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JOINT STATISTICS BETWEEN TEMPERATURE AND THE TEMPERATURE DISSIPATION COMPONENTS IN A TURBULENT ROUND JET

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ABSTRACT

The joint statistics between the temperature fluctuation θ and the components, $(\partial\theta/\partial x_i)^2$, of the temperature dissipation are investigated in the self-preserving region of a slightly heated turbulent round jet. The correlation between θ and $(\partial\theta/\partial x_i)^2$ is found to be quite small. The assumption of statistical independence between θ and its dissipation appears to be more closely approximated in this flow than in other turbulent shear flows. Not unrelatedly, local isotropy is also more closely approximated in the present flow.

INTRODUCTION

The scalar dissipation $\epsilon_\theta = \alpha[(\partial\theta/\partial x_1)^2 + (\partial\theta/\partial x_2)^2 + (\partial\theta/\partial x_3)^2]$ — where θ is the scalar fluctuation, x_i ($i = 1,2,3$) are the spatial co-ordinates, and α the molecular diffusivity — and its dependence on θ have been found to be important in turbulent combustion modeling (e.g. Bilger, 1989). Firstly, the mean scalar dissipation $\overline{\epsilon}_\theta$ features in second order models of turbulent flows : e.g. for determining the time scale ratio $(\overline{q^2}\overline{\epsilon}^{-1}/(\overline{\theta^2}\overline{\epsilon}_\theta^{-1})$, where $\overline{q^2}$ is the average turbulent energy and $\overline{\epsilon}$ is the average dissipation of $\overline{q^2}$. Secondly, a knowledge of the interdependence of θ and ϵ_θ may reflect on the possible relationship between small scale and large scale structures. Thirdly, the joint probability density function (jpdf) of θ and ϵ_θ can be correlated to the average rate of creation or destruction of chemical species in both premixed (Bray, 1980) and diffusion (Bilger, 1980) flames since the average reaction rate is proportional to the expectation of ϵ_θ conditional on the stoichiometric value of θ (Bilger, 1989).

Measurements of $\overline{\epsilon}_\theta$ have been made in some non-reacting turbulent flows : e.g. a quasi-homogeneous shear flow (Tavoularis and Corrsin, 1981), a self-preserving plane jet (Antonia and Browne, 1983), a self-preserving plane wake (Antonia and Browne, 1986), a turbulent boundary layer (Krishnamoorthy and Antonia, 1987), a developing round jet (Namazian et al., 1988), and a self-preserving round jet (Antonia and Mi, 1992a). However, little information is available for joint statistics of θ and ϵ_θ. Iso-jpdf contours and correlations between the temperature fluctuation θ and an approximation to the temperature dissipation ϵ_θ obtained by Anselmet and Antonia (1985) in the preserving region of a slightly heated turbulent plane jet provided approximate support for the assumption of independence between θ and ϵ_θ. This assumption allows the jpdf $p(\theta, \epsilon_\theta)$ of these two quantities to be written as a product of the marginal probability density functions (pdf), viz.

$$p(\theta, \epsilon_\theta) = p(\theta)p(\epsilon_\theta) \quad . \tag{1}$$

Anselmet and Antonia noted that Eq. (1) seemed to be more reasonably supported by the data as the distance from the jet centreline increases. Using boundary layer measurements of $p\{\theta, (\partial\theta/\partial t)^2\}$ [where $(\partial\theta/\partial t)^2$ is the squared time derivative of θ], Anselmet et al. (1991) pointed out that the assumption is valid only in regions where $p(\theta)$ is symmetrical. However, Namazian et al.'s (1988) measurements, in the developing region of an isothermal methane round jet, showed that θ and ϵ_θ are almost uncorrelated on the axis [$5 \leq x_1/d \leq 17$, where $p(\theta)$ is not necessarily symmetrical], but highly correlated away from the axis (where the mean strain rate is significant). These results seem to suggest that the degree of correlation between ϵ_θ and θ is more likely to depend on the degree to which $\overline{\epsilon}_\theta$ approximates isotropy (in Namazian et al.'s experiment, $\overline{\epsilon}_\theta$ satisfies isotropy only on the axis) rather than on whether the pdf of θ is symmetrical.

In this paper, we present measurements of the joint statistics of θ and the three components of ϵ_θ in the self-preserving region of a slightly heated turbulent round jet, a flow which has been found to conform more closely with isotropy than the previously mentioned flows (see Figure 1 and Antonia and Mi, 1992a). The main aim of the present investigation is to assess the validity of Eq. (1) for the round jet, and identify the factors on which the correlation between θ and ϵ_θ may depend.

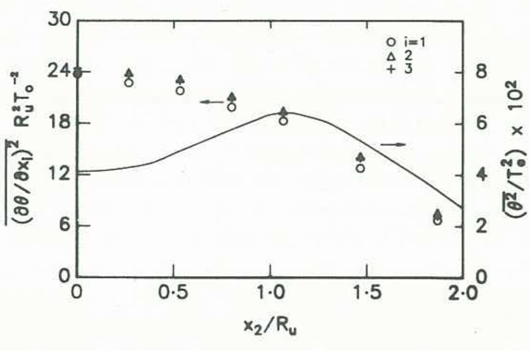

Figure 1 Distributions of $\overline{(\partial\theta/\partial x_i)^2}$ and $\overline{\theta^2}$.

EXPERIMENTAL SETUP AND TEST CONDITIONS

An axisymmetric nozzle with a 10 : 1 contraction ratio was used. The air supply was heated by an electrical fan heated (2.4 kW) located at the blower entrance. At the nozzle exit (diameter d of 25.4 mm), the velocity U_j and the temperature T_j were 11 m/s and 32°C (above ambient), respectively. The Reynolds number based on d was about 1.9×10^4.

All measurements were made at $x_1/d = 30$ (self-preservation was reached at $x_1/d \gtrsim 15$ in this flow; Chua, 1989) and were restricted to the nearly fully turbulent region ($0 \leq x_2/R_u \lesssim 1$) to avoid flow reversal and high local turbulence intensities. On the axis, the mean velocity U_0 and mean temperature T_0 were 2.1 m/s and 4.8°C (relative to ambient). The turbulence Reynolds number R_λ, based on the Taylor microscale λ ($= U_0^{-1}\overline{u_1^2}^{\frac{1}{2}}/\overline{u_{1,t}^2}^{\frac{1}{2}}$, where $u_{1,t} \equiv \partial u_1/\partial t$), was approximately 150 and the Kolmogorov length scale η [$\equiv (\nu^3/\overline{\epsilon})^{\frac{1}{4}}$, where ν is the kinematic viscosity of air] was about 0.17 mm. The Péclet number P_λ [$\equiv \overline{u_1^2}^{\frac{1}{2}}\lambda_\theta/\alpha$, where $\lambda_\theta = \overline{\theta^2}^{\frac{1}{2}}/\overline{(\partial\theta/\partial x_1)^2}^{\frac{1}{2}}$] was equal to 83. The half-radii R_u and R_θ, derived from mean velocity and mean temperature profiles, were 75 mm and 90 mm respectively. The ratio Gr/R_0^2 ($Gr \equiv gR_u^3 T_0/\nu T_a$ is the Grashof number, T_a is the absolute ambient temperature, $R_0 \equiv U_0 R_u/\nu$ is the local Reynolds number) is about 0.0027, indicating that the effect of buoyancy is negligible and justifying the use of temperature as a passive contaminant.

Spatial instantaneous derivatives of the temperature fluctuation θ were obtained using two parallel cold wires. Wollaston (Pt-10% Rh) wires of nominal diameter $d_w \simeq 0.63\mu$m were operated by in-house constant current circuits supplying 0.1 mA to each wire. At this current and for the experimental conditions outlined above, the velocity contamination of the temperature signal had a negligible effect on the statistics presented in this paper. The wires, with a nominal length ℓ_w of about 0.4 mm, were perpendicular to the flow direction. Each wire was carefully checked under a microscope for straightness immediately prior to the experiments. Care was taken to ensure that the etched portion of each wire was central and parallel so as to minimise the uncertainty in the measurement of Δx_i, the separation in the x_i direction (i = 1, 2 and 3 denote the axial, radial and azimuthal directions respectively) between the wires. Following a detailed investigation of the effect of Δx_i on $\partial\theta/\partial x_i$ (Antonia and Mi, 1992b), the separation Δx_i was chosen equal to about 3η since, for this value of Δx_i, the correction which had to be applied to obtain reliable values of $\overline{(\partial\theta/\partial x_i)^2}$ was relatively small. Also, this value of Δx_i is sufficiently large to avoid the relatively large uncertainty due to the electronic noise (Antonia and Mi, 1992a).

The diameter d_w and length ℓ_w of the wires were chosen so that the ratio ℓ_w/d_w ($\simeq 700$) was sufficiently large to avoid possible attenuation at low wavenumbers (Paranthoen et al., 1982) while the ratio ℓ_w/η was as small as practicable. At $x_1/d = 30$ and $x_2/R_u = 0$, the value of ℓ_w/η ($\simeq 2.6$) was small enough to avoid making a wire length correction [Wyngaard's (1971) calculations show that for a wire length of about 3η, $\overline{\epsilon}_\theta$ is attenuated by about 10%]. Only the central part of the Wollaston wires was etched to avoid difficulties associated with fully etched wires. For a given wire length, Paranthoen et al. (1982) found that the signal for a fully etched wire is more attenuated than that from a partially etched wire. Estimates of the temperature coefficient of the cold wires were made by mounting both wires at the jet exit using a 10 Ω platinum resistance thermometer operated in a Leeds and Northrup 8087 bridge (with a resolution of 0.01°C).

The cut-off frequency f_c of the low-pass filter was selected by viewing the time derivative spectra on the screen of a real-time spectrum analyser (HP3582A). Special attention was given to the degree of correlation between the two signals and to the time derivative spectra of these signals. If the spectra looked different, one or both of the wires were replaced by newly etched ones until no observable difference could be seen. The values of f_c were identified with the frequencies at which the derivative spectra were about 2–3 dB higher than those corresponding to the frequencies at which electronic noise first became important. These settings were determined at each measurement location and found to be the same for the two wires. At $x_2 = 0$ and $x_2 = R_u$, f_c was 2.2 kHz and 1.2 kHz respectively, while the Kolmogorov frequency f_k ($= \overline{U}/2\pi\eta$, $\overline{U}$ is the local mean velocity) was about 2.0 kHz and 0.8 kHz.

After filtering, the signals from the two wires were passed through buck and gain units to offset the D.C. components and provide suitable amplification prior to digitising the signals with a 12 bit A/D converter (RC Electronics) on a personal computer (NEC 386). A sampling frequency f_s equal to $2f_c$ was used in all cases and the record duration was 50 sec. The digital data were directly transferred from the personal computer to a VAX 8550 computer using an ETHERNET (fibre optic cable) link. The spatial and temporal squared derivatives were formed on the VAX computer.

RESULTS AND DISCUSSION

The jpdf of $(\partial\theta/\partial t)^2$, the quantity common to all the experiments and $(\partial\theta/\partial x_i)^2$ [i = 1, 2 or 3] is shown in Figure 2. Clearly, there are important differences between $(\partial\theta/\partial x_1)^2$ and either $(\partial\theta/\partial x_2)^2$ or $(\partial\theta/\partial x_3)^2$ as already noted by Anselmet and Antonia (1985) for a plane jet. Figure 2a implies a relatively high degree of correlation

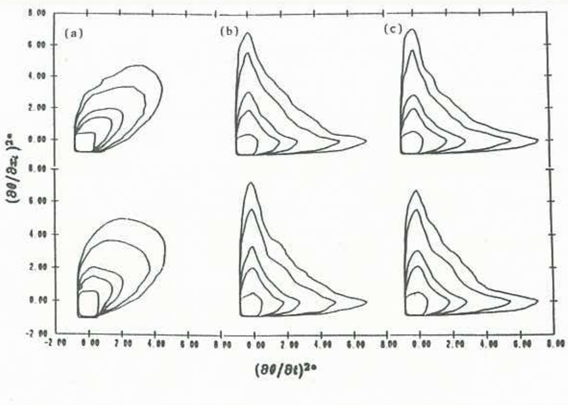

Figure 2 Normalised iso-jpdf contours between $(\partial\theta/\partial t)^2$ and $(\partial\theta/\partial x_i)^2$. (a) $p[(\partial\theta/\partial t)^{2*}, (\partial\theta/\partial x_1)^{2*}]$; (b) $p[(\partial\theta/\partial t)^{2*}, (\partial\theta/\partial x_2)^{2*}]$; (c) $p[(\partial\theta/\partial t)^{2*}, (\partial\theta/\partial x_3)^{2*}]$. Outer to inner contours : 0.0005, 0.001, 0.005, 0.01 and 0.1. Upper and lower contours are for $x_2/R_u = 0$ and 0.53, respectively.

Table I Correlation Coefficients Between θ and the Temperature Dissipation Components. The Temperature Skewness S_θ is Also Shown.

		$\overline{\theta^*(\partial\theta/\partial x_1)^{2*}}$			$\overline{\theta^*(\partial\theta/\partial x_2)^{2*}}$			$\overline{\theta^*(\partial\theta/\partial x_3)^{2*}}$		
x_2/R_u	S_θ	Total	$\theta^* \leq 0$	$\theta^* > 0$	Total	$\theta^* \leq 0$	$\theta^* > 0$	Total	$\theta^* \leq 0$	$\theta^* > 0$
0	−0.254	−0.019	−0.06	0.017	−0.023	−0.065	0.016	−0.023	−0.061	0.012
0.27	−0.246	−0.021	−0.066	0.022	−0.014	−0.032	0.003	−0.014	−0.036	0.004
0.53	−0.240	−0.002	−0.038	0.033	−0.011	−0.032	0.009	−0.025	−0.054	−0.001
0.80	−0.144	0.008	−0.017	0.032	−0.004	−0.018	0.009	−0.01	−0.032	0.01
1.07	0.205	0.072	−0.059	0.086	0.012	−0.018	0.043	0.002	−0.047	0.049

between $(\partial\theta/\partial t)^{2*}$ and $(\partial\theta/\partial x_1)^{2*}$ [the asterisk denotes that the variable is centered and normalised by the rms value]. The correlation coefficient between these two variables is in fact greater than 0.8 at all x_2 locations. Figures 2b and 2c show a close coincidence between large values of $(\partial\theta/\partial t)^{2*}$ and small values of both $(\partial\theta/\partial x_2)^{2*}$ and $(\partial\theta/\partial x_3)^{2*}$ and vice versa, indirectly suggesting that periods of strong activity in $(\partial\theta/\partial x_2)^{2*}$ and $(\partial\theta/\partial x_3)^{2*}$ may occur almost simultaneously and correspond to quiescent periods in $(\partial\theta/\partial x_1)^{2*}$.

A zero correlation between two variables may result from the independence between these variables but it can also be due to the symmetry of their jpdf (e.g. Tennekes and Lumley, 1972). Estimates of the correlation coefficients between θ and $(\partial\theta/\partial x_i)^2$ have been made both for $\theta \leq 0$ and $\theta > 0$. If the correlation coefficient is zero in both cases and $p[\theta^*, (\partial\theta/\partial x_i)^{2*}]$ is not symmetrical about $(\partial\theta/\partial x_i)^{2*}$ then $(\partial\theta/\partial x_i)^2$ should be independent of θ. In this flow, $p[\theta^*, (\partial\theta/\partial x_i)^{2*}]$ is not symmetrical about $(\partial\theta/\partial x_i)^{2*}$, e.g. the skewness of $(\partial\theta/\partial x_i)^2$ is in the range 10–20. As shown in Table I, the correlation coefficients are very nearly zero for both $\theta \leq 0$ and $\theta > 0$ at all flow locations. This implies a very weak dependence between θ and $(\partial\theta/\partial x_i)^2$ or between θ and ϵ_θ. This in turn suggests that Eq. (1) is approximately valid; direct checks of this equation (Figure 3) support this suggestion.

Table I indicates that the correlation coefficients $\overline{\theta^*(\partial\theta/\partial x_i)^{2*}}$ increase slightly as x_2/R_u increases, as observed by Anselmet and Antonia (1985) in the plane jet. However, the magnitude of these coefficients is much smaller than in the plane jet. Note that the temperature skewness S_θ (see Table I), which provides a measure of the degree of symmetry, exhibits a trend similar to that of the correlation coefficient, implying a possible dependence of $\overline{\theta^*(\partial\theta/\partial x_i)^{2*}}$ on S_θ.

Anselmet et al. (1991) concluded that the absence of correlation between θ and $(\partial\theta/\partial t)^2$ at $y^+ = 95$ (in a boundary layer) was mainly related to the symmetry of $p(\theta)$ — strictly, it should be related to the symmetry of $p[\theta, (\partial\theta/\partial t)^2]$ — since the contribution from $\theta > 0$ to the correlation was nearly balanced by that from $\theta < 0$. However, these authors also noted that each contribution is significant; tending to invalidate their claim of independence between θ and $(\partial\theta/\partial t)^2$. Note that Figure 3 indicates that Eq. (1) appears to be as closely validated on the axis (where the departure from symmetry is not small) as at $x_2/R_u = 0.8$ (where S_θ is negligible). Similarly, the lack of correlation between θ and ϵ_θ obtained by Namazian et al. (1988), in the developing region of a round jet, is associated with non-negligible asymmetry in $p(\theta)$ — or in $p(\theta, \epsilon_\theta)$ — on the axis ($|S_\theta| \simeq 0.2 - 0.4$, cf. Lockwood and Moneib, 1980; and Saetran, 1984). It would seem that for

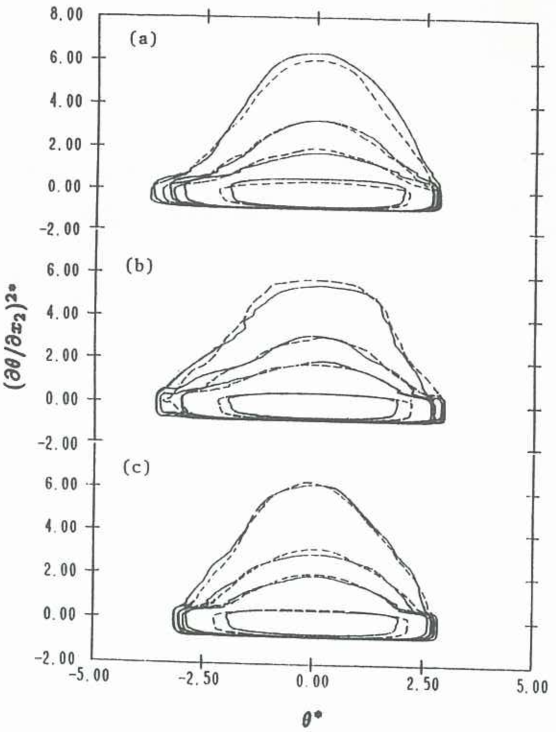

Figure 3 Independence check for the jpdf of θ and $(\partial\theta/\partial x_2)^2$. (a) $x_2/R_u = 0$; (b) 0.53; (c) 0.8. ----, $p[\theta^*, (\partial\theta/\partial x_2)^{2*}]$; ——, $p(\theta^*)p[(\partial\theta/\partial x_2)^{2*}]$. Outer to inner contours : 0.0005, 0.002, 0.006, 0.05.

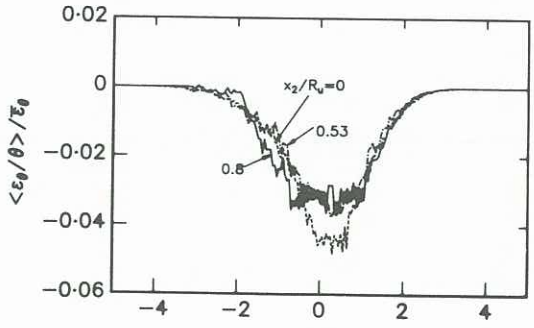

Figure 4 Averages of ϵ_θ conditioned on θ and normalised by $\overline{\epsilon}_\theta$.

the latter case, as in the present experiment, the degree of correlation between θ and ϵ_θ is mainly influenced by the degree to which $\bar{\epsilon}_\theta$ departs from isotropy (cf. Figures 5 and 6 in Namazian et al., 1988). Figure 1 indicates that $\bar{\epsilon}_\theta$ is nearly isotropic on the axis of the present jet, with only a small departure away from the axis. There are however other indicators of anisotropy in the present flow, e.g. the skewnesses of $\partial\theta/\partial x_1$ and $\partial\theta/\partial x_2$ are not zero. This anisotropy may be sufficient for the independence between θ and $(\partial\theta/\partial x_i)^2$ to be only approximate (Table I and Figure 2). The expected value of ϵ_θ (shown in Figure 4), conditioned on θ, indicates a small, but nonetheless measurable, departure from zero, $<\epsilon_\theta/\theta>$ tending to reach a maximum near $\theta = 0$.

CONCLUSION

The assumption of statistical independence between θ and $(\partial\theta/\partial x_i)^2$ or between θ and ϵ_θ is more closely validated in the present flow than in either the plane jet or the boundary layer. The assumption appears to depend more on the degree of local isotropy — implying a loss of dependence of the small scales on the large scales — than on whether the probability density function of temperature is symmetrical.

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