ship length and wavelength.

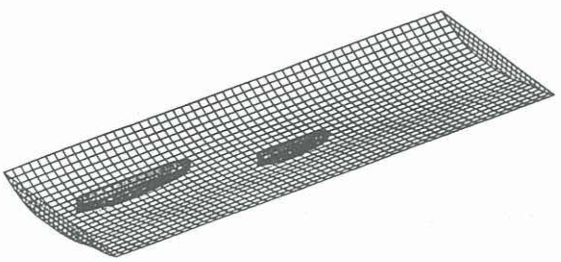

Figure 5 – Discretization of two ships in a channel of irregular cross-section. For subsequent time steps the larger ship moves past the smaller ship. The channel ends are in the far field of the smaller ship to minimize truncation effects in the prediction of the force on the smaller ship.

Various extensions are possible, including the evaluation of sinkage and trim on a ship in a restricted channel, the interactions between two (or more) moving ships, and curved trajectories. Free-surface effects can be accommodated by using the time-domain Green function as in §3. One effect which has not been included here is the lifting effect of oblique flow on a moving ship, which leads to shedding of a vortex wake. Computational models can be developed along these lines, with a Kutta condition imposed at the ship's stern, but estimates based on slender-body theory suggest that this may overestimate the lifting effect for practical ship hulls.

5. A B-SPLINE PANEL METHOD

Higher-order panel methods have been developed in the past with linear or quadratic polynomials used to represent the panel surface and the unknown potential or source strength. Usually these polynomials are in powers of Cartesian coordinates in a plane tangential to the panel. For curved panels the integrals for the singularity distributions cannot be expressed in closed form, and difficulties arise in the representation at the intersections of adjacent panels. Efforts to assess the efficiency of these methods have been hampered by differences between individual programs, and by different choices of the body geometry and hydrodynamic parameters.

A more general scheme, based on the use of B-splines, has been widely adopted for surface definition in computer-aided design. In panel methods the same basis functions can be used to represent both the geometry and the unknown potential or source strength. (This duality is analogous to isoparametric finite-element methods.) We have explored this approach for two-dimensional flow problems, and summarize the results briefly here. Further details are given by Hsin *et al* (1993).

The simplest problem is considered, for the two-dimensional streaming flow past a body defined by a closed contour in the complex z-plane. The normal velocity is zero on the body and the fluid extends to infinity in all directions.

Defining the body geometry by B-splines is equivalent to expressing the values of z on each panel by a polynomial of specified degree n in a parametric coordinate t. The degrees $n = 1, 2, 3$ correspond to piecewise linear, quadratic or cubic representations. The corresponding polynomials of B-spline basis functions preserve continuity in value, slope, and curvature, respectively. The parametric coordinate t increases monotonically along the contour, corresponding approximately to the arc length. The values of t at the intersections of adjacent panels are prescribed by the so-called 'knot vector'. The body shape is determined by specifying values of t at these points, and the coefficients of the basis functions. With N panels a total of $N + n$ coefficients are required. The same representation is used for the velocity potential on each panel, with $N + n$ unknown coefficients. Bodies with corners, and singularities in the potential, can be accommodated by using multiple knots.

Green's theorem provides an integral equation for the velocity potential, which is discretized in terms of the panels and solved by collocation. Some freedom exists in the number and choice of the collocation points. With cubic polynomials ($n = 3$) we use three collocation points on each panel. This gives $3N$ equations for $N + 3$ unknowns, an overdetermined linear system which is solved by least-squares.

The set-up of the linear system requires the evaluation of integrals over each panel of the source potential and its normal derivative, multiplied respectively by the B-spline representations of the normal velocity and potential. Analytic relations have been derived for this purpose including closed-form expressions valid when the collocation point is on the panel itself, and multipole expansions which are effective when the collocation point is greater than one panel-width away from the panel. When neither of these conditions is satisfied, *e.g.* when the collocation point is on an adjacent panel, the original panel is subdivided into smaller sub-panels such that the restriction for using the multipole expansion is satisfied for each sub-panel.

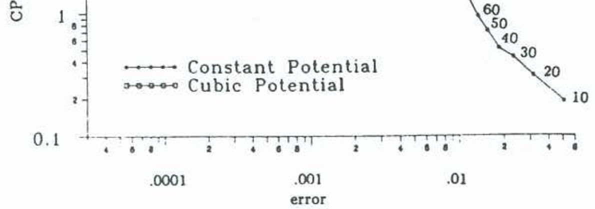

Figure 6 – Comparison of errors in the circulation on a Karman-Treftz foil using constant and cubic basis functions. The number of panels N is shown next to each symbol.

Two test cases have been studied, a square and a Karman-Treftz foil at an angle of attack. For the square, continuity of the potential is imposed across each corner. For the lifting foil, a point vortex is added at the trailing edge; its strength is determined iteratively to satisfy the Kutta condition of continuous pressure. For both cases comparisons have been made with the analytic solutions, using different degrees n and varying numbers of panels. These comparisons confirm the efficiency of the cubic representation, in terms of CPU time as well as storage requirements. The lifting case is illustrated in Figure 6. Current efforts are devoted to the three-dimensional extension of this method.

6. OTHER APPLICATIONS

In §§2-3 the free-surface Green functions are used to avoid integration over the domain of the free surface. This is effective in cases where the Green function is known, and practical to compute. An alternative is to utilize the simpler Rankine Green function ($1/R$), and include the domain of the free surface in the integral equation. This alternative is followed in two different contexts, for wave interactions with moving ships where a variety of different free-surface conditions are employed, and in the solution of fully-nonlinear wave problems.

The 'Rankine panel method' is applied to steady and time-harmonic ship-wave problems by Nakos and Sclavounos (1990). Quadratic B-splines are used to represent the potential on the ship hull and free surface. Attention is given to the numerical errors (dispersion and damping) associated with discretization of the free surface. Alternative free-surface conditions are applied including linearization about the free stream (the classical condition), and about the double-body flow (appropriate for small Froude numbers). An illustration of the steady results is shown in Figure 7.

Fully nonlinear three-dimensional wave effects are considered by Xü and Yue (1992), based on the mixed Eulerian-Lagrangian formulation. Periodic boundary conditions are imposed in the two horizontal directions, corresponding physically to waves propagating in a rectangular tank. Quadratic polynomials are used to represent the unknown potential on the exact free surface. The solution is carried out in a sequence of time steps, starting from prescribed initial conditions, and a pressure is applied on the free surface to energize the waves. A striking feature of the solution is the occurrence of transverse propagation of the highest wave elevation along the crest.

Contrary to the conventional estimates of computational cost in §1, Nabors *et al* (1992) demonstrate a panel method where the cost is $O(N)$ for sufficiently large numbers of panels. The panels are grouped in a cubical hierarchy such that the largest cube, enclosing the body, contains eight smaller cubes, and the subdivision is continued until the smallest cubes contain approximately the same number $C << N$ of panels. Spherical harmonic expansions are used to evaluate the influence of each cube, and to construct local expansions appropriate to each collocation point. The resulting matrix is pre-conditioned in an effective manner which exploits the cubical structure. After pre-conditioning the linear system for the N singularity strengths is solved iteratively. Computational evidence is given to demonstrate that the cost is linear in N for sufficiently large numbers of panels. Current work is devoted to applying this method to nonlinear wave problems, where the singularities are distributed on both the body and free surface.

Panel methods have been applied to a variety of lifting bodies in marine hydrodynamics, including propellers and yacht keels. Two examples are illustrated in Figures 8 and 9. The trailing vortices are replaced by a discretized sheet of normal dipoles, with the moments determined by the Kutta condition at the trailing edge and by the requirement of zero pressure jump across the sheet. Imposing the nonlinear Kutta condition is numerically troublesome, and requires an iterative approach. For screw propellers the wake panels are distributed on a helix. Truncation at a finite distance downstream is necessary, with the influence of the far wake approximated by an actuator disk. Various extensions include the representation of ducted propellers, nonuniform inflow leading to unsteady effects, and cavitation near the leading edge. A recent review is given by Kerwin & Keenan (1991).

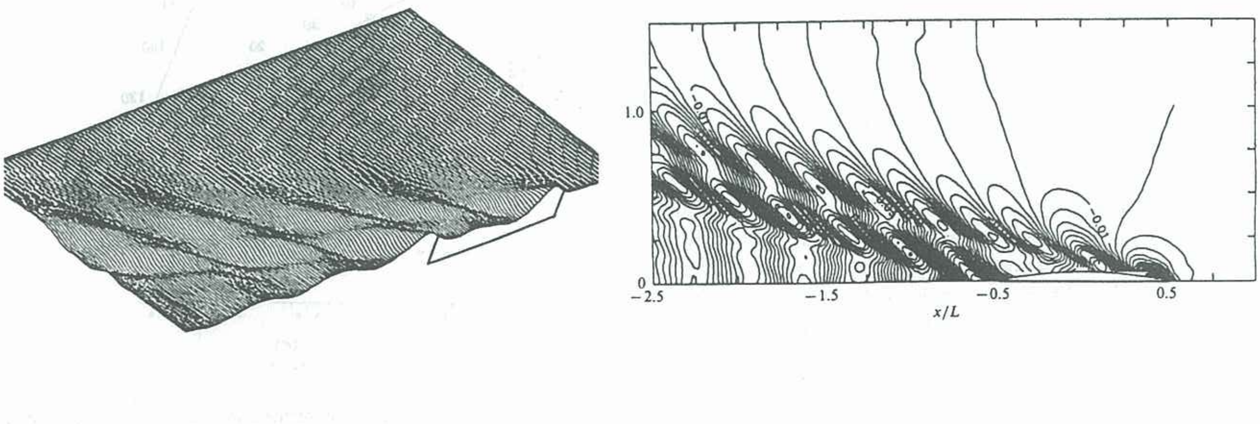

Figure 7 – Steady ship-waves generated by a Wigley hull with Froude number 0.32, computed by the Rankine panel method of Nakos and Sclavounos (1990). The hull is represented by 240 panels and the free surface by 3920 panels. In the abscissa of the contour plot the ship's bow and stern are at ±0.5.

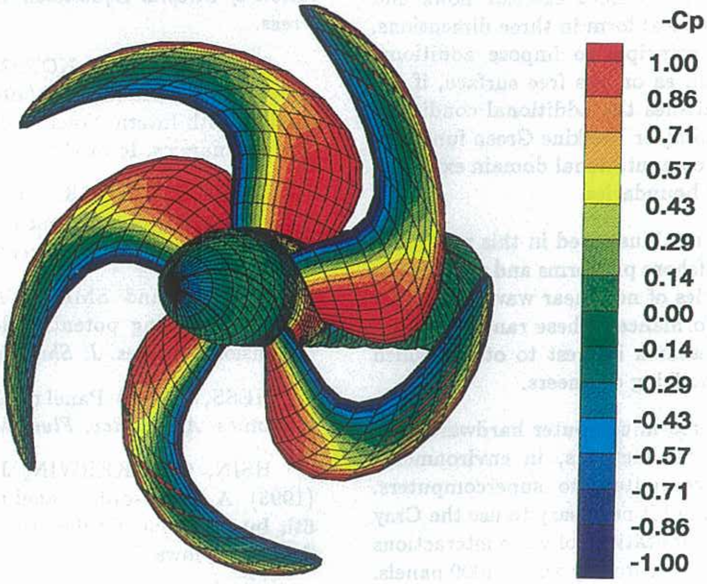

Figure 8 – Perspective view of a marine propeller with five blades. Each blade is represented by 450 panels and the trailing vortex sheet (not shown) by 330 panels. The corresponding sector of the hub is represented by 312 panels, giving a total of 5460 panels for the complete configuration. The pressure distribution is represented by different color shades.

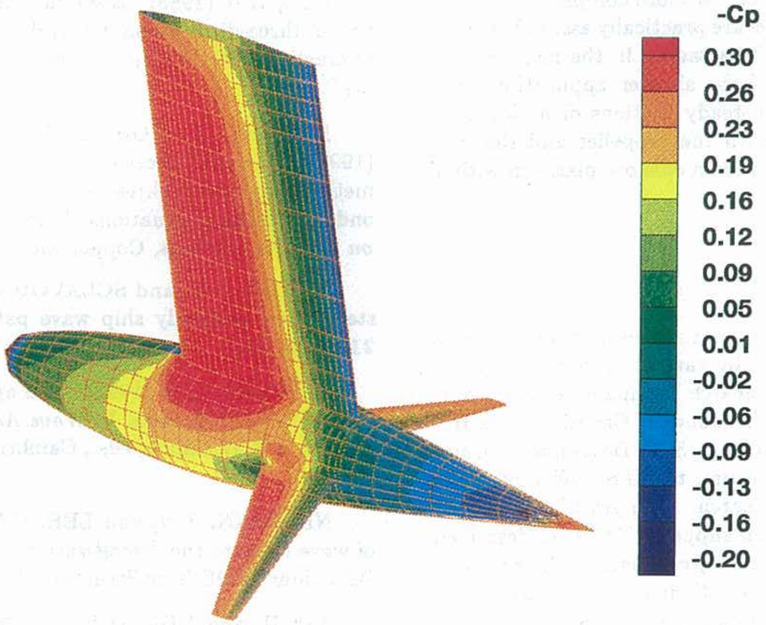

Figure 9 – Perspective view of an America's Cup yacht keel, oriented at a leeway angle of 3°. A total of 2220 panels are located on the keel, bulb, and winglets, with 390 additional panels (not shown) on the trailing vortex sheet. The pressure distribution is represented by different color shades.

7. CONCLUSIONS

Panel methods provide a versatile framework for the numerical solution of potential-flow problems in marine hydrodynamics. The principal advantages of this approach are its abilities to represent exterior flows and bodies of arbitrary geometrical form in three dimensions. It is straightforward in principal to impose additional boundary conditions, such as on the free surface, if the Green function which satisfies the additional conditions is known. Alternatively, simpler Rankine Green functions can be retained with the computational domain extended to include the additional boundaries.

Various applications are illustrated in this paper, including wave effects on offshore platforms and ships, ship-to-ship interactions, studies of nonlinear waves, and predictions of propeller performance. These range from applications primarily of research interest to others which are currently used by practicing engineers.

The rapid developments in computer hardware have facilitated the use of panel methods, in environments ranging from personal computers to supercomputers. Only four years ago we found it necessary to use the Cray to perform benchmark computations of wave interactions with offshore platforms represented by about 4000 panels. Today the same computations are performed routinely on PC's. Future advances can be assumed, which will enhance our ability to solve more complicated problems.

The advances in numerical analysis are equally important. Special algorithms for evaluating the influence functions and solving the linear system have improved the performance and reliability of panel methods. New techniques, such as the higher-order B-spline and order-N methods, offer the possibilities to match future advances in hardware with comparable improvements in software. Thus the numerical solutions of more complicated three-dimensional potential flows are practically assured. Likely problems to be successfully attacked in the next several years are combinations of the simpler applications, for example the steady or unsteady motions of a ship hull including the effects of both the propeller and the free surface, or the interactions of an offshore platform with a nonlinear wave system.

8. ACKNOWLEDGEMENT

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