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CURVATURE ANALYSIS OF SPILLWAY PROFILES

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SUMMARY: A study of the relation between the radius of curvature r and length of arc s in spillway profiles indicate a well defined trend for successful profiles.This trend is the basis of a profile synthesis using the r-s intrinsic coordinates. On this basis a table of overflow spillway coordinates has been compiled for high dams with negligible velocity of approach. This profile satisfies the criteria of continuity of radius of curvature and higher derivatives.

1. INTRODUCTION:

Since Muller (1908) suggested that an appropriate shape for an overflow spillway, one that will avoid subatmospheric pressures at the face of the dam, is that of the underside of the nappe springing freely from a sharp crested weir, see figure 1-a, there have been many attempts at defining accurately this ideal weir shape.

Muller recommended a spillway profile based on the careful experiments of Bazin (1890), which also served as the base for Creager's profile (1917). The widespread acceptance of this idea led to further experimental determination of the overflow profile by a number of researchers, especially de Marchi (1928), Scimemi (1930), Randolph (1938), and the US Bureau of Reclamation (1932,1948) and Rouse (1949).

Potential flow solutions of the sharp crested weir flow problem have been obtained by numerical means by Mc Nown, Hsu and Yih (1953), Strelkoff (1964), and using the electric analogy by Hay and Markland (1958). These profiles have small but significant differences with the experimental profiles, in particular near the crest of the weir, where the potential flow profile departs tangentially from the weir wall. Most of the experiments show a departure angle of about 27°, due to the unavoidable boundary layer effects on the flow upwards the weir wall, figure 1.

Tests of dams constructed with the Bazin profile have shown repeatedly that as long as the head h in the dam does not exceed the head on the sharp crested weir that originated the profile, the pressures on the face of the dam remain positive, with the exception of a zone near the crest of the dam, where negative pressures are commonly observed, figure 2.

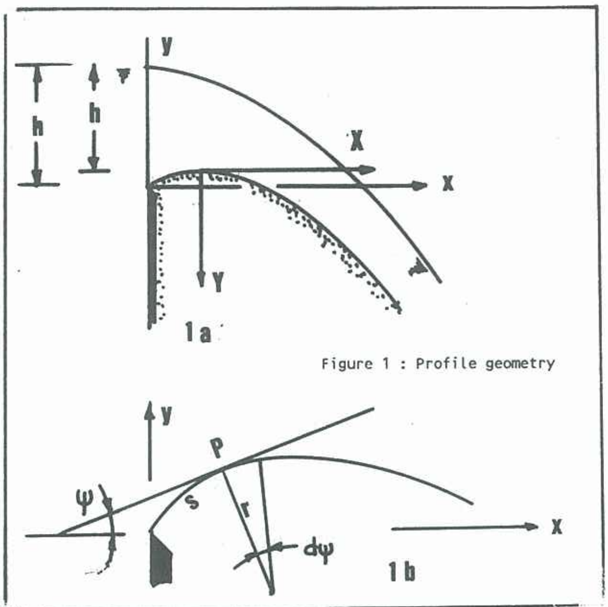

Figure 1 : Profile geometry

1.1 This phenomenon was noticed by the early researchers and it was found that very small variations of the dam profile near the crest eliminated or reduced considerably the negative pressures in that region.

Thus Rouse and Reid (1935) found in tests of Bazin profile spillways that by sanding down the upstream vertical wall of their model dam by .5 mm (equivalent to .007 of hd), the initially negative pressures on the crest were eliminated. The very small displacement of the vertical face had, apparently, the effect of increasing the radius of curvature at the junction with the crest profile.

Grzywienski (1951) noted the same effect on testing a modification of the Bazin profile.Whilst conserving the Bazin profile downstream of the crest apex, he replaced the forward part, comprised between x/h=0 to .3 by an elliptical quadrant, with axes (Normprofil II) of a=.300 h and b=.250 h.This simple variation raised substantially the crest pressures, figure 2. This modification of Bazin profile has been developed further by Reese and Maynord (1987).

The forward part of the crest region of Bazin's profile had been approximated by two arcs of circle by Randolph (1938), a form found also in the Waterways Experimental

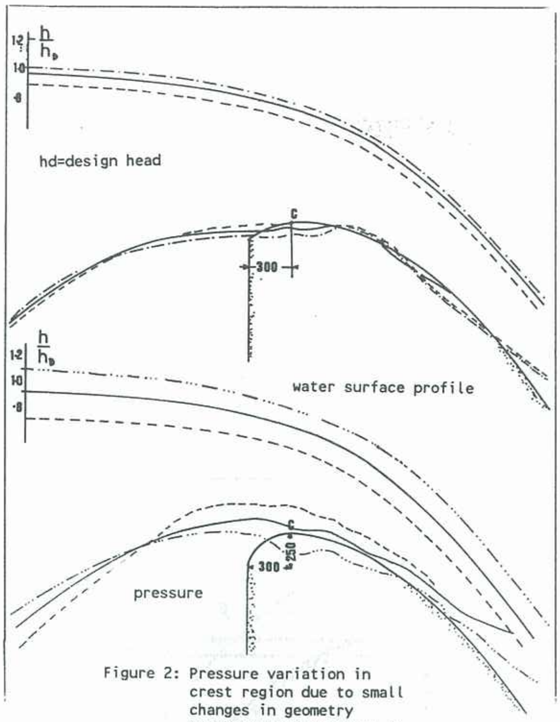

Figure 2: Pressure variation in crest region due to small changes in geometry GRZYWIENSKI NORMPROFIL II

Station (WES) profile (1952). This simplified representation led to a sharp discontinuity in the first derivative of the profile at the junction with the vertical face of the dam, as well as three discontinuities in the radius of curvature. It was shown by Abecasis (1962) that incorporation of a gradual transition at this point led to a substantial improvement in the crest pressures, even when curvature radii as small as .04= h were used for the transition. This modification has been adopted in the latest WES profiles.

1.2 The above mentioned examples reflect the powerful effect of small variation of the crest profile on the local pressures, a feature not sufficiently recognized in the early geometrical description of the spillway profile.

2. A REVIEW OF SOME CONTINUOUS SPILLWAY PROFILES:

Several authors have recognised that any sudden variation in the radius of curvature near the crest should be avoided, and have proposed continuous profiles with continuity of first and second derivatives for the the crest region and the early part of the parabolic tail.

- Knapp (1960) proposed, "after many fruitless trials" a continuous spillway profile that is described by the equation:

$$Y+ = X + -\ln(X+) \qquad (1)$$

where: $X+ = \frac{x}{rc} \qquad Y+ = \frac{y}{rc}$

are crest centered ordinates, and rc is the radius of curvature at the crest, assumed to be rc=.612 h. This profile applies to the crest region and early portion of the tail.

-A spillway profile, without discontinuities in the radius of curvature has been proposed and tested by Hager (1987,1992). Its equation is:

$$Y^* = - X^* \ln X^* \qquad (2)$$

where $Y^* = 2.7050\, y/h'$
$X^* = 1.3055\, x/h'$

The profile equation, when referred to sharp crested weir coordinates (which have been chosen here as the norm) is :

$$\frac{y}{h} = -.4828\,\frac{x}{h}\,(.3831 + \ln\frac{x}{h}) \qquad (3)$$

with a radius of curvature:

$$\frac{r}{h} = -2.0721\,\frac{x}{h}\,(1 + .2327\,(1.3831 + \ln\frac{x}{h}) \qquad (4)$$

It exhibits a vertical asymptote at x/h=0. Its highest point occurs for x/h=.25 , which is also the crest location in the WES 2 and 3 arc profiles.The radius of curvature at the crest is r/h=.520.

Other profiles with different equations to define the crest and the tail regions have been proposed by the US Army Engineers (1965), and by Reese and Maynord (1987).

3. CORRELATION OF THE RADIUS OF CURVATURE WITH THE LENGTH OF ARC:

An important property of the spillway profile is the relation of the radius of curvature with the length of arc measured from the beginning of the profile. Figure 3 illustrates this variation for certain cases mentioned in section 1. The data by Scimemi (1930) and Lane (Davis,1952) were found to be most consistent.

The experimentally derived data are in good agreement in respect of the variation of r with the arc s at the tail of the profile. The differences are slightly more pronounced of the profile.

4. ALTERNATIVE DERIVATION OF SPILLWAY PROFILE COORDINATES:

As the ideal spillway profile without negative pressures at a design head has been derived from measurements of a continuous physical process, the radius of curvature of this profile must also be continuous. It seems logical then to base improved profile coordinates on this simple premise.

Consider the geometry of the profile as defined on figure 1-b. If psi is the angle formed between the local tangent and a horizontal axis, then the element of arc ds and the radius of curvature are linked by:

$$r = \frac{ds}{d\psi}$$

Thus the angle is:

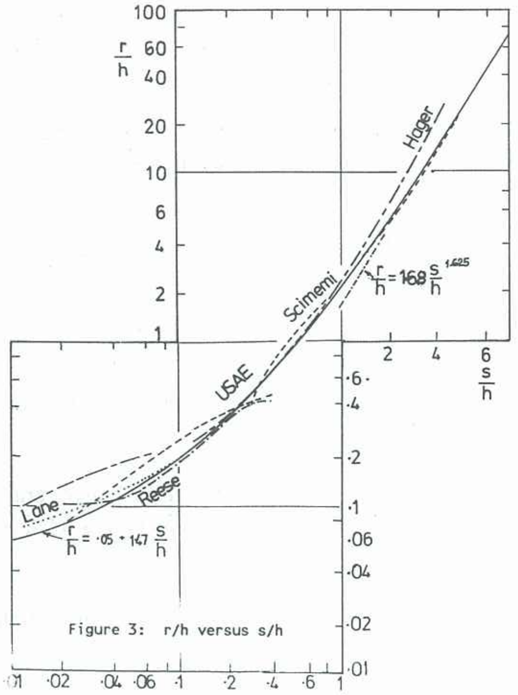

Figure 3: r/h versus s/h

$$\psi = \psi_0 + \int_0^s \frac{ds}{r} \qquad (5)$$

Where the integration constant psi0 is the profile angle at s=0, that is the angle of inclination of the wall with the horizontal.

The coordinates x and y follow from the relations dx=ds cos psi and dy = ds sin psi. After integration:

$$x = \int_0^s \cos\left(\psi_0 + \int_0^s \frac{ds}{r}\right) ds \qquad (6)$$

$$y = \int_0^s \sin\left(\psi_0 + \int_0^s \frac{ds}{r}\right) ds \qquad (7)$$

where the constants of integration for x and y have been adjusted to coincide with the point at which s=0.

This definition of the coordinates x,y depends on the relation between the radius of curvature and the arc. If such a relation is available, then a double quadrature provides the desired profile coordinates. They will have continuity of first and second derivatives even if the definition of r as a function of s is only piecewise continuous. If a continuous mathematical function links r and s the continuity of at least the third derivative is assured.

4. DETERMINATION OF THE RELATION BETWEEN r AND s FOR THE SPILLWAY:

Such a relation will be determined by interpolating the ensemble of data presented in figures 3 by continuous, smooth relation. Desirable properties of the interpolating function is fidelity to the trend of the r-s relationship even beyond the range of the original data. This makes undesirable the simple alternative of polynomial interpolation.

The trend of the r-s variation at low values of s is linear, the data being adequately represented by:

$$\frac{r}{h} = .05 + 1.47\,\frac{s}{h} \qquad (8)$$

(8) is the lower asymptote rl of the data.At higher values of s, an upper asymptote, ru, is found to be :

$$\frac{r}{h} = 1.68\,\left(\frac{s}{h}\right)^{1.625} \qquad (9)$$

An interpolation formula that assures a smooth variation between these two asymptotic values is :

$$\frac{r}{h} = rl\,\left(1 + \left(\frac{ru}{rl}\right)^{m}\right)^{\frac{1}{m}} \qquad (10)$$

The value of m is not critical and after some experimentation it was found that values in the range m = 2 to 3 were acceptable, m = 2.625 worked best. It may be noted that the initial value of r/h, for s=0 is almost precisely that suggested by Abecasis in his modification of the 2-arc WES profile.

5. RESULTS:

Equations 7 and 8 were integrated numerically and provided the spillway coordinates shown in Appendix I. They are compared in figure 6 with the ensemble of Bazin and Scimemi's data. There is close agreement throughout the whole experimental range. As the nose profile is so important, further comparison with the profiles of Hager and of USAE reveals very good agreement, with the only difference that the present profile has the maximum at x/h=.260, rather than at x/h =2.50 as in the case of Hager and USAE.

6. CONCLUSIONS

The relationship between the profile radius of curvature and the length of arc has been analysed for several of the current overflow spillway profiles, for the case of negligible velocity of approach. There is good agreement between them and a single, well defined trend can be extracted. An asymptotic interpolation of the r-s trend was made, and this relation is used to define the coordinates x,y of the spillway profile.

The advantages claimed for this method are the following:

-The profile obtained is continuous to at least third order in the derivatives.

-By imposing the condition that the profile must start tangential to the wall of the dam, the effect of boundary layer development along the vertical wall in distorting the true "zero pressure" profile is eliminated. Thus a profile is obtained that embodies naturally the empirical modifications of Grzywienski and of Reese and Maynord to the Bazin profile.

-As the trend of r with s is so clearly defined for high values of s, the profile coordinates can be obtained with

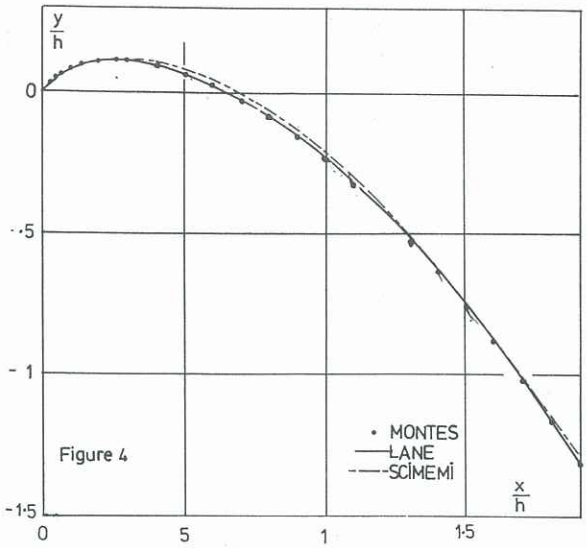

Figure 4: Comparison of present results with profiles of LANE and SCIMEMI

confidence for values of s exceeding the limit s/h=10 of figure 3.

-The method can be easily extended to the case of lower dams where the velocity of approach is important.

REFERENCES:

Abecasis, F.M. "Soleiras Descarregadoras", Laboratorio Nacional de Engenharia Civil, Lisbon, 1961.

Creager,W. "Engineering for Masonry Dams ", John Wiley and Sons, New York, 1917, pp.105.

Davis,C.V.(ed.) "Handbook of Applied Hydraulics", McGraw Hill, 1952 Section 7, Spillways and Stream Protection Works, by EW Lane and CV Davis.

DeMarchi,G. "Ricerche Sperimentali sulle dighe tracimanti" Annali dei Lavori Publici, Rome, 7, 1928.

Grzywienski,A. "Anti-vacuum profiles for spillwys of large dams "Transactions of the 4th Congress on Large Dams, 2, pp.105-124, New Delhi, 1951.

Hager,W.H. "Continuous Crest Profile for standard Spillway" Journal of Hydraulics Engineering, 113, 11, November 1987, pp.1453-1457.

Hager,W.H. "Experiments on Standard Spillway flow" Proceedings, Institution of Civil Engineers, Part 2,91, September 1991, pp. 399-416

Hay,N and Markland,E. "The determination of discharge over weirs by the electrolytic tank "Proceedings, Institution of Civil Engineers, 10, May 1958, pp.59-86

"Hydraulic Design of Spillways", Engineer Manual EM 1110-2-1603, Headquarters, Department of the Army, Office of the Chief Engineer., March 1965

Knapp,F.H. "Ausfluss, Uberfall and Durchfluss im Wasserbau" Verlag G. Braun, Karlsruhe, 1960.

McNown,J, En-yun Hsu and Chia-Shun Yih "Application of the Relaxation Technique in Fluid Mechanics" Proceedings, ASCE, Separate 223, July 1953.

Randolph,R.R. "Hydraulic Tests of the Spillway of Madden Dam" Transactions ASCE, Vol 103, 1938, pp. 1112.

Rouse,H. (Ed.) "Engineering Hydraulics", Ch.9 on Weirs and Controls, McGraw-Hill, 1949.

Rouse,H and Reid,L. "Model Research on Spillway Crests" Civil Engineering, 5, 1, p.1014
Scimemi, E. "Sulle forme delle vene tracimant" L'Energia Elettrica, April 1930, pp.293-305.

Strelkoff,Th.S. "Solution to highly curvilinear gravity flows" Journal of the Engineering Mechanics Division, ASCE, EM3, June 1964, pp.195-221.

US Army Corps of Engineers "Hydraulic Design Criteria", Waterways Experimental Station, Vicksburg, Miss. 1952.

U.S. Department of the Interior, Bureau of Reclamation, "Boulder Canyon Final Report" Bulletin 3, "Studies of crests for Overfall Dams", 1948.

Appendix I:
Profile coordinates for a high dam (negligible velocity of approach)

x/h	y/h	X	Y
0.0000	0.0000	-0.2600	-.1380
0.0200	0.0502	-0.2400	-.0878
0.0400	0.0715	-0.2200	-.0666
0.1000	0.1086	-0.1600	-.0294
0.2000	0.1346	-0.0600	-.0034
0.2600	0.1380	-0.0000	-.0000
0.3500	0.1319	0.0900	-.0061
0.4000	0.1232	0.1400	-.0147
0.5000	0.0965	0.2400	-.0415
0.6000	0.0586	0.3400	-.0790
0.7000	0.0106	0.4400	-.1273
0.8000	-0.0467	0.5400	-.1847
0.9000	-0.1127	0.6400	-.2507
1.0000	-0.1870	0.7400	-.3250
1.2000	-0.3595	0.9400	-.4974
1.4000	-0.5620	1.1400	-.7000
1.6000	-0.7933	1.3400	-.9313
1.8000	-1.0527	1.5400	-1.1908
2.0000	-1.3386	1.7400	-1.4766
2.2000	-1.6510	1.9400	-1.7890
2.4000	-1.9891	2.1400	-2.1271
2.6000	-2.3523	2.3400	-2.4903
2.8000	-2.7401	2.5400	-2.8781
3.0000	-3.1517	2.7400	-3.2897
3.5000	-4.2847	3.2400	-4.4227
4.0000	-5.5586	3.7400	-5.6966