11th Australasian Fluid Mechanics Conference
University of Tasmania, Hobart, Australia
14-18 December 1992

9C-1

THERMALS, PUFFS AND MASS SOURCES I - SIMILARITY AND VARIABILITY

B.R. MORTON, R.W. CRESSWELL and K.C. NGUYEN

Centre for Dynamical Meteorology
Department of Mathematics, Monash University
Clayton, VIC 3168, AUSTRALIA

ABSTRACT

The simplest ideas of similarity have played an essential role in both laboratory and theoretical studies of thermals and puffs, and yet values for the non-dimensional constants of proportionality in these cases have shown a disconcerting degree of variability. We review briefly some of the more important contributions in order to reassess past successes and failures, and we identify a class of hydrostatic instabilities as responsible for introducing indeterminacy into the behaviour of thermals as normally defined. We report also the realization of potential mass source flows in viscous fluids.

1. INTRODUCTION

In the following three papers we report on a class of flows generated by rapid axisymmetrical or two-dimensional release from sources compact about a point or line, respectively, of masses of fluid into a stationary, fully miscible fluid environment. Our discussion covers a number of unsteady, compact, advecting flows generated in homogenous or stably stratified environments from sources of buoyancy and momentum separately or in combination, and from mass sources free from buoyancy and net momentum. *Thermals* are generated by the rapid release from rest of a volume of buoyant fluid in a homogeneous or stably stratified environment; *forced thermals* from sources of buoyancy and vertical momentum simultaneously into neutral or stable environments; *puffs* from impulsive sources of momentum only in homogeneous environments; and potential outflows from sources of mass but neither buoyancy nor net momentum. Thermals and weakly forced thermals suffer an initial period of acceleration to maximum translational velocity through an entry length extending to a few source diameters (1.5 for axisymmetric thermals from a hemispheric cup according to Scorer, 1957) before entering their final phase of deceleration caused by turbulent entrainment. Similarity arguments apply only to the latter phase.

Strong source impulses may generate laminar vortex rings or pairs which may or may not be buoyant, and these vortices exhibit a heirarchy of similarity states which we shall discuss elsewhere. Here we are concerned for the most part with turbulent thermals and puffs, as commonly observed, although this turbulence may develop in an entry length of reduced entrainment. The advecting flows that we consider are relatively well-mixed and may be represented by a single length scale and either mean or gross buoyancy or momentum. Full analytic solutions are seldom available and an important technique has been to seek similarity conditions in which the pattern of motion is invariant at least on average and only the scales of length and velocity change with time or distance.

In this paper we review briefly selected earlier papers and discuss some basic ideas of similarity, including the departures from similarity which laboratory workers since Scorer (1957) have failed to rationalize (or even, in some cases, to accept), the relevance of two-dimensional solutions and the laboratory experiments of Richards (1963) and Tsang (1971), and the important role of a hydrostatic instability in the behaviour of thermals. In the second paper we describe the use of a large eddy simulation model to obtain numerical solutions for thermals and puffs, which we use in a series of experiments on similarity and departures from similarity with results that go some way towards explaining the variability observed in laboratory studies. Finally we describe a new series of laboratory experiments which shows that two-dimensional thermals are a fiction of unsuitable illumination, which give some support to the argument that variability in the similarity constants is inevitable, and which show that the flow from two-dimensional sources breaks immediately into three-dimensional motion but is constrained by the two-dimensional nature of the source.

2. PREVIOUS RESULTS

Scorer (1957) has reported a series of experiments on thermals in which a small hemispherical cup was turned by hand about a horizontal axis in or parallel to the water surface in a tank to provide rapid discharge of masses of downwardly buoyant solutions or suspensions free from initial momentum (though not from initial disturbances). He observed that descending masses grew in volume by incorporating ambient fluid as they fell, and immediately rejected an earlier model that successive layers of warm air would be eroded from the cap of a rising atmospheric thermal into its wake until the original volume was exhausted. It is now clear that entrainment and not erosion is characteristic of compact turbulent flows, but it is a great deal less clear whether these flows can possess any kind of real wake. Scorer had expected that the continuing growth of thermals released from small containers would be largely independent of initial configuration, but using appropriate visualization he found that the front surface of a thermal became covered with protuberances immediately on release and that there was a strong tendency for thermals to preserve their shapes as they grew, even down to patterns of eddies, but that there were almost as many different structures as there were thermals. Some grew in a grossly asymmetric way, but these he rejected as due to residual internal motions of either source fluid or tank at the moment of release.

In the simplest dimensional analysis of the motion of thermals their *scale* is determined by one length, say the maximum mean radius r in horizontal sections, and their *strength* is determined by the mean relative buoyancy $g\bar{B} = g\overline{(\rho_o - \rho_i)}/\rho_o$, where the density ratio $\bar{B}$ is averaged over the volume of the thermal with inside and outside densities ρ_i and ρ_o. The vertical velocity of the front of the thermal must then be of the form

$$w^2 = C^2 g\bar{B}r,$$

where C^2 is a Froude number, being a ratio of inertia to buoyancy forces. As *all* lengths vary in proportion, the distance z from virtual source to the front of the thermal must be

$$z = nr,$$

where n is a second proportionality constant and determines the semi-angle of the cone swept out by the growing thermal and with vertex at the virtual source. Scorer defined a thermal volume, $V = mr^3$, where m is a third constant of proportionality, and as the total buoyancy is constant in a uniform environment, he found for an axisymmetric thermal

$$kz^2 = t,$$

where $k = m^{1/2}/2nC(gB_oV_o)^{1/2}$, and gB_oV_o is the constant gross buoyancy. We cannot guarantee in advance that a particular class of realizations will exhibit this similarity structure, although n, C and m or k should certainly be universal constants if it does so. Bearing in mind that Scorer's measurements were on the crude side, his results are intriguing. In any particular realization, excluding the initial region in which the turbulent intensity was growing as the flow accelerated and the upper region in which flow may be influenced by the walls of the tank, individual realizations satisfied his scale relationships, but over many realizations the 'constants' n, C, m and k showed quite unexpected variability. Thus in one set of 18 realizations he found $2.9 < n < 5.0$. Bearing in mind that Scorer had already rejected "grossly asymmetric" thermals, one is apparently left with two possibilities: that the experiments were insufficient to define the motions; or that the similarity assumption did not adequately encapsulate the physics of the flow.

Woodward (1959) continued the work of Scorer and obtained displacement patterns within a thermal released in a water tank recorded on $16mm$ film using particles of near neutral density in a thin sheet of light through the axis of propagation. She assumed similarity and prepared a single plot of scaled velocities in a scaled cross-section of the thermal combining succesive $16mm$ frames from each of five experimental realizations. For each frame she determined a value for Z, the distance of the cap of the thermal from the deemed position of the virtual source, and then scaled all distances relative to Z and velocities to $W = (2kZ)^{-1}$. This is equivalent to projecting the velocity field for each frame from each of the five realizations onto a standard frame, and not surprisingly it produced a consistent mean pattern. It should be noted however that an important feature of flows of the kind under consideration is that the velocity fields due to vorticity distributions ranging from a thin cored vortex ring to a Hill spherical vortex show surprisingly little difference in the patterns of streamlines relative to the advecting flow. All generate a compact region of advecting fluid with closed streamlines within and outer streamlines from $+\infty$ coming round the core and receding to $-\infty$ with only small differences in shape. Thus it is almost impossible to identify a particular vorticity distribution from rough velocity fields available from experiments of the quality described, and with the degree of averaging that has been involved it may well be virtually impossible to identify similarity.

Richards (1963) has reported an extensive set of observations on cylindrical thermals with both excesses and deficiencies of density on release from single troughs rotated about horizontal axes, from double troughs which could be opened in the tank interior and from "cut troughs" which were not further specified. In some cases the source fluid was effectively released from rest 'at an instant', but in others it emerged from the source as a thin laminar plume which drained into the rear of the nascent thermal. Some thermals were created in the interior of a tank and others against a wall. The observations were analysed using Scorer's two-dimensional scale analysis for cylindrical thermals released from rest,

$$z = nR, \quad z^{3/2} = C\left(\frac{Mg}{\rho}\right)^{1/2} t,$$

where $2R$ is the cross-sectional width, z and t the distance between virtual source and thermal front and the time since release at the virtual source, respectively, and M the mass excess per unit length. Richards found good agreement with n and C approximately constant during each realization but varying appreciably between realizations, with

$$1.49 < n < 3.17 \quad \text{and} \quad 1.05 < C < 3.33.$$

Richards (1965) carried out a further set of experiments with non-buoyant puffs generated from a puffer that could generate either axisymmetric or two-dimensional turbulent compact flows in a uniform tank of water. These flows again exhibited variability with n values for axisymmetric and cylindrical puffs,

$$2.2 < n < 6.2, \quad 3.4 < n < 5.6$$

respectively.

We refer finally to the experiments of Tsang (1971), who released line (or cylindrical) thermals in a uniform environment from an immersed horizontal tube which was initially filled with buoyant fluid and then withdrawn rapidly along its axis leaving free source fluid at rest under the action of buoyancy alone. This procedure is more nearly reproducible, but is not free from initial disturbance. Tsang considered that his measurements had no more than normal experimental error, but an analysis of his results suggests that

$$3.1 < n < 3.7 \quad \text{and} \quad 1.7 < C < 2.3,$$

a level of variability that is very similar to our own numerical simulations. Thus, Tsang's experiments have significantly improved control, but we suggest that much of the remaining and by no means insignificant variability may be inherent in the motion and not just an error of realization or observation.

3. SIMILARITY AND VARIABILITY

Experiments on unsteady convection are not easy to carry out accurately and it is inevitable that some experimental error should remain even in well planned experiments. We are, however, struck by the level of variability in all reported experiments. Moreover, thermal watchers will be aware that the front of a buoyant thermal immediately grows 'protuberances' and that these develop broadly in proportion with other length scales, with the result that each realization has its own geometry and while some are close to the average, others depart sufficiently to affect the conical spread angle and by inference the entrainment rate. The observational evidence is that almost all (if not all) thermals suffer conical development, supporting the postulate that there is a single length scale, but that the semi-angles of these cones vary considerably. In formal terms we can say that the thermals are *self-similar* in the sense that their geometry of evolution changes little with time and that they are characterized by a single length scale which provides a measure of that growth with time. However, in spite of their single length scale, each realization exhibits its own geometric development in which the pattern of growth is laid down very early and survives to a very large degree unchanged throughout that lifetime. In these circumstances it seems that we should regard thermals as a self-similar but not strictly similar family of flows.

4. THE STABILITY OF THERMALS

The initial interface between thermal and ambient fluid, whether for a heavy 'thermal' that will flow downwards or a light one that will rise, has denser fluid overlying less dense and is hydrostatically unstable at the instant of release. If we consider two infinite regions of fluid separated by a horizontal interface free from surface tension, when the upper fluid is more dense than the lower it may be shown readily that the interface is unstable to waves of all wavenumbers (wavelengths). In more restricted geometries the instability may be restricted to a discrete family of eigenmodes with a fundamental wavenumber setting the lower limit (or higher limit of wavelength) and a series of higher wavenumbers (shorter wavelengths) all suffering exponential growth. The surface separating source and ambient fluid at the moment of release of a thermal, by whatever mechanism, will also be gravitationally unstable for a family of eigenmodes with fundamental wavelength of order the diameter of the source and an infinite series of progressively shorter wavelengths.

The special property of the thermal lies in the fact that at the moment of release it is at rest and subject to buoyancy operating on the scale of the source. In energy terms, the freshly released thermal possesses potential energy by virtue of its difference in density from its surroundings, and that energy can flow equally into motion on the scale of the source, or into any one or more of the unstable modes of smaller length scale. The partitioning of energy acrossthese scales will depend in detail on the Fourier structure of the disturbance at release and the way in which it matches unstable source modes. We have seen that all release mechanisms contribute to this initial disturbance and there will be other contributions according to the nature and site of the experimental rig. Because its motion starts from rest and it is virtually impossible to eliminate disturbances caused by the release mechanism, the thermal is particularly sensitive at its earliest stages of motion, each thermal receiving a pattern of growth structure as the available potential energy flows into the permitted modes of kinetic energy during the earliest stages as motion develops.

We give two illustrations of variability in the motion of line (cylindrical) thermals.

(i) During a brief visit by Professor Baines in December 1990, we carried out a series of laboratory experiments on line thermals generated by releasing dense source fluid from an overturned trough in the surface of a tank of water. Initially we illuminated the thermals using a parallel beam of light along the line of the trough axis with shadowgraph visualizationon the end wall of the tank. We recorded what we took to be the straight-sided wedge of growth of a two-dimensional thermal. We then illuminated dyed thermals using thin vertical sheets of light at right angles to the trough and discovered that the motion was highly three-dimensional with a considerable proportion of the 'observed wedge' free from dye. That series of experiments has continued and will be discussed in our third paper. Meanwhile we have modelled line thermals with our LES code, and find that these too break immediately into cells along the source axis, and that these cells grow and merge as they ascend (or descend). Figure 1 shows isotherms in a vertical plane through the source axis of a line thermal $56s$ after the release of a horizontal cylinder of warm water with Gaussian temperature profile in a homogenous tank of cool water. Motion is vertically upwards and our supposed line thermal is highly three-dimensional.

(ii) Figure 2 shows a series of curves of height of the front of a line thermal above the base of the tank (Z_b) against maximum horizontal radius (R). Each has the same source strength (initial heat) but the curves represent motion of thermals initiated with a variety of levels of imposed small scale disturbance ranging from zero in the lowest curve to 0.23% in the highest. The two broken lines have slopes (values of n) 3 and 4, respectively, and it may be seen that these relatively small variations in the source disturbance at release significantly affect the spread angles of the resulting thermals throughout their lifetimes.

We have stressed the role of hydrostatic instability in the evolution of thermals, but we have seen that puffs exhibit variability too; this must originate in a hydrodynamic instability which is likely to occur at appreciably higher Reynolds numbers. This view is supported: (i) by the fact that *weak* puffs calculated with our LES numerical model exhibit little 'eddy structure'; and (ii) while relatively strong puffs do develop a structure of resolved eddies, they are much weaker than those of the LES thermals.

Figure 3 shows a section of a line puff in a vertical plane through its axis of release after $18s$. In the case illustrated the source had a top hat profile of initial vertical velocity $75mms^{-1}$ and an initial disturbance of 1.5% of the source velocity was imposed. The contours shown are for a passive marker.

5. POTENTIAL FLOW FROM A SOURCE

Our final contribution in what might have been treated more as a discussion of sources than flows is to return to the potential flow solution for a mass source. The mass sink is well known and, provided that the sink orifice is far from walls, produces an excellent approximation to the flow of a viscous fluid towards a small orifice through which fluid is extracted steadily. The corresponding solution for a steady mass source from an orifice through which fluid is discharged has been regarded as one of the less successful contributions of potential theory. Real viscous fluids must satisfy the no-slip condition at the walls of the pipe or channel supplying the source. Fluid flowing near the wall will suffer frictional loss of total head so that it is unable to expand through the orifice into the full solid angle but separates at some line in the orifice to form a directed jet with both mass and momentum fluxes. The potential solution is fatally flawed, not for lack of vorticity as it develops vortex sheets enveloping its boundaries and these correspond with a limiting representation of the boundary layers that exist at lower Reynolds numbers, but because in the absence of viscous diffusion vortex sheets cannot separate from the boundaries. However, we are in error in criticizing the solution which was never intended to represent a jet, and we should rather criticize the realization of that solution in which viscous dissipation near the boundary walls is responsible for flow separation to produce a jet rather than a source flow.

We now seek to modify the experimental realization of the potential source flow rather than the source solution and to do this it will help to reconsider the boundary conditions. The boundary layer at a fixed wall in a viscous flow may be regarded as the mechanism through which real fluids match the wall condition of zero velocity with the reality that flow at the outer edge of the boundary layer is almost parallel to the wall (i.e. that normal velocity is almost zero at the outer edge of the boundary layer for large Reynolds numbers). The corresponding condition for *inviscid* flow is that the velocity is zero at the wall *under the vortex sheet*, whereas at the outer edge of the vortex sheet the normal component of velocity is precisely zero while the tangential component cannot be specified but is determined by the outer flow. To produce a 'potential source flow' in a viscous fluid we must reproduce boundary conditions in the pipe or channel to the source orifice corresponding to those on the 'fluid side' of a vortex sheet/thin boundary layer, that is zero normal velocity and the same tangential velocity as the adjacent fluid. Figures 1 and 2 show an illuminated cross-section of two long narrow channels open at the bottom, each containing a strip piston. The piston on the left in each case is driven down by a rod from above and that on the right is pulled down by two sheets of plastic film attached at either side of the piston, passing down the respective channel walls, around the bottom edges and up through a central cavity contained within each wall. In an experiment the channels are immersed in a tank of water which has been seeded with particles of neutral density. The two pistons are then driven/pulled down at identical speeds and the ensuing motion photographed using a thin vertical sheet of light normal to the channels. In the run illustrated the pistons were driven down through $110mm$ in about $0.25s$ discharging a volume $2.2 \times 10^5 mm^3$. The pistons were then stopped impulsively. Figure 4 shows a time exposure over the period of descent and Figure 5 shows a second time exposure of duration $0.25s$ started $\approx 0.25s$ after the pistons had stopped. Both flows in Figure 4 are like source flows, in part because viscous flows always *start* like inviscid flows; but in Figure 5 the two are decidedly different with the reversible irrotational potential flow on the right brought instantly to rest, while the rotational part of the left hand flow survives as a vortex pair which propagates slowly to the bottom of the tank.

REFERENCES

Richards J.M.
Experiments on the motion of isolated cylindrical thermals through unstratified surroundings. *Int. J. Air Wat. Poll.*, 7:17, 1963.

Richards J.M.
Puff motions in unstratified surroundings. *J. Fluid Mech.*, 21:97, 1965.

Scorer R.S.
Experiments on convection of isolated masses of buoyant fluid.
J. Fluid Mech., 2:583, 1957.

Tsang G.
Laboratory study of line thermals. *Atmos. Environ.*, 5:445, 1971.

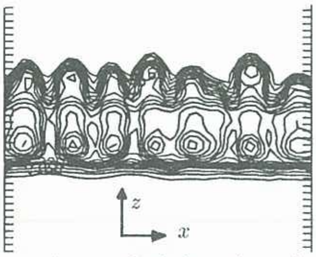

Figure 1: Isotherms in a vertical plane through the source axis showing horizontal structure of a line thermal $56s$ after release

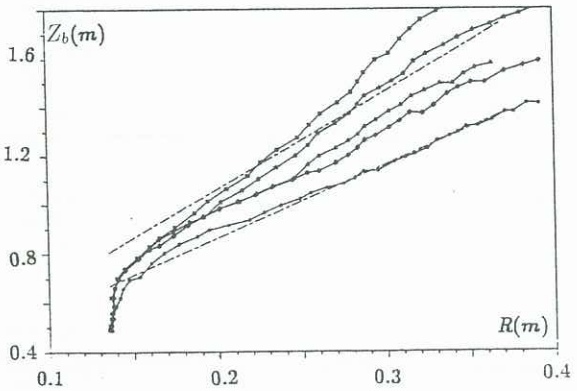

Figure 2: Relationship between leading edge displacement, Z_b, and halfwidth, R, for gaussian line thermals for levels of initial disturbances up to 0.23%.

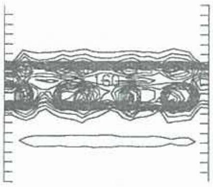

Figure 3: Section of a line puff in a vertical plane through its axis of release at $t = 18s$.

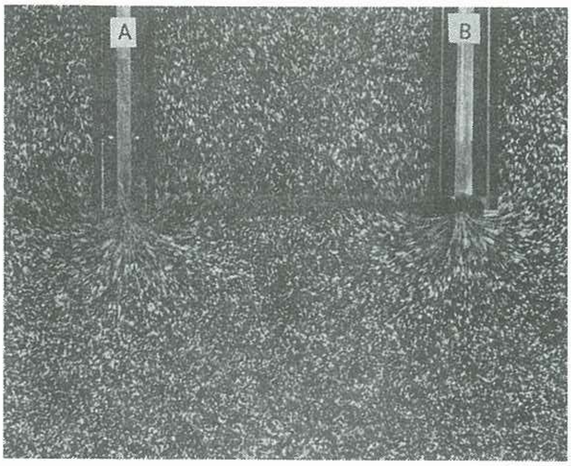

Figure 4: Time exposure over the period of descent for line pistons with (a) static wall boundaries, and (b) moving wall boundaries

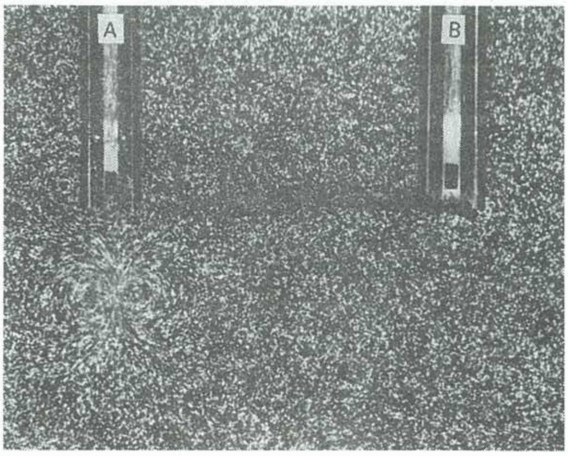

Figure 5: Time exposure of $0.25s$ taken approximately $0.25s$ after piston motion had ceased.