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IMPROVEMENTS IN THE COMPUTATION OF THE WAVE FUNCTION IN THE NEUMANN-KELVIN SHIP WAVE PROBLEM

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ABSTRACT

Following a brief introduction to the physical nature of wave drag, a particular form of the relevant Green's function used in the Neumann-Kelvin problem recently shown to be well suited to computation is introduced and discussed.

The heart of this representation is in two complementary infinite Neumann series. An improved method for assigning an upper limit to the number of terms practically required is discussed and the improvement to the matching between these two series when an even more recent "missing term", as presented by the original and present author, is included is demonstrated. The order of numerical integration required to evaluate the panel influence coefficients is briefly discussed.

The wave drag of a submerged spheroid is taken as a test case, and the non dimensional wave drag calculated via standard Panel Pressure summation (PPS) is shown to be in good agreement with the results from an analytical study.

NOTATION

a_n	coefficient in Fourier series
a, b, c	semiaxes, focal length of spheroid $c = \sqrt{(a^2 + b^2)}$
D	$= \sqrt{Y^2 + Z^2}$
Fr	Froude number $Fr = U/\sqrt{gL}$
g	acceleration due to gravity
G	the Kelvin wave Green's function
i	$\sqrt{-1}$, {-1, 0, 0}
J, K, I, Y	Bessel functions J, K, I and Y
L	Length of ship
M	$= X^2/4D$
N	Number of panels, upper summation limit
R_w	$= R/\rho V^2 L^2$
U	Ship velocity
$\underline{x}$	= {x, y z} source point
$\underline{x}'$	= {x, y, -z} source image point
$\underline{X}$	= {X,Y,Z} field to source image point vector
$\underline{X}'$	= {X, Y, Z'} field to source point vector
ß	$= \tan^{-1}(Y/Z)$
φ	perturbation potential, $U = -\underline{i} + \nabla\varphi$
ρ	water density
Σ'	summation with first term multiplied by 1/2
ξ	= {ξ, η, ζ} field point

Subscripts
BP, F indicate a value in Baar and Price's or Farell's system.
x, y or z indicates partial differentiation with respect to this coordinate.

INTRODUCTION

The wave drag of surface ships has long been investigated; Michell (1898) is given credit for the first computational presentation of his thin ship analysis, and the Froudes developed much of the practical and theoretical precepts upon which tank testing is still based.

Froude's Hypothesis (1889) "allows" the separation of a ship's viscous and wave resistance to the effective end that analysis of the wave resistance can be presented as depending on a single dimensionless parameter, the Froude number, $Fr = U/\sqrt{gL}$. Thus the variation of drag with speed for a typical ship may be schematically presented as in figure 1, which plainly indicates the importance of the wave resistance at higher speeds; a typical operating range is Fr = 0.1 to 0.3.

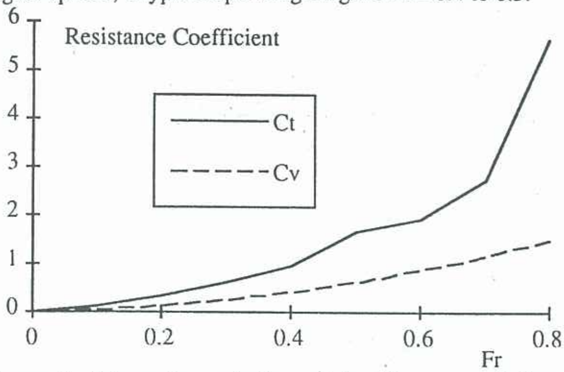

Figure 1: Schematic variation of the Viscous and Total resistance coefficients with Froude number.

This paper seeks to present a discussion of various details within a scheme for solving the Neumann-Kelvin problem computationally. The algorithms used are those of Baar and Price (1988a, b), Newman (1987) and Ursell (1988); the test case is compared with an "exact" analysis (Farell 1973).

THE NEUMANN-KELVIN PROBLEM

The Neumann-Kelvin problem has become a standard ship wave problem; it requires that the Neumann "no-flow" boundary condition is satisfied on the actual hull surface, whereas the free surface boundary condition is simplified by linearising the wave elevation so that the combined kinematic and dynamic boundary condition is applied on the known surface z = 0 rather than on the unknown deformed surface.

Here we adopt the Green's function approach of Hess and Smith (1966), with the complication that the fundamental singularity used is one which will automatically satisfy the linearised Kelvin (wave) boundary condition (BC). The physical conditions such a fundamental singularity must satisfy are apparent from observation:

1. Fluid must not be created nor destroyed.
2. Fluid on the water surface must stay on the surface.
3. The free surface is a constant pressure surface.
4. Waves are present only downstream of the source.

In a coordinate system with x in the direction of travel, z up, and y completing a right handed coordinate system, and with Baar and Price's non-dimensionalising scheme in terms of L, ρ and V (so that the nondimensional acceleration due to gravity is Fr^{-2}) these become

$$\nabla G^2 = 0 \qquad (1)$$

$$Fr^2 G_{xx} + G_z = 0 \ \text{ on } z = 0 \qquad (2)$$

$$G = \begin{cases} O(1/|x-\xi|) \text{ as } x-\xi \to -\infty \\ o(1) \text{ as } x-\xi \to +\infty \end{cases} \qquad (3)$$

The major alternative representations for this function are discussed by Noblesse (1981). Baar and Price use that by Peters (1949) which can be written:

$$G = -1/R + (1/D + N(\underline{X}) + W(\underline{X}))/Fr^2 \qquad (4)$$

where the vector $\underline{X}$ is usefully defined with reference to figure 2 and $R = |\underline{X}'|$, $D = |\underline{X}|$ where

$$\underline{X} = \{x-\xi, y-\eta, z+\zeta\}/Fr^2 \qquad (5)$$

$$\underline{X}' = \{x-\xi, y-\eta, z-\zeta\} \qquad (6)$$

$$N(\underline{X}) = \frac{2}{\pi}\int_{-1}^{1} \mathrm{Im}\left(e^A E_1(A)\right)dt \qquad (7)$$

$$W(\underline{X}) = -u(X)4\int_{-\infty}^{\infty} \sin\left((X+Yt)\sqrt{1+t^2}\right)\exp(-Z(1+t^2))dt \qquad (8)$$

where $A = \left(-Z\sqrt{1-t^2} + Yt + i|X|\right)\sqrt{1-t^2}$, E_1 is the complex valued exponential function (Abramowitz and Stegun 1972) and u(X) represents a step zero for $X < 0$ and unity above.

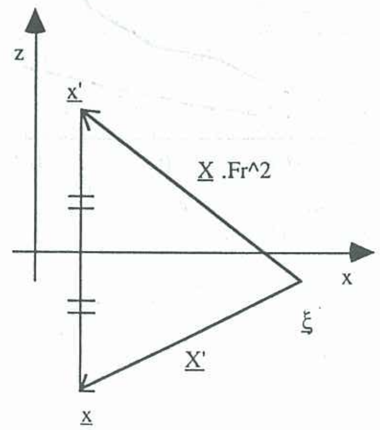

Figure 2: Vector definition sketch.

This provides a decomposition between (the first two) parts which (by themselves) enforce the free surface as a reflection plane, and a Kelvin perturbation represented as a local term symmetric in x and y, (N) and a wave part which only exists downstream as required (W).

With this function evaluable the problem can be posed in terms of a perturbation potential φ such that $\underline{U} = -\underline{i} + \nabla\varphi$ whereupon the wave drag is given most simply by the integration of the dynamic pressure over the body surface h

$$R_w = -\int_h (\varphi_x - 1/2|\nabla\varphi|^2)n_x da \qquad (9)$$

COMPUTATIONAL ASPECTS

The fundamental singularity defined above can be used, with the standard panelling scheme of aerodynamics, and the Neumann BC, to create a Fredholm integral of the second kind which we can solve exactly as per Hess and Smith (1966). Here though the influence coefficient kernel must be integrated numerically over each panel, and for surface piercing forms a waterline integral is also required.

The Rankine terms are easily treated by the Hess and Smith equations. The N component is evaluated here from a series of Chebyshev polynomials as presented by Newman (1987). (It should be noted that these are indeed evaluated as Chebyshev polynomials as advised by Press et al (1987), and as is more convenient when the derivatives are also required). The major computational burden, and numerical problem, is then in the evaluation of the W function.

Bessho (1964) showed that the above expression could be written as two complementary Neumann series, viz:

$$W(\underline{X}) = u(X)8e^{-Z/2}\sum_{n=0}^{\infty}{}'(-1)^n J'_{2n}(X)K_n(D/2)\cos(n\beta)$$

$$\approx -u(X)8\pi e^{-Z/2}\sum_{n=0}^{\infty}{}' Y'_{2n}(X)I_n(D/2)\cos(n\beta) \qquad (10)$$

with J' and Y' representing derivatives with respect to X.

Baar and Price (1988b) have shown that this is a useful form for numerical computation. They write:

$$W(\underline{X}) = \sum_{n=0}^{N}{}' a_n\cos(n\beta) \qquad (11)$$

then cleverly circumvent the numerical difficulties inherent in the use of Bessel functions by applying the relevant recurrence relationships in such a way that the coefficients in this Fourier series can be generated from ratios of Bessel functions of similar order, which are therefore close to unity.

FURTHER COMPUTATIONAL DEVELOPMENTS

The proceeding then presents the basic method developed by Baar and Price. The following presents the new aspects resulting from the present author's study which seek to more fully understand the behaviour of the two series, and from this to calculate them faster and more accurately.

This is in terms of an examination of the method of finding a truncation limit for the summation of eqn (11), and the analysis of a term appearing since their study was completed. The order of Gaussian Quadrature required to satisfactorily evaluate the panel influence coefficients is also briefly discussed.

The Truncation Point

In the summation of any infinite series a truncation point must be found, viz the N of eqn (11); this is also, here, very much related to the fact that a decision must be made to use either the JK or the YI series. Baar and Price (1988b) suggest:

Evaluate the J and K ratios up to N=75. As the J recurrence relationship must be applied in the reverse direction, it is necessary to assign an upper limit first.

Perform the JK series sum; if $|a_N| < \varepsilon$, 10^{-6} say, then this series has converged, and its result used. If this is not the case, use the YI series with N=25.

They mention too, that defining $M=X^2/4D$ provides an approximate split with points for which $M < 20$ using the JK series and the rest the YI one. This dependence can be explained by reference to Abramowitz and Stegun (1972) equations (9.3.1, 2) and (9.7.7, 8); writing the Neumann series in terms of these approximations yields an expansion in terms of M and 1/M for these respectively.

More accurate approximations (Abramowitz and Stegun 1972 eqn 9.3.11, 12) can be used most profitably to estimate any given a_i for either series. This allows the determination of which series to use and its truncation point to be easily made, as illustrated below for the point {X, Y, Z} = {10, 1, 1} for which the coefficients are shown in figure 3.

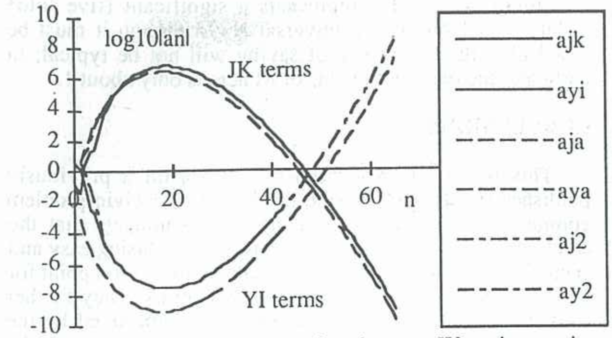

Figure 3: Fourier coefficients for the two W series at the point $\{X, Y, Z\} = \{10,1,1\}$. ajk, ayi are the exact values; aja and aya are calculated from Abramowitz and Stegun (9.3.1, 2 and 9.7.7, 8); aj2 and ay2 are from (9.3.11, 12 and 9.7.7, 8). The latter approximations are indistinguishable from the exact values above $n \approx 10$. Note that the exact YI series is truncated at $N \approx 40$; the approximations go to $N = 65$. As is typical for these curves $a_{iJK} \approx k/a_{iYI}$.

The latter approximations are obviously more than adequate to evaluate both when $|a_n| < \varepsilon$, and, equally important, whether the sum thus formed will have any significance; this last is determined by the maximum $|a_n|$ and the magnitude of the final answer, here approximated by

$$W_{max} \approx 4e^{-Z}\sqrt{(\pi/Z)} \qquad (12)$$

(Baar and Price 1988b).

With these approximations available the selection of a series and a truncation point is as follows:

Find, for the JK series, the maximum $|a_n|$, and $N\varepsilon$ for $|a_{N\varepsilon}| < \varepsilon$. If $N\varepsilon$ is too high, (eg > 130) the JK series cannot be used.

Use the estimate of W_{max} and $|a_n|_{max}$ to determine if the sum formed from this series will have any useable precision. If there will be insufficient significance remaining, the JK series cannot be used.

If the JK series is deemed acceptable, calculate the JK series with $N = N\varepsilon$, otherwise use the YI series with the equally easily calculated $N_{YI\varepsilon}$; owing to the asymptotic nature of this series the sum should stop at the bottom of the curve for maximum accuracy.

For the given example of $\{10, 1, 1\}$ this becomes:

$W_{max} \approx 2.6 \approx 10^1$ so to achieve a sum with six significant figures we must sum the JK series to $|a_n| < 10^{-5}$ which corresponds to $N = 58$.

The maximum term is $|a_{19}| \approx 10^7$; with double precision and hence about 16 significant figures, the final sum arrived at will still have about 9 significant figures left. Thus the JK series can be used at this point.

The Missing Term

The later paper by Baar and Price contains a proof note suggesting that Ursell may have recently derived a "missing term" required in certain regions for the YI series. This paper (Ursell 1988) reworks Bessho's analysis, and arrives at a term here called UA:

$$(UA)\exp(Me^{-i\beta}) = \int_{-\infty}^{\infty}\exp(-Mt^2).$$
$$\exp\left\{\frac{D^2e^{+i\beta}}{16M(1-ite^{+i\beta/2})^2}+\frac{De^{+i\beta}}{2(1-ite^{+i\beta/2})}\right\}\frac{e^{+i\beta/2}dt}{i(1-ite^{+i\beta/2})} \qquad (13)$$

which is described there as being important whenever β is close to $\pi/2$ or z is small. The leading order real term is also given there:

$$Re(UA) \approx \sqrt{\frac{\pi}{M}}\exp\left(-(M-\frac{D}{2})\cos\beta\right)\sin\left((M+\frac{D}{2})\sin\beta+\beta/2\right) \qquad (14)$$

Noting that this term must be included as $W_{YI} = W_{YI} + 4e^{-Z/2}Re(UA_x)$ to bring it into Baar and Price's system, it can be shown that this term is in fact important whenever M is high (so the YI series is in fact being used) and β is close to $\pi/2$ (ie y/z is large). When this is the case, this term makes an important contribution to the YI series, greatly improving the matching between the two series which may then fairly be described as complementary.

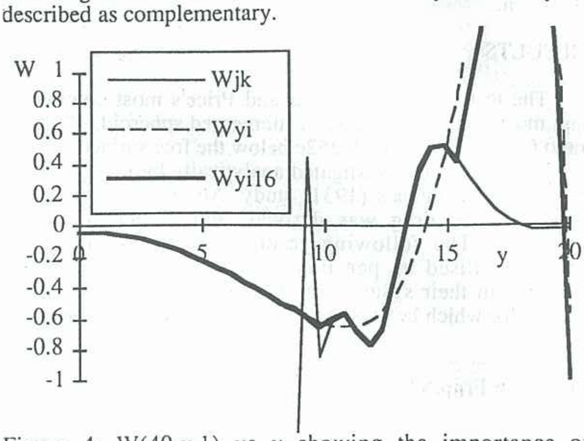

Figure 4: W(40,y,1) vs y showing the importance of including Ursell's extra term to a reasonable order.

Figure 4 shows W(40,y,1) for $0 \le y \le 20$; shown are W_{JK}, W_{YI} and $W_{YI} + UA_{16}$ (where UA_n indicates the first n terms of the series for evaluating UA given below are included). Inclusion of just the real first order term above does not give any significant improvement in this case; the "exact" answer from numerical integration is not distinguishable from the continuous curve made up from W_{JK} and $W_{YI} + UA_{16}$. The improved matching, and elimination of the area where neither series is correct, is easily evident.

With both the importance of this term, and the inadequacy of the leading order approximation verified by a brief numerical study which also indicated the foolishness of direct numerical integration (which can obviously be used for W itself), a better approximation to this term was required.

Following Ursell, the method of steepest descents (Marsden 1973, Kreyszig 1983) is used. Given:

$$F = \int_{-\infty}^{\infty} f(t)\exp(-kt^2/2)dt \qquad (15)$$

then if we can write

$$f(t) \approx \sum_{0}^{N} a_n t^n \qquad (16)$$

$$a_n = \frac{1}{n!}\frac{d^nf}{dt^n}\bigg|_{t=0} \qquad (17)$$

where the approximation for f(t) must hold near t = 0, then

$$F \approx \sqrt{\frac{2\pi}{k}}\left(a_0 + \frac{a_2}{k} + \frac{a_4.1.3}{k^2} + \frac{a_6.1.3.5}{k^3} + ...\right) \qquad (18)$$

The problem then becomes the derivation of $f^{(n)}$ for the appropriate kernels for UA, UA_x, UA_{xx}, UA_{xy} and UA_{xz}; this has been carried out for arbitrary n so that any order approximation may be used. In practice N=16 is used as a sensible limit with single precision complex arithmetic. As the resulting expressions are cumbersome, lengthy and numerous, they are not given here.

The Order of Gaussian Quadrature required for the Influence Coefficient Evaluation

As the G function is not in a form where it can be analytically integrated over an arbitrary panel a numerical technique is needed. Baar (1988) suggests 2 x 2 Gaussian Quadrature (GQ); with N panels this requires $4N^2$ W and N evaluations. As virtually all of the computational effort is in computing the W function and its gradients the GQ order has a direct effect on execution time.

As a preliminary investigation comparison is made with a 2 x 1 GQ scheme with the 2 points taken here along the flow direction.

RESULTS

The test case here is Baar and Price's most convincing and most accurate test case, a submerged spheroid, of aspect ratio 6 with its centroid 0.252c below the free surface.

This case was investigated analytically by Farell (1973), extending Havelock's (1931) study. An analytic expression for the wave drag was derived, and numerical results presented. The following results are all presented non dimensionalised as per Baar and Price, but the Froude numbers in their system are chosen to be those in Farell's system for which he presents R_w in a table. Conversion is:

$$Fr_F = Fr_{BP}.\sqrt{\frac{a}{c}} \qquad (19)$$

$$Rw_F = Rw_{BP}.\frac{V^2(2a)^2}{\pi g c^3} \qquad (20)$$

Figure 5 presents Farell's curve of Rw vs Fr together with results from the present study with 30 panels cosine spaced longitudinally and 10 evenly spaced circumferentially on half of the body. The displayed results are for 2x2 Gaussian Quadrature, but for this case 2x1 GQ gave R_w values which were the same to four significant figures (with half the computational effort).

Fr_F	Fr_{BP}	10^3R_w Present	10^3R_w Farell
0.35	0.348	0.667	
0.4	0.397	2.35	2.30
0.45	0.447	3.01	3.01
0.5	0.496	3.01	3.01
0.55	0.546	2.59	
0.6	0.596	2.12	2.14
0.65	0.645	1.71	
0.7	0.695	1.37	1.38
0.75	0.745	1.09	
0.8	0.794	0.883	0.895

Table 1: Farell and Baar and Price Froude numbers and Resistance values ($1000R_w$).

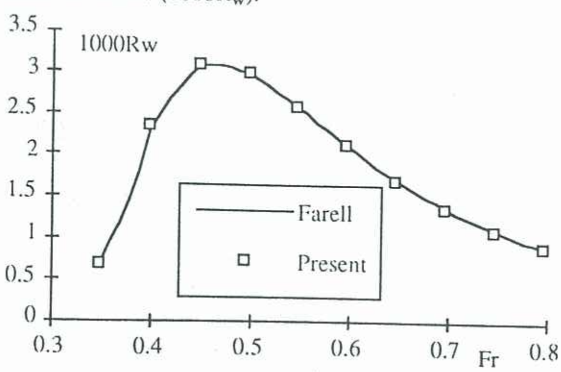

Figure 5: Farell's results and the present calculations.

For this test case all W evaluations are from the JK series; for all Froude numbers the average number of terms required was 13 with a (program imposed) minimum of 10 and a maximum of 26. This represents a significant (five fold) saving over assuming a universal N=75 though it must be noted that this magnitude of saving will not be typical; in particular the maximum value of M here is only about 1.

CONCLUSIONS

This paper has discussed details within a previously published method for solving the Neumann Kelvin problem computationally. The preliminary results indicate that the solution time may be substantially reduced by using easy and accurate approximations to find a suitable truncation point for the summation of the required infinite series. They further suggest that the solution accuracy can be improved by the inclusion to suitable order of a term appearing since the previous study was completed, and that reducing the order of the numerical integration required to find each panel's influence coefficients may be permissible for some cases.

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