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PREDICTION OF SOUND GENERATED BY A TWO-DIMENSIONAL JET IMPINGING ON A BODY

Osamu MOCHIZUKI, Masaru KIYA and Jin Woo PARK

Mechanical Engineering Department, Hokkaido University
Kita 13 Nishi 8, Kita-ku, Sapporo 060, JAPAN

ABSTRACT

The paper presents an analytical model to predict the sound generated by a two- dimensional jet impinging on a body. The model described in this paper is based on a composite pressure field of an incident pressure associated with vortices of the jet and a pressure reflected from the surface of the body. The composite pressure field yields a standing wave between a nozzle and the body whose positions correspond to the anti-nodes of the pressure fluctuation. The frequency f of the standing wave is obtained in terms of the convection velocity of the vortex U_c, the speed of sound c, and the distance l between nozzle and the body in the form $f=N(U_c/l)/(1+U_c/c)$, where N is an integer. This relation coincides with the result deduced from the feedback mechanism. The sound pressure level (SPL) can be predicted as a function of the amplitude of the standing wave at the nose of the body. The analytical model can explain the mechanism of which determines the frequency and the SPL. Moreover, the expression for the SPL involves the reflection coefficient of a material of the surface of the body so that it indicates a method to suppress the sound generated by the feedback mechanism by the surface of the body constructing by a sound absorption material.

1. INTRODUCTION

Aerodynamic sound has recently received much attention in industries because of the public concern about a quiet environment. The edge-tone is a typical example in the aerodynamic sound generated by free shear layers, thin jets, and wakes impinging on a body. Such flows characterized by the interaction between vortices and the body are observed in many kinds of engineering devices such as turbomachinery, heat exchangers, air conditioners and high speed vehicles. Several experimental (Karamcheti, et. al., 1969) and analytical (Crighton, 1992) studies aimed at understanding the relation between the fluid flow and the sound generated by the flow. The frequency of the sound has been studied in terms of the flow structures and instability of the jet. An attempt to predict the magnitude of the edge-tone was made by Powell (1961), considering the feedback mechanism. He showed that the gain of the feedback loop consists of the coefficient η_s related to the strength of the effect at the edge, the transmission effectiveness η_t of the disturbance from the edge, the communication effectiveness η_d between the shear layer of the jet and the disturbance, and the amplification q at the edge. These coefficients are combined with velocity fluctuations at each location where each coefficient is defined. He concluded that the effectiveness coefficients must satisfy the relation of $q\eta_s\eta_t\eta_d=1$ to sustain the edge-tone. His theory for the gain of the feedback loop is developed by means of a dipole source associated with the fluctuating fluid force acting on the edge. However, to obtain the gain it is required the frequency which is determined by the different expression from that of the gain.

Ho and Nosseir (1981) obtained details of the feedback mechanism of a jet impinging on a normal plane. They showed that disturbances propagating in the upstream direction at the speed of sound was generated in the impingement region. Kaykayogle and Rockwell (1986a,1986b) showed that the pressure fluctuation on the surface of the body is strongly related to the secondary vortex induced by the impinging jet vortex. These results support the feedback mechanism, and suggest that the pressure should be taken into account to formulate the sound pressure level (SPL).

This paper presents an analytical model to predict the frequency and the SPL generated by a jet impinging on a body. The formulation is based on composing an incident pressure wave due to the jet vortices and a reflected pressure wave generated by the impingement of the jet vortices on the surface of the body. The composite pressure field yields a standing wave between the nozzle and the body whose locations correspond to the anti-nodes of the pressure fluctuation. The frequency can be determined so as to form the standing wave. The magnitude of the standing wave is regarded as the amplitude of the near field pressure fluctuation (NFPF) measured at a position above the nose of the body. The composite pressure model can describe both the frequency and the magnitude of the NFPF.

2. FORMULATION

In experiments on the edge-tone a sinusoidal fluctuation of pressure is observed not only in the far field but also in the near field. Therefore, we should focus on the near field pressure fluctuation produced as a result of the interaction between vortices and a body. If we observe the pressure at a location where the vortices pass regularly, we will obtain the pressure fluctuation with a frequency depending on the convection velocity and an interval, namely a wavelength, of the vortices. On the other hand, the disturbance generated by the vortices impinging on the surface of the body propagates as a sound wave in all directions at the speed of sound, if the flow is subsonic. Therefore the interaction between the pressure waves due to the vortices and the disturbances is supposed to occur spatially. Assuming that the vortices travel only in the x-direction, the flow can be considered as a quasi-one-dimensional flow as sketched in Fig. 1. Thus the pressure fluctuation is a function of time t and a location x. An incident pressure $p_v(t,x)$ due to vortex of the jet moves toward the body located at x=0 with a velocity U_c which coincides with the convection velocity of the vortex. A reflected pressure $p_d(t,x)$

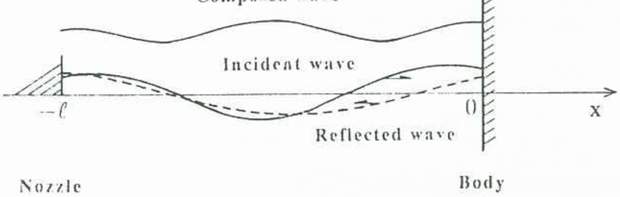

Fig. 1 Schematic of incident, reflected and composite pressure waves.

generated by the vortices impinging on the body propagates in the upstream direction with the speed of sound c. If these pressure change sinusoidally with a frequency ω (=$2\pi f$), these are written as follows.

$$p_v(t,x)=P_v\sin(\omega t-k_v x+\varphi_v) \quad (1)$$

$$p_d(t,x)=P_d\sin(\omega t+k_d x+\varphi_d) \quad (2)$$

where φ_v and φ_d are phases, and P_v and P_d are amplitudes. The wave number k_v and k_d are written as

$$k_v=2\pi f/U_c\text{, and } k_d=2\pi f/c \quad (3)$$

Adding Eqs. (1) and (2), the composite pressure p(t,x) is obtained as

$$p(t,x)=P(x)\sin(\omega t+\varphi(x)) \quad (4)$$

Here P(x) and $\varphi(x)$ are amplitude and phase functions written as

$$P(x)=[P_v^2+P_d^2+2P_vP_d\cos\{(k_v+k_d)x-(\varphi_v-\varphi_d)\}]^{1/2} \quad (5)$$

$$\tan\varphi(x)=\{P_v\sin(-k_vx+\varphi_v)+P_d\sin(k_d+\varphi_d)\} /\{P_v\cos(-k_v+\varphi_v)+P_d\cos(k_d+\varphi_d)\} \quad (6)$$

Frequency

The amplitude of the standing wave given by Eq. (4) attains the maximum at a location $x=-x_p$ when the following condition is satisfied.

$$\cos\{-(k_v+k_d)x_p-(\varphi_v-\varphi_d)\}=1 \quad (7)$$

Hence,

$$(k_v+k_d)x_p+(\varphi_v-\varphi_d)=2N\pi \quad (8)$$

where N is an integer. Since k_v and k_d are related to f by Eq. (3), the frequency is written as

$$f=(U_c/x_p)\{N-(\varphi_v-\varphi_d)/(2\pi)\}/(1+U_c/c) \quad (9)$$

Since the boundary must be the anti-node of the pressure of the standing wave, the location where the amplitude of the standing wave attains a peak value coincides with the location of the exit of the nozzle and the nose of the body. To satisfy the condition at the nose of the body ($x_p=0$), the phase difference ($\varphi_v-\varphi_d$) in Eqs. (7), (8), and (9) must be zero. Supposing that the nozzle locates at $x_p=l$, the frequency of the standing wave formed between the nozzle and the body is obtained by Eq.(9) as follows.

$$f=N(U_c/l)/(1+U_c/c) \quad (10)$$

This equation shows that the period of the standing wave is expressed by the sum of the time of the vortices moving toward the body with the convection velocity U_c and the time of the disturbance propagating distance l in the upstream direction with the speed of sound. This is essentially same as the idea of the feedback mechanism. Therefore, the feedback mechanism is found to be equivalent to the formation of the standing wave by the incident and the reflected pressure.

Amplitude

The amplitudes of the incident and the reflected pressures may change corresponding to as functions of the distance x measured from the nose of the body. The changes are assumed to be expressed by the following forms

$$P_v(x)=P_{v,0}f(x) \quad (11)$$

$$P_d(x)=P_{d,0}g(x) \quad (12)$$

where f(x) and g(x) are arbitrary functions of x with f(0)=1 and g(0)=1 at the nose of the body, and $P_{v,0}$ and $P_{d,0}$ are the amplitudes of the incident pressure wave and the reflected pressure wave at the nose of the body which is located at a distance x'=l from the nozzle. The function f(x) is supposed to express the strength of the vortex changing in the x-direction. The function g(x) expresses an attenuation of the acoustic intensity during its propagation. Moreover, $P_{v,0}$ is a function of x', since this depends on the strength of the vortex at the location of the nose of the body if the body were absent. In the similar manner as the acoustic reflection at the boundary, we can define the reflection coefficient α by

$$\alpha=J_d/J_v=U_cP_{d,0}^2/\{cP_{v,0}^2\} \quad (13)$$

where J_v $\{=P_{v,0}^2/(2\rho U_c)\}$ and J_d $\{=P_{d,0}^2/(2\rho c)\}$ are the intensity of the pressure fluctuations per unit area and unit time. The reflection coefficient α depends on the property of the material of the surface. Substituting Eq. (13) into Eq. (12), we have

$$P_d(x)=\eta_i(\alpha c/U_c)^{1/2}P_{v,0}g(x) =\eta_rP_{v,0}g(x) \quad (14)$$

where η_i is defined as the transfer coefficient of the disturbance from the pressure induced by the vortex at the surface, and $\eta_r=\eta_i(\alpha c/U_c)^{1/2}$ is redefined as the reflection coefficient at the surface of the body to the pressure waves with the different convection velocities. This reflection coefficient η_r can take the value larger than unity if $\alpha=1$ and $\eta_i=1$, since the ratio of c/U_c is larger than unity at low Mach number. Equation (14) suggests a method to suppress the disturbance to the incident pressure by using the sound absorption material, since the reflection coefficient α is related to the sound absorption coefficient α_a by $\alpha=1-\alpha_a$. Substituting Eqs. (11) and (14) into Eq. (5), the amplitude of the composed pressure is written as

$$P(x)=P_{v,0}[f(x)^2+\{\eta_rg(x)\}^2+2\eta_rf(x)g(x)\cos\{(k_v+k_d)x -(\varphi_v-\varphi_d)\}]^{1/2} \quad (15)$$

At the nose of the body (x=0) where f(0)=g(0)=1 and ($\varphi_v-\varphi_d$)=0, Eq. (15) becomes

$$P(0)=P_{v,0}(1+\eta_r) =\eta_sP_{v,0} \quad (16)$$

where η_s (=$1+\eta_r$) is the collision coefficient which is equivalent to the strength of the effect at the edge given by Powell.

From Eqs. (11) and (12) the amplitudes at the nozzle (x=-l) are written as

$$P_v(-l)=P_{v,0}f(-l) \quad (17)$$

$$P_d(-l)=P_{d,0}g(-l) =\eta_rP_{v,0}g(-l) \quad (18)$$

From Eq. (15), the amplitude of the composite pressure at the nozzle becomes

$$P(-l)=P_{v,0}[f(-l)^2+\{\eta_rg(-l)\}^2+2\eta_rf(-l)g(-l)]^{1/2} \quad (19)$$

Substituting Eq. (17) into Eq. (19), we have

$$P_v(-l)=P(-l)f(-l)/[f(-l)^2+\{\eta_rg(-l)\}^2+2\eta_rf(-l)g(-l)]^{1/2} =\eta_dP(-l) \quad (20)$$

where $\eta_d=f(-l)/[f(-l)^2+\{\eta_rg(-l)\}^2+2\eta_rf(-l)g(-l)]^{1/2}$ is a receptivity coefficient, being equivalent to the communication effectiveness given by Powell.

Finally, we can define the effectiveness coefficient η of the feedback loop in the following form.

$$\eta=P(-l)/P(0)$$
$$=f(-l)/(\eta_s\eta_d)$$
$$=[f(-l)^2+\{\eta_r g(-l)\}^2+2\eta_r f(-l)g(-l)]^{1/2}/(1+\eta_r) \qquad (21)$$

This equation is the most important result of this paper. Supposing that the value of g(-l) is unity (that is, the disturbance is not attenuated through propagation), Eq. (21) becomes

$$\eta=\{f(-l)+\eta_r\}/(1+\eta_r) \qquad (22)$$

Hence, the value of the amplitude function f(-l) is obtained by

$$f(-l)=\eta(1+\eta_r)-\eta_r \qquad (23)$$

Since f(-l) represents the amplitude of the pressure at the nozzle, we have $f(-l)\geq 0$. Thus, the effectiveness coefficient must take the value satisfying the following condition.

$$\eta \geq \eta_r/(1+\eta_r) \qquad (24)$$

From Eq. (24), the reflection coefficient must satisfy

$$\eta_r \leq \eta/(1-\eta) \qquad (25)$$

3. DISCUSSION

Comparison with experimental results

Amplitude

The near field pressure fluctuation (NFPF) was measured in a two-dimensional jet impinging on a circular cylinder. The height H of the nozzle was 15mm. The diameter D of the circular cylinder was 30mm, being twice H. The cylinder was located at l=4.8H downstream of the nozzle. The velocity U_0 of the jet was 40m/s at the nozzle, the velocity distribution at the nozzle being flat. The frequency of the sound was 270Hz, and the sound pressure level (SPL) was 110 dB at a position 20H above the cylinder.

Figure 2 shows the contour map of the cross correlation coefficient of the NFPF and the surface pressure fluctuation at this phase where a vortex of the jet located in the vicinity of the position at which the r.m.s. surface pressure attained the maximum on the surface of the cylinder. The value of the cross correlation coefficient exceeds 0.8 in the region just above the nose of the cylinder. This region corresponds to the location of the vortex. This result supports the idea that the pressure wave associated with the vortex can be expressed by Eq. (1), and the nose of the cylinder corresponds to the anti-node of the composite pressure. The effective value of the NFPF outside the jet plotted against the x-direction attained the maximum at the nose of the cylinder as shown in Fig. 3. The efficiency η of the feedback loop in this flow is obtained as 0.59 from Eq. (21).

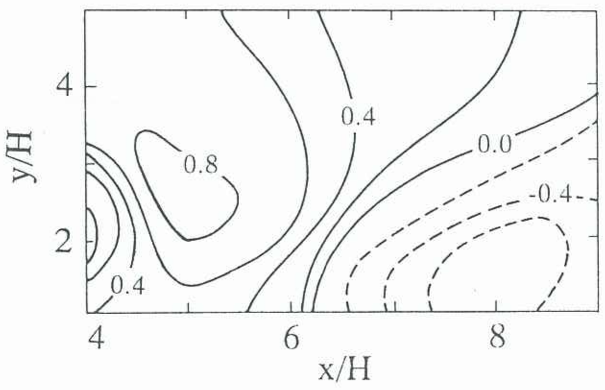

Fig. 2 Contour map of cross correlation coefficient of NFPF and surface pressure fluctuation on circular cylinder in experiment.

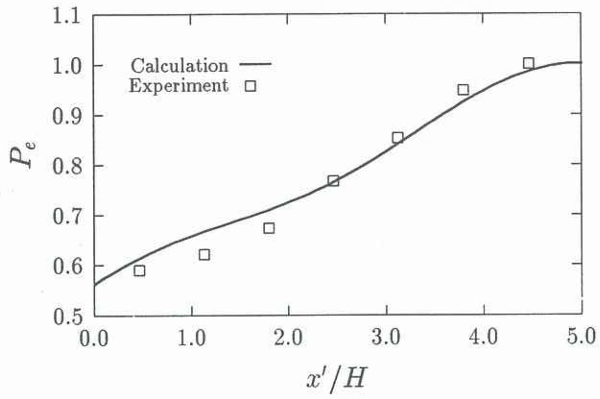

Fig. 3 Variation of effective value P_e of standing wave between nozzle and circular cylinder, being normalized by maximum effective amplitude and l_m: —, calculation; □ , experiment.

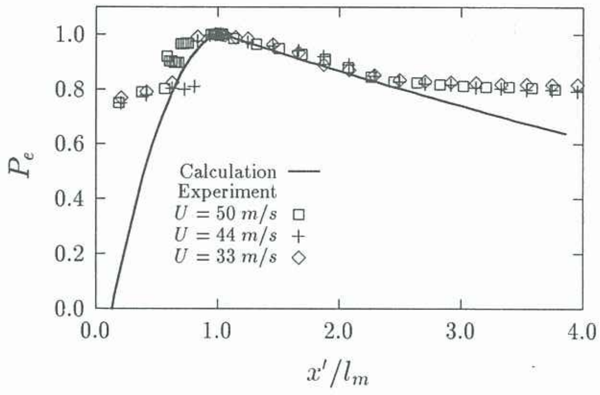

Fig. 4 Effective value of calculated amplitude of standing wave at nose of body and SPL measured at location 20H above circular cylinder against to distance x' from nozzle, being normalized by maximum effective amplitude and l_m: —, calculation; □+◇, experiment.

One considers the functions f(x) and g(x) in view of the experimental result. Figure 4 shows the SPL plotted against to the distance l from the nozzle to the cylinder. Normalizing the SPL by its maximum value and l by the distance l_m at which the SPL attained the maximum, the results obtained for different values of U_0 collapses onto a single curve. The distance l_m coincides with the position where the potential core of the jet disappears. The vortex of the jet grows linearly, becomes largest tat the end of the potential core and subsequently decays by the viscous diffusion. Thus we can assume the amplitude $P_{v,0}$ at the nose of the body which is located at a distance x'=l from the nozzle changing as follows.

$$P_{v,0}= \exp\{-k_c(x'-l_m)^2\} \quad \text{on} \quad \{0\leq x'\leq l_m\},$$
$$(x'-l_m+1)^{-1} \quad \text{on} \quad \{l_m\leq x'\} \qquad (26)$$

where k_c is a constant to be adjusted to have agreement with experimental results. Supposing that the amplitude function $P_v(x)$ given by Eq. (11) changes in the same manner as $P_{v,0}$, f(x') is written as

$$f(x')= \exp\{-k_c(x'-l_m)^2\} \quad \text{on} \quad \{0\leq x'\leq l_m\},$$
$$(x'-l_m+1)^{-1} \quad \text{on} \quad \{l_m\leq x'\} \qquad (27)$$

The disturbance propagating as a sound wave decreases with increasing distance from the body as a result of attenuation due to the molecular friction. Supposing the attenuation to be the same as the absorption of the plane sound wave, the reflected pressure wave at a distance x from the cylinder is expressed by

$$P_d(x)=P_{d,0}\exp(-k|x|) \quad (28)$$

where k is an attenuation coefficient given by

$$k=(33+0.2\varphi)\times10^{-12}f^2 \ [m^{-1}] \quad (29)$$

where φ (%) is the relative humidity. Comparing Eq. (28) and Eq. (12), the function g(x) is obtained

$$g(x)=\exp(-k|x|) \quad (30)$$

In view of the distribution of NFPF in Fig. 3 and $\eta=0.56$ we adopted the values of $\eta_r=0.05$ by Eq. (25) and $k_c=110.2$ by Eqs. (23) and (27). The solid line in Fig. 3 shows the calculated amplitude of the standing wave between the nozzle and the nose of the cylinder as a function of x. Good agreement with the experiment result is obtained. Figure 4 shows the variation (with respect to l) of the effective amplitude of the NFPF obtained experimentally at the position 20H above the cylinder and the calculated one of the standing wave at the nose of the cylinder. The solid line in Fig. 4 shows the calculated one, being normalized by the maximum effective amplitude and l_m to be compared with the experimental result. Though the calculation estimates larger values than the experimental ones, the change of the calculated effective amplitude of the standing wave seems to correspond to that of the NFPF. The attenuation of the NFPF depending on the frequency accounts for the difference between the analysis and the experiment. Taking into account of the simple forms of the functions f(x) and g(x), the analytical model based on the standing wave is appropriate to estimate the SPL of the sound generated by a two-dimensional jet impinging on a body. Thus, it is possible to obtain more precise estimation of the SPL if adequate functions for f(x) and g(x) are adopted to describe the flow.

<u>Frequency</u>

In order to obtain the frequency change with respect to the distance l between the nozzle and the cylinder by means of Eq. (9), one estimates the convection velocity of the vortex. Since the frequency f_c at the position l_m is known from the experiment, the convection velocity U'_c is written as, from Eq. (9) with N=1,

$$U'_c=l_m f_c/(1-l_m f_c/c) \quad \text{on } \{l\le l_m\} \quad (31)$$

This convection velocity is supposed to be applicable if the cylinder locates at the position $l \le l_m$, and to decrease according to the following manner if the cylinder locates at $l > l_m$. Since the velocity of the rectangular jet decreases in inverse proportion to the distance (Rajaratnam, 1976), we have

$$U_c=U'_c/(l-l_m) \quad \text{on } \{l>l_m\} \quad (32)$$

Substituting Eqs. (31) or (32) into Eq. (10), the frequency can be calculated.

In general it is a common feature that the frequency jumps occur at a certain distance between the nozzle and the body at a fixed main stream velocity. To realize this phenomenon by our model it is necessary to introduce a criterion of the frequency jump. To determine the criterion is equivalent to determine the integer N appearing in the frequency relation of Eq. (9). Supposing that the frequency jumps occur at the location of l_N ($=l_1, l_2, l_3,\ldots$, the suffix means the stage), the ratio of l_N to l_m appears to have the following relation from the experimental data.

$$l_N/l_m=2.4^N \quad (33)$$

The variation of the frequency given by this equation is shown in Fig. 5 with the experimental data. A tolerable agreement is obtained between the analysis and the experiment.

4. CONCULUSION

The analytical model has been proposed to describe the NFPF generated by a two-dimensional jet impinging onto a body by means of the composite pressure field of an incident pressure wave moving with the vortices of a jet and a reflected pressure wave as disturbances from a body. The results of the model are consistent with the feedback mechanism. The frequency of the sound and the effectiveness coefficient related to the sound pressure level is obtained by the model. The frequency is given by

$$f=N(U_c/l)/(1+U_c/c)$$

and the effectiveness coefficient is given by

$$\eta=[f(-l)^2+\{\eta_r g(-l)\}^2+2\eta_r f(-l)g(-l)]^{1/2}/(1+\eta_r)$$

where N is an integer and means the stage of the mode, and f(-l) and g(-l) are values of arbitrary functions calculated at the nozzle. The formulation of the SPL involves the reflection coefficient of a material of the surface of the body so that it indicates a method to suppress the sound generated by the feedback mechanism by constructing the surface of the body by a sound absorption material.

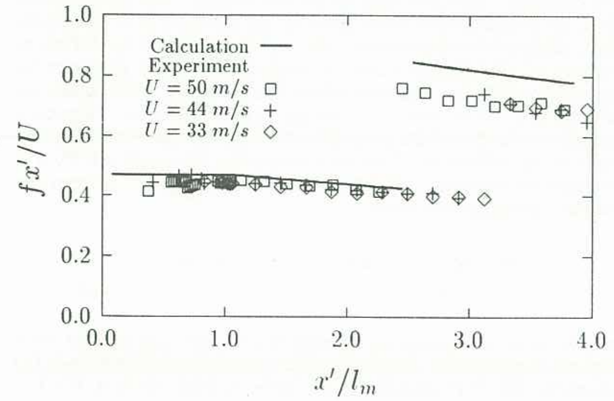

Fig. 5 Variation of fx'/U against to x'/l_m: —, calculation; □+◇, experiment.

REFFERENCES

Powell, A. (1961) On the edgetone. <u>J. Acous. Soc. Am. 33-4</u>, pp. 395-409.

Karamcheti, K., Bauer, A. B., Sields, W. L., Stegen, G. R. and Wooly, J. P. (1969) Some features of an edge-tone flow field. <u>NASA HQ Conf., Basic Aeronaut. Noise Res., NASA Spec. Publ. 207</u>, pp. 275-304.

Rajaratnam, N. (1976) Turbulent jets. Elsevier Scientific Publishing Co.

Rockwell, D. and Naudascher, E. (1979) Self-sustained oscillations of impinging free shear layers. <u>Ann. Rev. Fluid Mech. 11</u>, pp. 67-94.

Ho, C. M. and Nosseir, N. S. (1981) Dynamics of an impinging jet. Part 1. The feedback phenomenon. <u>J. Fluid Mech. 105</u>, pp. 119-142.

Kaykayoglu, R. and Rockwell, D. (1986a) Unstable jet-edge interaction. Part 1. Instantaneous pressure fields at a single frequency. <u>J. Fluid Mech. 169</u>, pp. 125-149.

Kaykayoglu, R. and Rockwell, D. (1986b) Unstable jet-edge interaction. Part 2. Multiple frequency pressure fields. <u>J. Fluid Mech. 169</u>, pp. 151-172.

Crighton, D. G. (1992) The jet edge-tone feedback cycle; linear theory for the operating stages. <u>J. Fluid Mech. 234</u>, pp. 361-391.