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RAINDROPS IN THE SEA I - GENERATION OF VORTICITY AND VORTEX RING PRODUCTION

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ABSTRACT

The effects of rain on the ocean are manifold and a large number of processes depend on the dynamics of the drop-surface interaction for their characteristics. An important part of this interaction is the development of vortex rings by impacting drops. This phenomenon has been reported widely in the literature and we re-examine the problem here from the point of view of vorticity. We review the extensive literature on drop-formed vortex rings and demonstrate that at no time has the appearance of vorticity been explained correctly. We conclude that a finite surface force must act during the coalescence process resulting in acceleration of the surface relative to the underlying fluid and generating the necessary vorticity. Also this force must be related in some way to the surface tension forces in the drop and the receiving fluid.

A better physical understanding of processes linking the atmosphere and oceans is important both to improve the accuracy of atmospheric and especially coupled ocean atmospheric climate models, and for remote sensing of the sea to determine surface winds and rainfall. The behaviour of rain and spray drops as they strike the sea surface has long been considered significant in transport processes between air and water, and hence as important in correctly modelling the ocean-atmosphere interface. However, the lengthscales of a few millimetres for drops and a few tens of kilometers for thunderstorms are difficult to inter-relate and neither can be resolved in broadscale models, so that we seek a sufficient understanding of these processes to ensure an effective parameterization. Osborne Reynolds (1875) proposed more than a century ago that vortex rings produced by the impact of raindrops on the sea surface cause vertical mixing of horizontal momentum, thereby dampening waves of short wavelength and reducing surface wind drag leading to reduced amplification of longer waves. The role of rain in producing surface calming of oceans has been discussed by a number of authors (including Sainsbury and Cheeseman, 1950; Manton, 1973; and Tsimplis and Thorpe, 1989) without the introduction of any satisfactory alternative damping mechanism, and our purpose is to reassess more than a century of rather variable contributions directing attention more than hitherto to the role of vorticity in droplet/surface interaction. It may be noted that impacting droplets also play an important role in gas exchange between atmosphere and ocean by trapping air bubbles and carrying them down several centimetres below the surface (Blanchard and Woodcock, 1957; Esmalizadeh and Mesler, 1986). These bubbles dissolve readily even in saturated water due to their excess vapour pressure, thus providing a mechanism for gas exchange between atmosphere and oceans that seems to have been little discussed in the meteorological/climate context. Further, the underwater noise spectrum of rain (Nysten, 1985; Prosperetti et al, 1989; Oguz and Prosperetti, 1990) appears to be dominated by resonant oscillation of air bubbles produced during impact, either by entrapment between drop and water surface or by 'pinching off' of a bubble at the base of the impact crater. Thus an understanding of the mechanics of the impact event is essential if we are to employ the underwater noise spectrum of rain as a means of monitoring rainfall over the oceans. Finally, the possibility of measuring rainfall directly by radar depends heavily on the splash dynamics of the impact process, the amount of backscatter 'clutter' depending on both the calming of the gravity waves and the generation of capilliary waves (Hansen, 1986).

Harlow and Shannon (1967a,b) show numerical calculations of the drop impact problem which ignore both surface tension and viscosity. Their results, whilst realistic in appearance, are later criticized by Carroll and Mesler (1981) as not representing reality because the results show no vortex rings. Oguz and Prosperetti (1990) show calculations of the impact dynamics in which they model surface tension effects ignored by Harlow and Shannon. Using a boundary element method they assume potential flow beneath the surface. This is explicitely irrotational and cannot reproduce the behaviour of the vortex ring. It seems unlikely that a phenomenon as dramatic as the production of a vortex ring during coalesence will not be a factor in surface deformation.

Thomson and Newall (1885) were the first to report that vortex rings form when drops fall into miscible liquid, and that the depth of penetration varies cyclically with the height of fall, linking the penetration to the oscillation of the falling drop and hence to its shape on impact. They observed that downwardly buoyant rings subdivide progressively into linked smaller vortex rings. They did not handle vorticity with confidence, and assumed a sheet of vorticity encompassing the drop after it has penetrated the water surface.

Sainsbury and Cheeseman (1950) found that the probability of ring formation was decreased significantly both with decreasing angle of incidence and with increasing proximity of neighbouring drops. Chapman and Critchlow (1967) related drop fall height to drop penetration and concluded that maximum penetration was achieved for drops which were spherical on impact and oscillating from oblate to prolate so as to offset flattening of the drop on impact. Their model for vortex ring formation was based on pressure gradients due to surface tension, but did not identify the source of vorticity nor explain why higher velocity impacts fail to produce vortex rings. Keedy (1967) found a relationship between fall height and penetration depth similar to that of Chapman and Critchlow, but did not attempt to explain ring formation.

Manton (1973) identified three mechanisms for rain attenuation of sea waves: vertical mixing of horizontal momentum by vortex rings; horizontal mixing of momentum by rings moving at an angle to the vertical; and an effective vertical pressure gradient due to decrease in ring momentum with depth. He concluded that only the first of these is significant in wave damping and calculated an expression for wave attenuation using an eddy viscosity representation for the effective stress due to the propagating rings. Hallett and Christensen (1984) catagorized the impacts (e.g. vortex ring formation, splash with crown, splash with Rayleigh jet, etc) in terms of drop diameter and an effective impact velocity obtained by adding the drop surface energy to the impact kinetic energy. They catagorized both laminar and turbulent vortex rings according to the size (and hence velocity) of droplets at terminal velocity.

Rodriguez and Mesler (1985, 1988) observed 'bouncing' and 'floating' drops, and identified drops which coalesced with the surface (low velocity, vortex rings formed) and drops which splashed (high velocity, no vortex rings). They reported that maximum penetration of an oscillating drop occurs when the the drop is prolate at impact, in conflict with Chapman and Critchlow (1967) and Keedy (1967).

Hsiao, Lichter and Quintero (1988) differed from Hallett and Christensen in their catagorization of drops, using Weber number (We, the square root of the ratio of drop kinetic energy to surface energy) rather than adding the surface energy to the impact kinetic energy as Hallett and Christensen had done. This led Hsiao et al to conclude that there was a critical value $We_c \approx 8$ below which vortex rings are formed on impact. The direct implication of this observation on the effects of rain over the sea is that the majority of raindrops have Weber numbers above this critical value and therefore will not produce vortex rings on impact. However, it has been observed that a characteristic of the splash event is the formation of a rebound column, or 'Rayleigh jet' in which the majority of the original droplet fluid is contained in the tip of the column (Kuo, Ichiye and Ichiye, 1975) which commonly breaks off to form a secondary drop. This secondary drop then falls from a height of a few centimetres, re-entering the water with a Weber number below the critical value.

DISCUSSION

The aforementioned work contains several inconsistencies and questions which need to be resolved. First and foremost is the question of the generation of vorticity. Only Thomson and Newall (1885) and Chapman and Critchlow (1967) have attempted explanations of the source of vorticity. These unsatisfactory explanations have either been accepted by other workers, or the question has been ignored.

Consider first the explanation of Thomson and Newall in 1885. They stated that at impact the drop fluid has a vortex sheet between itself and the receiving fluid which subsequently diffuses out to form the vortex ring. Oguz and Prosperetti (1990) calculated the magnitude of this vortex sheet and stated that it was insignificant, justifying their assumption of potential flow. The idea of a vortex sheet encompassing the drop is based on the entry of a solid sphere into a liquid and if correct would imply that larger impact velocities would lead to stronger vorticity, and that the reduction in surface tension by the addition of surfactants would have no effect on the generation of vorticity, the latter being contradicted by experimental observations. The concept of a vortex sheet due to the difference in velocity potentials between drop and receiving fluid before impact can be shown to be incorrect by the following argument. The Helmholtz vorticity equation for a homogenous fluid is:

$$\frac{D}{Dt}\left(\frac{\vec{\omega}}{\rho}\right) = \left(\frac{\vec{\omega}}{\rho} \bullet \vec{\nabla}\right)\vec{u} + \frac{\vec{\nabla}\rho \wedge \vec{\nabla}P}{\rho^3} + \frac{\nu}{\rho}\nabla^2\vec{\omega}.$$

In the absence of density variations the equation contains no generation terms in the interior of a fluid; all vorticity must come from the boundaries. For instance, for a rigid boundary vorticity can only be generated by a pressure gradient along a surface or by tangential acceleration of that boundary (Morton, 1984). Consider next the instant after contact is made between the drop fluid and the surface (figure 1). The point of contact may be considered to be locally flat and the only velocity gradients are normal to the streamlines. At this time the two fluids are part of the same continuous homogenous media, and there are no density variations through the fluid. The total circulation around the closed loop shown in figure 1 can easily be seen to be zero. From Stokes' theorum, the circulation of a vector around a closed loop is equal to the

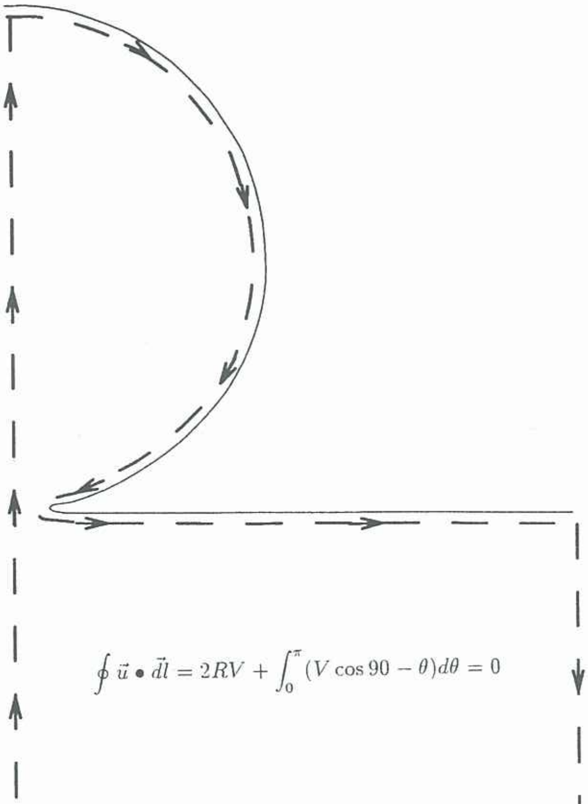

Figure 1 : Circulation in drop/surface system
immediately after contact

flux of the curl of that vector through a surface bounded by that loop, i.e. the total amount of vorticity present is zero. It easily shown that for one body of fluid to move through another, vorticity must be present. This means that from the moment of contact onwards, the fluid will maintain a total circulation of zero and the drop will be unable to penetrate into the receiving fluid *unless vorticity is generated at the boundary of the problem, i.e. at the free surface.*

This brings us to the second proposed mechanism for the vorticity generation put forward by Chapman and Critchlow. They stated tentatively that the pressure gradient due to surface tension effects which exists along the boundary of the fluid and causes the drop fluid to be accelerated downwards into the surface generates the vorticity. The pressure gradient is derived from the fact that surface tension effects increase the internal pressure of the drop relative to that in the receiving fluid. However, pressure is a force per unit area which remains constant as one approaches a vanishingly thin layer. The situation is never reached in which a finite force acts on an infinitesimally small mass and so the surface layer is not accelerated relative to the body of the fluid by a pressure gradient along a free surface. Neither is the body of the fluid accelerated relative to the surface, as would be the case for a presure gradient along a rigid boundary.

In addition to the lack of a satisfactory mechanism for the generation of vorticity, there is also no explanation offered by any previous workers of the observations that the droplet shape on impact affects the penetration depth of the resultant vortex ring, and that the lowering of the surface tension by the addition of surfactants also decreases the likelihood of vortex ring formation. To the first of these questions there is not even agreement between workers as to the observations, with Keedy (1967) and Chapman and Critchlow (1967) stating that the best vortex rings form when the drop is spherical on impact, and Rodriguez and Mesler (1988) claiming the drop should be prolate on impact for maximum penetration. The oscillation of drop shape observed in the laboratory is in general due to the conditions at the moment of drop release but these observations have relevence to the behaviour of real raindrops as these are also seen to oscillate (Jones,1959).

CONCLUSIONS

It is seen that the phenomenon of vortex ring production by rain over oceans has far reaching implications for the parameterization of precipitation in coupled ocean-atmosphere models both directly, where the wind drag on the ocean and the ocean-atmosphere fluxes must be estimated, and also indirectly in our ability to remotely measure precipitation over the oceans. The work over the last hundred years or so has concentrated on observations and empirical deductions based on those observations without leading to much understanding of the processes involved. We have demonstrated that all previously reported explanations of the source of vorticity are flawed. The conclusion is that the formation of the vortex ring can only occur if there is generation of vorticity during the coalescence process. Due to the lack of density gradients in the system after contact, vorticity must be generated at the free surface by some finite (non-vanishing) force acting at the surface of the fluid which, from evidence previously reported in the literature, must be linked with surface tension.

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