11th Australasian Fluid Mechanics Conference
University of Tasmania, Hobart, Australia
14-18 December 1992

7E-2

A TRANSITION FUNCTION FOR ARTIFICIAL STRIP ROUGHNESS IN OPEN CHANNELS

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ABSTRACT

The use of rectangular strips, aligned orthogonally to the direction of flow and positioned at regular intervals, for artificial roughening of open channel boundaries in laboratory investigations or model studies has a number of practical advantages. However, a disadvantage has been that formulae for evaluating artificial strip roughness have been limited to the fully developed *completely rough* regime.

The paper will describe a study of strip roughness and present results that include a new formula that applies for all regimes: *smooth*; *transition*; and *fully rough*.

Interpretation of results also offers some explanations as to why some researchers have noted a substantial scatter in experimental friction factors for hydraulically smooth channels.

NOTATION

d	circular conduit diameter	[m]
f	Darcy-Weisbach friction factor	
f_b	bed friction factor	
g	gravitational acceleration	[m/s^2]
H	head; flow depth in main channel	[m]
k	roughness height	[m]
k_s	equivalent Nikuradse's sand roughness height	[m]
R	hydraulic radius	[m]
R_b	bed hydraulic radius (= H)	[m]
$\mathbf{R_e}$	Reynolds number (= $4UR/\nu$)	
$\mathbf{R}_{eb}$	Reynolds number for the bed (= $4UR_b/\nu$)	
S_f	slope of the energy grade line	
u_*	local friction velocity (= $\sqrt{\tau_0 / \rho}$)	[m/s]
u_{*b}	local bed friction velocity (= $U\sqrt{f_b / 8}$)	[m/s]
U	depth averaged velocity	[m/s]
χ	roughness parameter for strip roughness	[m]
κ	Von Kármán turbulence coefficient	
λ	roughness strip spacing	[m]
ν	kinematic viscosity	[m^2/s]
ρ	fluid density	[kg/m^3]
τ_0	local boundary shear stress	[Pa]
HRD	High Roughness Density	
LRD	Low Roughness Density	

INTRODUCTION

Laboratory investigations in open channels have made extensive use of strip type elements to develop artificial roughness. The elements are attached traversely across the bed of the channel at regular intervals. This form of roughness provides a number of practical advantages over other artificial roughening techniques which include:

- reduced effort in placing roughness elements where a regular pattern is required;
- reduced effort in reconfiguring such patterns;
- precise description of the geometric properties of boundary roughness; and
- potential for planning strip layouts to provide a wide range of roughness characteristics for a series of experiments.

Although a number of researchers have examined the technique and it is widely used, calibration still remains an essential prerequisite to application. A further limitation is that the theoretical roughness function, that has previously been applied to research by others, is only applicable to fully developed turbulent (*completely rough*) flow. This has therefore made it difficult for researchers to properly investigate the relatively low Reynolds number regime such as flow in shallow channels

THEORY

Background

Roughness functions have traditionally been defined in terms of friction laws developed for pipes roughened with sand. According to Schlichting [1979], in the region of laminar flow all rough pipes have the same resistance as a smooth pipe and can be said to be *hydraulically smooth* with resistance depending on hydraulic radius alone. As the Reynolds number of flow increases the resistance function deviates from that for a smooth pipe and approaches the region of the quadratic resistance law where resistance depends on relative roughness (k_s/R). Hence the hydraulic characteristics of flow may be classified in terms of three distinct regimes:

hydraulically smooth → *transition* → *completely rough*

with the extent of each regime region being defined by the dimensionless term $u_* k_s / \nu$.

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[1] A full review of this earlier work is given in Macintosh [1990].

Colebrook [1939], in a joint effort with C.M. White, established a resistance function which correlates the whole *transition* region from *hydraulically smooth* to *completely rough* flow. This well known function may be expressed as:

$$\frac{1}{\sqrt{f_b}} = -2 \log \left(\left(\frac{k_s}{14.8 R_b} \right) + \frac{2.51}{\mathbf{R_e} \sqrt{f_b}} \right); \qquad (1)$$

Sayre & Albertson [1963] undertook a series of detailed experimental investigations into the artificial roughening of channels. This lead them to the conclusion that the variation of the resistance function, to a relative roughness parameter (c), was logarithmic in nature and could be described by an equation of the form:

$$\frac{1}{\sqrt{f_b}} = \frac{2.30}{\kappa \sqrt{8}} \log \left(\frac{R_b}{\chi} \right); \qquad (2)$$

for *completely rough* two-dimensional flow (wide channel, $R_b = H$). They also found that a value of k equal to 0.38 was appropriate for artificial roughness investigations in open channel flow.

Application of equation (2) to the methodology applied by Colebrook [1939] for the formulation of the resistance function, equation (1), then gives a general resistance function for strip elements:

$$\frac{1}{\sqrt{f_b}} = -2 \log \left(\left(\frac{\chi}{R_b} \right)^{1.07} + \frac{2.51}{\mathbf{R_e} \sqrt{f_b}} \right); \qquad (3)$$

where κ has been set equal to 0.38.

For $\chi \rightarrow 0$ this equation transforms into a resistance function for *hydraulically smooth* flow. For $\mathbf{R_e} \rightarrow \infty$, it transforms into equation (2) for the *completely rough* flow regime.

A comparison of equations (1) & (3) shows a near linear proportionality between the χ and ks roughness parameters:

$$\frac{k_s}{14.8\ R} = \left(\frac{\chi}{R_b} \right)^{1.07}. \qquad (4)$$

However, despite the similarity between these equations, interpretation of calibration results has also shown a dependence of friction factor on the spacing (or density) of strip roughness elements. This dependence is not apparent when dealing with a uniform surface roughnesses having a relatively high density (ie. packed sand grains).

GEOMETRY

Artificial roughness elements were formed from aluminium strips, rectangular in cross-section and 10 mm wide by 3 mm high (k = 3 mm). These were aligned orthogonally across the channel and spot glued in position at a regular centre to centre spacing, λ.

Roughness was conveniently varied between calibration runs by the systematic removal of the strip elements on an alternate basis (starting with l = 30 mm).

CALIBRATION

Apparatus

Calibration experiments were undertaken in a rectangular channel, 1065 mm wide and approximately 25 m long. The channel was constructed from perspex and set to a uniform slope of 0.10%.

Regulated flows to the channel were supplied by a variable speed pump. Low flows were measured using a V-notch weir located in the return water circuit. High flows were measured using a Dall-tube mounted in the discharge line of the pump.

Tailwater levels in the channel were controlled using an adjustable free over-fall sluice gate. Water levels were measured using static pressure tappings which were installed in groups of four aligned transversely across the width of the channel. Tapping group were spaced at 1.2 m intervals over full length of the channel and each group manifolded to a common manometer.

Methodology

A total of six calibration series were undertaken with each series having a specific strip roughness configuration as listed in Table I. Artificial strip roughness elements were fixed only to the bed of the channel (ie. smooth side walls).

TABLE I. — Calibration Series

Series No.	I	II	III	IV	V	VI
λ (mm)	30	60	120	240	480	∞
λ/*k*	10	20	40	80	160	∞

Each series comprised nine individual runs covering a range of flow depths from approximately 20 mm to 100 mm, in 10 mm increments.

The general experimental procedure comprised the following steps:

- set up roughness elements to the desired configuration;
- establish uniform flow conditions at approximately the target depth;
- measure flow rate and actual depth;
- compute f_b from measured data and include corrections for side walls[2] and datum plane shift (Adachi [1984]); and
- compute χ using the general resistance function, equation (3).

It should be noted that computation of c using equation (3), contrasts to the method applied by Sayre & Albertson [1963] who ensured that all their data fell within the *completely rough* flow regime. This allowed direct computation of χ parameters from equation (2). Although Knight & Macdonald [1979] also applied this approach there is some doubt as to whether all of their data actually lay within the *completely rough* regime.

REVIEW OF RESULTS

Roughness Parameters

Calibration χ values were derived for each λ/*k* experimental series by averaging values obtained from each run within a series (9 runs per series). Results are listed in Table II. A comparison between predicted roughness values obtained from equation (3) (using calibrated χ values) to experimental data gave a standard error in f_b of 5.9% (average error of 0.0%). Values of equivalent Nikuradse sand roughness (k_s), the roughness Reynolds number ($u_{*b} k_s / \nu$) and flow regime classification are also listed in Table II for reference.

It will be noted from Table II that equivalent Nikuradse sand roughness values are dependent on both flow depth and λ/*k* and therefore encompass a range of values for each experimental series.

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[2] Friction factor side wall correction based on the concepts of Einstein, Johnson and Vanoni & Brooks as presented by French [1986] and modified by the author to include the work of Knight et al [1984].

TABLE II — Artificial Strip Roughness χ Parameters

Series	I	II	III	IV	V	VI
λ/k	10	20	40	80	160	∞
χ (mm)	1.418	1.177	0.542	0.232	0.102	0.004
k_s (mm)	15 - 18	12 - 15	5.0 - 8.2	2.0 - 4.0	0.6 - 2.4	0.0 - .08
k/χ	2.116	2.549	5.582	12.95	29.35	761.0
$\upsilon^*_b k_s/\nu$	200-450	180-370	92 - 150	37 - 54	15 - 28	.13 - 1.6
Regime	rough	rough	rough	transit'n	transit'n	smooth

Variation of χ with λ is presented in Figure 1. For comparison, c parameters after Knight & Macdonald [1979] are also shown. It should be noted that Knight & Macdonald's calibration refers to roughness strips which were 3 mm square (in contrast to that used for the current research) and therefore produce slightly different characteristics. It is interesting to note that some consistency between the two sets of data is apparent:

- the decrease in c with increase in λ, with a near logarithmic variation for $l/k > 20$; and
- a maximum artificial roughness at a relative spacing (λ/k) of around 10.

The apparent shift in Knight & Macdonald's calibration function also suggests that their strip roughness elements are capable of producing a given artificial roughness at a lower strip spacing density than that required for the strips used in current research.

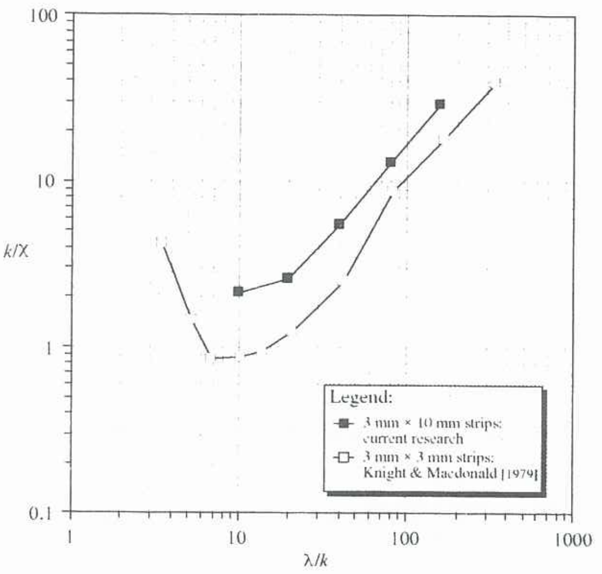

FIGURE 1. — Artificial Strip Roughness χ Parameters

Regime Limits

According to Schlichting [1979], flow regime classification may be delineated in terms of the following dimensionless limits:

- *Hydraulically Smooth:* $\dfrac{u_{*b}k_s}{\nu} < 5$; (5)
- *Transition:* $5 \le \dfrac{u_{*b}k_s}{\nu} \le 70$; and (6)
- *Completely Rough:* $70 < \dfrac{u_{*b}k_s}{\nu}$. (7)

Application of these regime limits to the calibration parameters listed in Table II show that only the three most dense roughness spacing configurations (l/k = 10, 20 & 40) produced *completely rough* flow, while the remaining two spacing configurations (λ/k = 80 & 160) produced roughness in the *transition* flow regime. The general resistance equation (equation (3)) was therefore used for the evaluation of all calibration parameters for all flow regimes.

Transition Function

A transition function for open channel flow, based on the general resistance function (equation (3)), is presented in Figure 2. This function represents an open channel adaptation of the transition function for pipe flow. Accordingly the figure presents those relationships which express the difference between *actual* and *completely rough* resistance in the form:

$$\Delta\left(\frac{U}{u_{*b}}\right) = \Phi\left(\frac{u_{*b}\chi}{\nu}\right); \tag{8}$$

where Φ is a function defined by:

$$\Delta\left(\frac{U}{u_{*b}}\right) = \frac{U}{u_{*b}} - \frac{2.30}{\kappa}\log\left(\frac{R_b}{\chi}\right); \tag{9}$$

in which the first right-hand side term represents the *actual* resistance, and the second term gives the *completely rough* resistance.

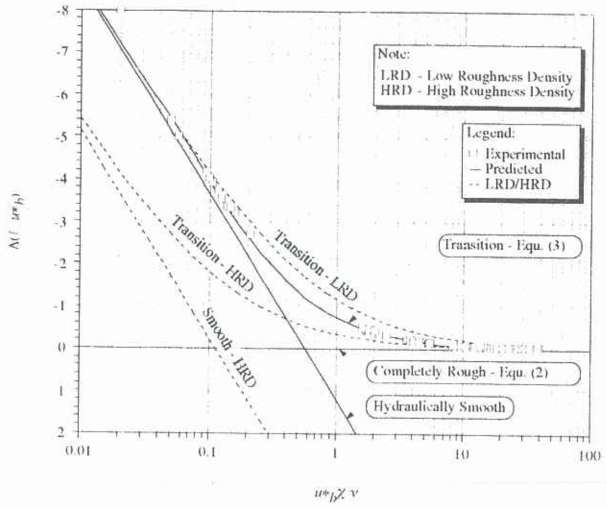

FIGURE 2. — Open Channel Flow Transition Function

Substitution of *hydraulically smooth*, *transition* (equation (3)), and *completely rough* (equation (2)) resistance functions into the first right-hand term of equation (9), and setting κ equal to 0.38 (Sayre & Albertson [1963]), then gives the transition functions presented in Figure 2:

- *Hydraulically Smooth:*

$$\Delta\left(\frac{U}{u_{*b}}\right) = -6.06\log\left(\frac{R_b}{\chi}\left(\frac{2.51}{\mathbf{R}_{\mathbf{e}_b}\sqrt{f_b}}\right)^{0.935}\right); \tag{10}$$

- *Transition:*

$$\Delta\left(\frac{U}{u_{*b}}\right) = -5.66\log\left(1 + \frac{2.51}{\mathbf{R}_{\mathbf{e}_b}\sqrt{f_b}}\left(\frac{R_b}{\chi}\right)^{1.070}\right); \text{ and} \tag{11}$$

- *Completely Rough:*

$$\Delta\left(\frac{U}{u_{*b}}\right) = 0\,. \tag{12}$$

A number of ancillary curves are presented in Figure 2 in addition to the basic transition function (equation (11)) and associated experimental data. These additional curves represent functions associated with: *completely rough*; *hydraulically smooth*; *hydraulically smooth - high roughness density (HRD)*; *transition - HRD*; and *transition - low roughness density (LRD)*. A curve for *hydraulically smooth - LRD* is not shown as this very nearly overlies that for *hydraulically smooth.*

Reference to a typical transition function for pipe flow (White [1939]) will only show one set of curves comprising that of the *transition* curve with its associated *completely rough* and *hydraulically smooth* tangents. These functions implicitly refer to resistance characteristics associated with a uniform high density roughness configuration, such as that developed when using a uniform sand coating. However, the use of strips for artificial roughness elements creates a non-uniform roughness density which will vary in accordance with the strip element spacing, λ.

In Figure 2 the curves associated with *HRD and LRD functions* represent bounding limits between which the actual strip roughness transition function will lie. The limiting HRD case would comprise narrow, closely spaced strip elements (equivalent to say a surface treatment of sand) whereas the *LRD limit* could be taken to comprise an isolated strip element.

For a given roughness value (χ) Figure 2 also shows that the effective boundary friction will increase with a decrease in strip element density (increase in λ). This characteristic has previously been discussed in relation to the data after Knight & Macdonald [1979], (Figure 1).

It is also interesting to note that *LRD* characteristics suggest that small irregularities in the bed of a channel (such as that produced by any misalignment in the joints of a laboratory channel, or poor surface finish) would be likely to result in a relatively large increase in bed friction. This observation therefore offers some explanation as to why some researchers have noted a substantial scatter in experimental friction factors for channels which are meant to be hydraulically smooth.

Resistance Diagram

The general resistance function (equation 3)) has been used, in conjunction with calibrated c values, to construct a resistance diagram for the particular type of strip roughness element used in this research, Figure 3. This diagram covers a $\mathbf{R}_e$ range from approximately 5×10^3 to 250×10^3, and a λ/k ratio range from 10 to ∞ (*hydraulically smooth*).

As noted in Table II experimental friction factors for: λ/k equal to 10, 20 & 40 lie within the *completely rough* regime: λ/k equal to 80 & 160 lie within the *transition regime*; and λ/k equal to ∞ lies within the *hydraulically smooth* regime.

Associated experimental data points are shown in Figure 3 and resulted in an overall standard error to predicted fb of 5.9%. These findings have been taken to depict satisfactory performance of the transition function.

CONCLUSIONS

The use of rectangular strips for the generation of artificial roughness have been found to be well suited to open channel experiments where the strips are aligned orthogonally to the flow. Calibration experiments have shown that variation of strip spacing (λ) may be related to the strip roughness parameter (χ) by a unique continuous function. Comparison with research by others has shown this function to be dependent upon strip cross-sectional geometry.

The proposed general strip element resistance formula has been shown to satisfactorily correlate the whole *transition* regime from *hydraulically smooth* to *completely rough.*

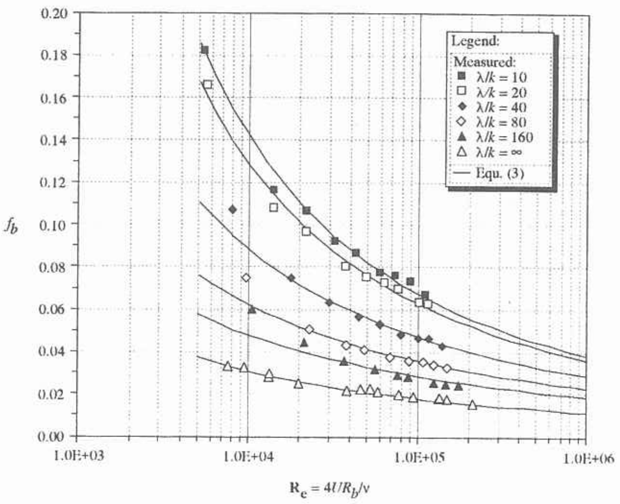

FIGURE 3. — Strip Element Resistance Diagram

ACKNOWLEDGMENTS

The work presented in this paper was undertaken by the principal author in the hydraulics laboratory of the Department of Civil Engineering, University of Queensland, Australia. It formed part of a PhD research programme.

REFERENCES

Adachi, S. (1964): 'On the Artificial Strip Roughness', Disaster Prevention Research Institute, Kyoto University, Tokyo, Japan, Bulletin Nº 69.

Colebrook, C. F. (1939): 'Turbulent Flow in Pipes with Reference to Transition Region Between Smooth and Rough Pipe Laws', Journal, Institute of Civil Engineers, Vol. 11, February, Paper Nº 5204, pp. 133-162.

Colebrook, C.F. & White, C.M. (1937): 'Experiments with Fluid Friction in Roughened Pipes', Proceedings of the Royal Society, Vol. 161, August, Paper Nº 906, pp. 367-381.

French, R.H. (1986): Open-Channel Hydraulics, McGraw-Hill Book Company, New York.

Keulegan, G.H. (1938): 'Laws of Turbulent Flows in Open Channels', Journal of Research, U.S. National Bureau of Standards, Vol. 21, December, pp. 707-741.

Knight, D.W. & MacDonald, J.A. (1979): 'Hydraulic Resistance of Artificial Strip Roughness', Journal of the Hydraulics Division, American Society of Civil Engineers, Vol. 105, Nº HY6, Paper Nº 14635, pp. 675-690.

Macintosh, J.C. (1990): 'Hydraulic Characteristics in Channels of Complex Cross-section', Thesis presented to the University of Queensland, Australia, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Sayre, W.W. & Albertson, M.L. (1963): 'Roughness Spacing in Rigid Open Channels', Transactions, American Society of Civil Engineers, Vol. 128, Nº 1, Paper Nº 3417, pp. 343-427.

Schlichting, H.: Boundary Layer Theory, McGraw-Hill Book Company, New York, Seventh Edition, 1979.