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WAVE TRANSFORMATION AND DISSIPATION ON STEEP REEF SLOPES

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ABSTRACT

The modified refraction–diffraction equation, developed recently by the author, is applied to evaluate wave transformation and dissipation, and wave set–up on the steep reef slopes. Waves are assumed to be random and narrow frequency banded. Two dissipation mechanisms are considered, i.e. wave breaking and bottom friction. To demonstrate the applicability of the model, numerical calculations and comparison with experimental data are given.

INTRODUCTION

The water motion due to surface waves is the most dynamic factor observed on the continental shelf and in the coastal zone. When waves are approaching the shore line, their structure is constantly changing due to bottom shoaling. In the classical approach, the conservation of wave energy flux for slowly–varying bottom topography is used to predict an evolution of wave height with depth. This approach is easily extended to the three–dimensional bottom and wave –refraction models, based on tray tracing over the shallower coastal area, have been used for many years.

If the bottom slope becomes steeper, the ray method is no longer able to predict the wave characteristics. The Great Barrier Reef is an example. In the northern section of the GBR the reefs are long, two–dimensional structures. Seaward of the reef the water depth rises very rapidly, from approximately 1000 m to a shallow reef crest at low tide level (Gourlay, 1988; Young, 1989). For such topographies, the refraction and diffraction effects are substantial and cannot be neglected. At reef edge, waves loose their stability and break. Once the waves start to break, the breaking processes dominate the wave transformation.

In the previous author's paper (Massel, 1992) the extended refraction - diffraction equation for surface water waves was developed. In this equation, the higher order terms in bottom slope, bottom curvature, and the evanescent modes were included. This equation can be considered as an extention of the mild–slope approximation of Berkhoff (1972) towards steeper bottom slopes.

In the present paper, the extended refraction–diffraction equation is applied to describe the transformation of waves over reef–like steep slopes. The energy dissipation due to wave breaking and bottom friction and wave set-up mechanisms are incorporated in the model.

GOVERNING EQUATION

The nature of wave effects upon a reef depends upon which part of the reef is under consideration. On the continental shelf, in the vicinity of a reef, water depth is deep enough to neglect the refraction. However, due to partial reflection from the reef edge, incident as well as reflected waves contribute observed wave patterns.

On the reef slope, waves undergo substantial changes due to shoaling and dissipation caused by wave breaking and bottom friction. Finally, on a reef flat, the water depth is usually almost constant and only waves propagating from the reef edge are present.

Offshore reefs are usually subjected to action from locally generated wind waves, as well as from long swell propagated from the Coral Sea (Wolanski, 1986). For weak local wind circulation, swell predominates and the wave energy spectrum is narrowing. The case of narrow spectrum is considered in the present paper, i.e. $S(\omega) = \sigma_\zeta^2 \delta\left(\frac{\omega}{\omega_p} - 1\right)$, where frequency ω_p is a peak frequency and σ_ζ^2 is a variance of the spectrum.

The Rayleigh distribution is used to describe the random nature of wave heights (Massel, 1989):

$$pdf(H) = \frac{2H}{H_{rms}} \exp\left[-\left(\frac{H}{H_{rms}}\right)\right], \tag{1}$$

in which H_{rms} is a root–mean–square value of wave height.

Let the origin of a rectangular coordinate system $0(x, y, z)$ be taken in the mean free surface of the fluid, and the axes chosen so that the x and y coordinates are horizontal and y coordinate is extending alongshore. The z–coordinate is vertical and increasing upwards. Bottom contours are straight and parallel, i.e. $h = h(x)$. In this instance, no special assumption is made on bottom slope.

The total energy flux on reef slope is properly described by using an energy density spectrum with group velocity integrated over all frequencies and directions. Unfortunately, the knowledge on the evolution of energy fluxes for a broad frequency banded nonlinear wave field in shallow water, and particular in surf zone, is still very poor. Therefore, to evaluate the H_{rms} transformation, we use the fact that incident wave spectrum is very narrow and incident wave train can be represented as a regular wave train with the wave height H_{rms0} and frequency ω_p.

Using the extended refraction–diffraction equation for regular waves (Massel, 1992), and a representation of the dissipation term as suggested by Booij (1981) and

(Massel, 1991), we obtain the following equation for transformation of the complex wave height $\hat{H}_{rms}$, in a very narrow frequency banded wave train over a straight and parallel bottom contour:

$$\frac{d^2\hat{H}_{rms}}{dx^2}+\frac{\frac{d}{dx}(CC_g)}{CC_g}\frac{d\hat{H}_{rms}}{dx}+[r(x)+i\gamma k]\,\hat{H}_{rms}=0, \qquad (2)$$

in which:

$$r(x)=k^2-\chi^2+\frac{1}{2ph^2}\frac{kh}{\tanh(kh)}\cdot\left[R_1(kh)\left(\frac{dh}{dx}\right)^2+R_2(kh)\frac{\frac{d^2h}{dx^2}}{k_0}\right], \qquad (3)$$

$$C_g=pC, \qquad p=\frac{1}{2}\left(1+\frac{2kh}{\sinh 2kh}\right), \qquad (4)$$

$$\chi=k\sin\Theta=k_i\sin\Theta_i=const, \qquad (5)$$

in which γ is a damping factor and $i=\sqrt{-1}$ is an imaginary unit, C is a phase velocity, C_g is a group velocity and wave number in deep water $k_0=\frac{\omega^2}{g}$, Θ_i is an incidence angle at some water depth $h=h_i$. The wave number k is given by the dispersion relation:

$$\omega^2=g\,k\tanh(kh) \qquad (6)$$

The $R_1(kh)$ and $R_2(kh)$ are complicated functions of kh. They are given in another paper (Massel, 1992).

The effect of the dissipation term ($i\gamma k$) becomes more evident when (2) is presented in the form of transport equation. The complex wave height $\hat{H}_{rms}$ can be represented as:

$$\hat{H}_{rms}(x)=H_{rms}(x)\exp[iS(x)], \qquad (7)$$

Substitution of (7) into (2) and separation of real and imaginary terms yields the wave energy transport equation in the form:

$$\frac{d}{dx}\left[\left(\frac{dS}{k\,dx}\right)C_gH^2{}_{rms}\right]=-\gamma C_g\,H^2{}_{rms} \qquad (8)$$

or

$$\frac{d}{dx}\left[\left(\frac{dS}{k\,dx}\right)C_gE\right]=\frac{-1}{8}\rho g\gamma C_g\,H^2{}_{rms}=-<\epsilon>, \qquad (9)$$

in which:

$$E=\frac{1}{8}\rho gH^2{}_{rms} \qquad (10)$$

The phase $S(x)$ in (7) is a solution of the eikonal equation. From (9) is clear that term $<\epsilon>=-\frac{1}{8}\rho g\gamma C_gH^2{}_{rms}$ represents the average rate of energy dissipation (per unit area) due to wave breaking or bottom friction.

ENERGY DISSIPATION RATE ϵ

Energy dissipation in waves approaching and propagating over a shallow reef is governed by two principal processes: energy losses due to breaking and bottom friction, i.e. $<\epsilon>=<\epsilon_b>+<\epsilon_f>$. Using this representation in (9), we obtain:

$$\gamma=\gamma_b+\gamma_f, \qquad (11)$$

in which:

$$\gamma_b=\frac{8<\epsilon_b>}{\rho gC_gH^2{}_{rms}} \qquad and \qquad \gamma_f=\frac{8<\epsilon_f>}{\rho gC_gH^2{}_{rms}} \qquad (12)$$

Damping Factor γ_b (due to wave breaking)

To determine the quantity γ_b, the similarity between a breaking wave and the phenomenon of a bore is utilized as a basis for the analysis. The bore model has proved to be very useful in many previous studies (Battjes and Janssen, 1978; Gerritsen, 1981; Thornton and Guza, 1983; Massel, 1991; Massel and Belberova, 1991).

The rate of energy dissipated per unit area for each bore can be represented as:

$$<\epsilon_b>=\frac{\alpha\rho g\omega_p}{8\pi}\frac{\sqrt{gh}}{C}\frac{H^3{}_{rms}}{h}, \qquad (13)$$

in which α is a breaker coefficient of $O(1)$ (Massel and Belberova, 1991). The average rate of energy dissipation is obtained by multiplying the dissipation for a single broken wave of height H by the probability of wave breaking at each height $p_b(H)$. Thus we have:

$$<\epsilon_b>=\frac{\alpha\rho g\omega_p}{8\pi}\frac{\sqrt{gh}}{Ch}\int_0^\infty H^3\,p_b(h)\,dH \qquad (14)$$

Following Thornton and Guza (1983), we express the distribution of breaking wave heights as a weighting of the Rayleigh distribution for all waves

$$p_b(H)=W(H)\,pdf(H) \qquad (15)$$

The area under the distribution is equal to the percent of breaking waves, i.e. $A_b=\int_0^\infty p_b(H)\,dH$. The simple hypothesis that the waves break in proportion to the distribution for all waves, yields (Thornton and Guza, 1983):

$$W(H)=\left(\frac{H_{rms}}{\Gamma h}\right)^n \qquad (16)$$

The experimental data reported by Hardy et al (1990) showed $\Gamma\approx 0.3-0.5$. After substitution (1) and (15) into (14) we obtain:

$$<\epsilon_b>=\frac{3\alpha\rho g\omega_p}{32\sqrt{\pi}}\frac{\sqrt{gh}}{C}\frac{H^3_{rms}}{h}\left(\frac{H_{rms}}{\Gamma h}\right)^4 \qquad (17)$$

In (17) the value $n=4$ was used. Now, from (12) we obtain:

$$\gamma_b=\frac{3\alpha\omega_p}{4\sqrt{\pi}}\frac{\sqrt{gh}}{CC_g}\frac{H_{rms}}{h}\left(\frac{H_{rms}}{\Gamma h}\right)^4 \qquad (18)$$

Damping Factor γ_f (due to bottom friction)

The dissipation rate ϵ_f in the wave boundary layer is given by:

$$\epsilon_f=\frac{2}{3\pi}\rho f_w u_b^3, \qquad (19)$$

where f_w is the friction loss factor, and u_b is the velocity amplitude at the bed. Substitution (19) into (12) gives:

$$\gamma_f=\frac{16f_w}{3\pi}\frac{u_b^3}{gC_gH^2_{rms}} \qquad (20)$$

Again assuming, a very narrow-band wave spectrum, with central frequency ω_p and applying solution of the extended refraction-diffraction equation (2), the bottom velocity u_b is:

$$u_b=\frac{g}{2\omega\cosh kh}\frac{dH_{rms}}{dx} \qquad (21)$$

For rough turbulent flow, the friction factor is independent of the Reynolds number and is a function of the ratio $\left(\frac{a_S}{k_s}\right)$, in which a_δ is the maximum horizontal excursion of a water particle near the bottom from the mean position, and k_s is a bottom roughness parameter.

On a reef, a coefficient f_w is much higher than that on a beach. Gerritsen's field experiments (Gerritsen, 1981) suggest that on reef slope and reef flats $f_w \approx 0.1 - 1.0$.

Finally, after substitution (11), (18) and (20) into (2), we obtain a differential equation for transformation of wave height H_{rms} over the bottom contour. To solve this equation, boundary conditions for H_{rms} at certain water depths should be prescribed. Let the surrounding bathymetry be set to a constant depth $h = h(0)$, which is the depth of the continental shelf. At that point, a superposition of incident and waves reflected from a reef is assumed. On the reef platform, at some distance from the reef edge, the water depth is constant and waves, reformed as oscillatory, progressive waves, are assumed.

WAVE SET-DOWN AND SET-UP

So far the water depth $h(x)$ was described as a depth of the bottom below some reference plane, i.e. still water level. However, the spatial changes in the radiation stress, due to shoaling, refraction, diffraction and dissipation, result in changes of the sea level $\bar{\eta}$, according the following balance of momentum (Longuet–Higgins and Stewart, 1964):

$$\frac{dS_{xx}}{dx} + \rho g[h + \bar{\eta}]\frac{d\bar{\eta}}{dx} = 0, \tag{22}$$

in which:

$$S_{xx} = \left(\frac{3}{2}p - \frac{1}{2}\right) E + \frac{1}{2}Ep\cos 2\Theta, \tag{23}$$

where E and p are given by (10) and (4), respectively. After substitution (23) into (22) we obtain:

$$\frac{d\bar{\eta}}{dx} = \frac{-1}{8(h+\bar{\eta})} R(x) \tag{24}$$

in which:

$$R(x) = [(3+\cos\Theta)p - 1]\, Hrms\frac{dHrms}{dx} + \frac{1}{2}(3 + \cos 2\Theta)H^2_{rms}\frac{dp}{dx} - pH^2_{rms}\sin 2\Theta\frac{d\Theta}{dx} \tag{25}$$

Equations (2) and (24) form a system of two equations for unknown root-mean-square wave height H_{rms} and mean water level $\bar{\eta}$.

NUMERICAL SOLUTION

To solve a system of equation (2) and (24), the finite - difference method was used. As the damping factor γ and set-down(set-up) $\bar{\eta}$ are the functions of unknown H_{rms} wave height, a reccurent scheme was developed, in which the actual water depth was improved by computing $\bar{\eta}$, until the prescribed accuracy was satisfied.

Consider now an elongated reef situated in unrestricted open water and subjected to severe cyclone conditions. The cross-section of the reef is given in Fig. 1. The incident water depth is $h_i = 32m$ and water depth on the reef flat is $h_p = 2m$. The depth on the reef slope is changing according to the formulas:

$$h(x) = \frac{h_1 + h_p}{2} - \frac{h_i - h_p}{2}\tanh[p(x)] \tag{26}$$

$$p(x) = \frac{2}{n(h_i - h_p)}\left\{x - \left[x_0 + \frac{n(h_i - h_p)}{2}\right]\right\}, \tag{27}$$

in which $(1/n)$ is a tangent of a maximum slope (in our case: $1/n = 1/1$) at the abscissa $x = x_0 + \frac{n(h_i - h_p)}{2}$; x_0 is an adjusting parameter. It should be mentioned that this reef configuration is very similar to that considered by Nelson and Lesleighter (1985) in their physical model. However, a more realistic inclined slope was adopted instead of a vertical reef edge. Changing the parameter n in (26) results in the changing of this slope, assuming that depths h_i and h_p are constant.

Let us now select the incident H_{rms0} value to be equal $4.0m$ and the peak period $T_p = 8s$. The results of calculations are presented in Fig. 1 in a form of functions $H_{rms} = H_{rms}(x)$, $\bar{\eta} = \bar{\eta}(x)$ and $A_b = A_b(x)$. At the front of reef H_{rms} wave height shows oscillations due to interference of incident waves and waves reflected from the reef edge. The calculations have shown that the reflection coefficient $K_R = 18.8\%$ and transmission coefficient $K_T = 29\%$. It means that $\approx 88\%$ of energy was dissipated.

Waves start to break at water depth $h \approx 7m$. The breaker index $\xi = \frac{1}{n}\left(\frac{H}{L}\right)^{-\frac{1}{2}} \approx 5$; thus breakers belong to plunging and collapsing categories. The full breaker ($A_b = 100\%$) was developed at abscissa $x \approx 67m$ and was continued along the distance less than one wave length. Afterwards the intensity of breaking decreases. At $x \approx 190m$ the percentage of broken waves is $\approx 1.8\%$ and the waves reformed again as oscillatory waves.

The evolution of wave height results in changes of the mean water level $\bar{\eta}$. On the reef top the wave set–up $\bar{\eta}$ is equal to $\approx 16.8\%$ of the incident wave height. It should be noted that decreasing of wave height at the reef front (at water depth $h \approx 30m$), results also in a small set–up ($\approx 2cm$).

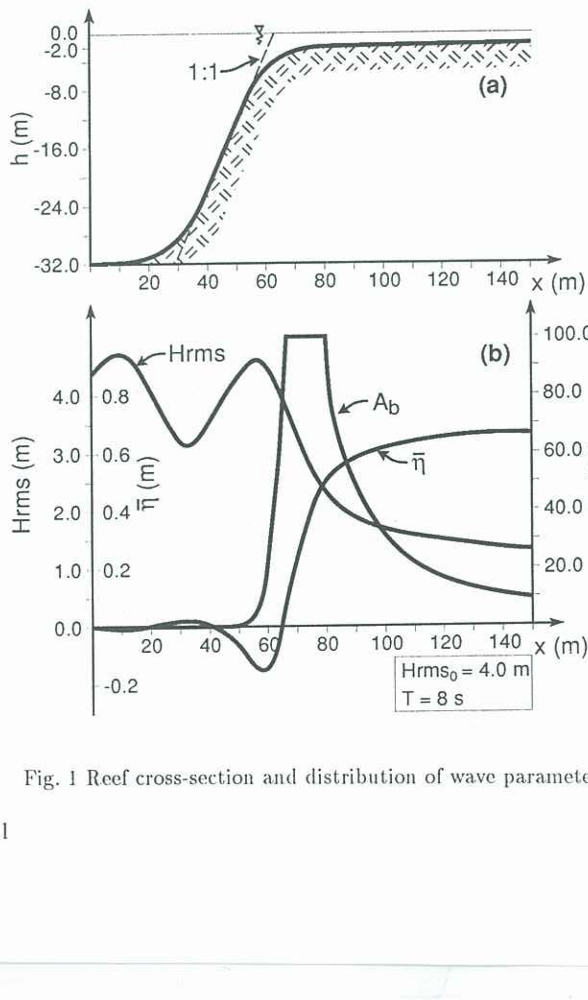

Fig. 1 Reef cross-section and distribution of wave parameters.

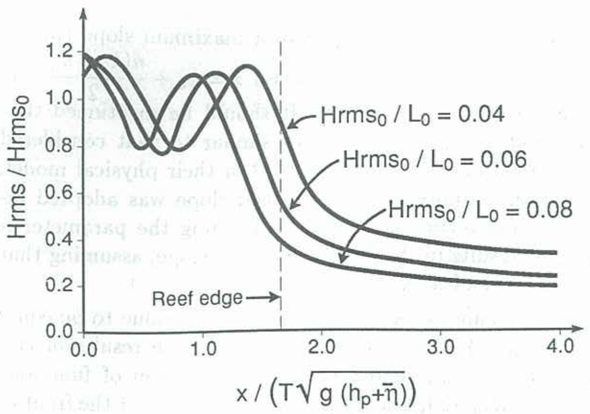

Fig. 2 Influence of wave steepness on waves on a reef.

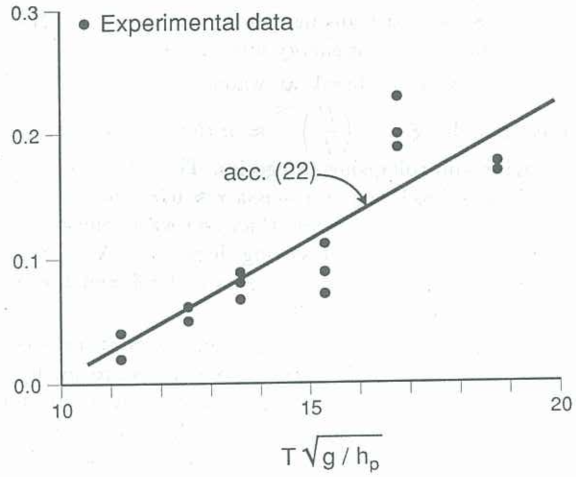

Fig. 3 Wave set-up.

Fig. 2 demonstrates the influence of the initial wave steepness on the wave transformation and dissipation. The steeper waves start to break early, at the greater distance from the reef edge, and their intensity of breaking is higher than that of smaller waves. Waves of the low steepness loose their stability closer to the reef edge and dissipate energy mostly on the reef platform. These results are in agreement with Nelson and Lesleighter's (1985) observations.

In Fig. 3 the relative wave set-up $\frac{\bar{\eta}}{H_{rms0}}$ is plotted against the ratio of shallow water wave length to reef platform still water depth $T\sqrt{\frac{g}{h_p}}$. In calculations, the following reef and wave parameters were adopted: $h_i = 35m, h_p = 5m, 1/n = 2$ and $8s < T_p < 13s$. The initial wave steepness was constant and equal to 0.03. Equation (22) gives almost linear dependence of wave set-up and it reflects properly the trend observed in the Nelson and Lesleighter (1985) experiments, which were carried out for vertical reef edge.

CONCLUSIONS

The extended refraction–diffraction equation, developed by the author, was applied for description of wave transformation and dissipation on the steep slope of the elongated reefs, with parallel isobats. The pattern of wave field is determined by two main mechanisms, i.e. wave shoaling and wave breaking. Under the assumption of the narrow spectrum of incident waves, the presented model gives realistic results, even for very steep reef slopes. The dissipation due to bottom friction was found to be negligible in comparison to wave breaking. The modified bore model has proven to be applicable in representing the energy losses in breaking waves. Its incorporation in the wave energy transportation equation is quite straight forward. Numerical solution of wave energy equation, supplemented by momentum balance equation, provides information on wave parameters at the arbitrary point on the reef, as well as on the wave set-up. The theoretical predictions were verified against the existing experimental data on wave set-up. However, more experimental evidence is needed for model calibration. Moreover, further theoretical works are needed to provide a better understanding of the hydrodynamical processes on the steep reef slopes. The following processes are of particular worth to future study; for instance, random wave action on an isolated reef structure (3D-model), interaction between spectral components and higher harmonics generation on the reef platform, and, wave-induced circulation on a reef top among others.

REFERENCES

BATTJES, J A, and JANSSEN, J P F M (1978) Energy loss and set-up due to breaking of random waves. Proc. 16th Coastal Engineering Conference, Hamburg, I, 569–587.

BERKHOFF, J C W (1972) Computation of combined refraction–diffraction. Proc. 13th Coastal Engineering Conference, Vancouver, I, 471–490.

BOOIJ, N (1981) Gravity waves with non-uniform depth and current. PhD Thesis, Delft Univ. of Tech., 130pp.

GERRITSEN, F (1981) Wave attenuation and wave set-up on a coastal reef. Look Laboratory Technical Report No. 48, 416pp.

GOURLAY, M R (1988) Coral cays: products of wave action and geological processes in a biogenic environment. Proc. 6th Inter. Coral Reef Symp., II, 491–496.

HARDY, T A, YOUNG I R, NELSON R C and GOURLAY M R (1990) Wave attenuation on an offshore coral reef. Proc. 22nd Coastal Engineering Conference, Delft, I, 330–344.

LONGUET-HIGGINS, M S and STEWART, R W (1964) Radiation stress in water waves: a physical discussion with applications. Deep Sea Res., 11, 529–562.

MASSEL, S R (1989) Hydrodynamics of coastal zone. Elsevier Science Publishers, Amsterdam, 336pp.

MASSEL, S R (1991) Inclusion of wave breaking mechanism in a modified mild-slope model. Proc. IUTAM, Breaking Waves Symp., Sydney (in press).

MASSEL, S R (1992) Extended refraction–difraction equation for surface waves. Coastal Engineering, (accepted for publication).

MASSEL, S R and BELBEROVA, D (1991) Parametrization of the dissipation mechanisms in the surface waves induced by wind. Arch. Mech., 42, 515–530.

NELSON, R C and LESLEIGHTER, E J (1985) Breaker height attenuation over platform coral reefs. 7th Australasian Conference of Coastal and Ocean Eng., Christchurch, I, 9–16.

THORNTON, E B and GUZA, R T (1983) Transformation of wave height distribution. Jour. Geoph. Res., 88, 5925–5938.

YOUNG, I R (1989) Wave transformation over coral reefs. Jour. Geoph. Res., 94C7, 9779–9789.

WOLANSKI, E, (1986) Observations of wind-driven surface gravity waves offshore from the Great Barrier Reef. Coral Reefs, 4, 213–219.