

## DIRECT MEASUREMENT OF BUOYANCY FLUX IN THE FIELD

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**Abstract:** This paper examines the statistical reliability of direct estimates of buoyancy flux. A relationship is determined, based upon field data for shear driven turbulence, for the length of data required if the buoyancy flux estimate is to be within a certain range of the actual buoyancy flux.

### INTRODUCTION

A significant problem in the ocean and other stratified water bodies is to determine the amount of mixing or buoyancy flux  $B = g/\rho \overline{\rho' w'}$  where  $\rho$  is density,  $g$  is gravity,  $w$  is vertical velocity, the primes denote fluctuating quantities and the overbar denotes an ensemble average.

It is only recently that attempts have been made to directly measure buoyancy flux in the field. Prior to this, estimates of turbulent fluxes were made indirectly based on measurements of  $\chi$  (dissipation of temperature variance),  $\epsilon$  (dissipation of turbulent kinetic energy), the Thorpe scale (a measure of the size of unstable patches) and  $N^2 = -g/\rho (\partial\rho/\partial z)$  the buoyancy or Brunt-Väisälä frequency squared where  $z$  is positive upwards.

The first indirect method is that of Osborn and Cox (1972) who assumed that the unsteady and advective terms were negligible in the temperature variance equation to obtain

$$B = D_T C_T N^2 \quad (1)$$

where  $D_T$  is the molecular diffusivity of heat and  $C_T$  is the temperature Cox number for turbulence and is defined by

$$C_T = \frac{\chi_T}{2D_T} / \left[ \frac{\partial \overline{T'}}{\partial z} \right]^2 = 3 \left[ \frac{\partial \overline{T'}}{\partial z} \right]^2 / \left[ \frac{\partial \overline{T'}}{\partial z} \right]^2 \quad (2)$$

where  $T'$  is the turbulent temperature fluctuation and  $\chi_T = 6D_T (\partial \overline{T'}/\partial z)^2$ .

Osborn (1980) assumed that the unsteady and advective terms were negligible in the turbulent kinetic energy equation, resulting in

$$P = B + \epsilon \quad (3)$$

where  $P$  is the production of turbulent kinetic energy. The flux Richardson number  $R_f$ , which is a measure of the mixing efficiency of the flow, is typically defined as  $R_f = B/P$ . This combined with (3) leads to

$$B = \frac{R_f}{1-R_f} \epsilon \quad (4)$$

Osborn (1980) assumed  $R_f \approx 0.15$  which results in

$$B = 0.2 \epsilon \quad (5)$$

Questions have been raised about the applicability of the indirect methods (e.g. Moum, 1980). For this reason there are several groups attempting to measure buoyancy flux directly. Moum (1980) has published some preliminary estimates of  $B$  based on vertical profiles, while Yamazaki and Osborn (1992) have made estimates of  $B$  from horizontal profiles.

The present study aims to present some preliminary estimates of the amount of data required to get reliable estimates of  $B$ .

### DATA ACQUISITION

Data was collected using the microstructure flux probe (MFP) which was developed at the Centre for Water Research, Perth, Australia, to measure vertical profiles of microstructure.

To achieve these aims, the MFP carries a two-axis laser Doppler anemometer and a pulse-to-pulse coherent doppler acoustic system in addition to fast response temperature and conductivity sensors. With a vertical probe velocity of  $0.1 \text{ ms}^{-1}$  and a sampling rate of 100 Hz, it was possible to directly measure horizontal  $u$  and vertical  $w$  velocities and measure temperature, salinity and their gradients to a vertical spatial resolution of approximately 1 mm. The fluctuating quantities  $u$ ,  $v$  and  $\rho$  can all be estimated within a volume of a few millimetres in extent.

The data used in this paper consist of two vertical profiles collected in a narrow 45 m deep channel connecting two sections of Lake Argyle in the North West of Australia. Fig 1 shows density and horizontal (along channel) velocity for each of the two profiles.

The horizontal velocity profiles show the presence of an intrusion centred at approximately 11 m depth. Associated with the intrusion are patches of unstable fluid; indicating the presence of shear driven turbulence. For further analysis, the region from 7 to 15 m depth was used as it fully encompasses the intrusion for both profiles.

### REYNOLDS DECOMPOSITION

Before the buoyancy flux can be evaluated from the density  $\rho$  and vertical velocity  $w$  measurements it is necessary to determine the fluctuating or turbulent parts

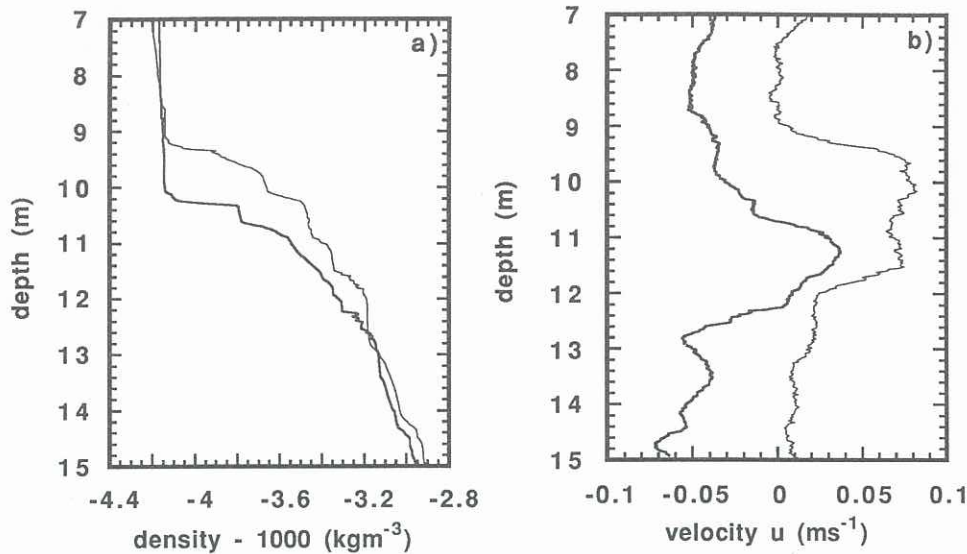


Fig. 1 Vertical profiles showing a) density at 1104 (thick line) and 1247 (thin line) and b) corresponding horizontal (along channel) velocity profiles showing the presence of an intrusion centred at around 11 m depth.

of  $\rho$  and  $w$  (i.e. a Reynolds decomposition). Typically, the fluctuating density and velocity would be defined as

$$\begin{aligned} \rho' &= \rho - \bar{\rho} \\ w' &= w - \bar{w} \end{aligned} \quad (6)$$

thus requiring the mean density  $\bar{\rho}$  and velocity  $\bar{w}$  to be known. For a long statistically stationary time series sufficient averaging can be performed to obtain  $\bar{\rho}$  and  $\bar{w}$  fairly accurately.

However, profiles from the MFP are generally not suitable for averaging either spatially or temporally. Averaging over large distances in the vertical is likely to 'smear' actual changes in  $\bar{\rho}$  and  $\bar{w}$ . Averaging consecutive profiles together to find  $\bar{\rho}$  and  $\bar{w}$  is only effective if the period between profiles being collected is much less than the timescale over which the system is evolving.

The effects of internal waves and other 'non-turbulent' signals were removed by applying an adaptive filter to the data. The filter was based on the Ozmidov length  $L_o$ , which is the largest lengthscale permitted by the stratification

$$L_o = \left[ \frac{\epsilon}{N^3} \right]^{1/2} \quad (7)$$

Since the longest timescale for turbulence  $\sim N^{-1}$  (i.e. that for an eddy of size  $L_o$ ) and the shortest timescale for internal waves  $\sim N^{-1}$ , a high-pass filter of length  $L_o$  should exclude internal waves but pass all turbulence. If it is assumed, for the purposes of filtering, that  $\epsilon$  is constant then (7) becomes

$$L_o \approx c N^{-3/2} \quad (8)$$

where  $c$  depends on some representative value of  $\epsilon$ . This equation forms a basis for the length of an adaptive filter. Fig 2 shows how the filter length can vary over a vertical profile. The same filter is used on the density and velocity signals to obtain  $\rho'$  and  $w'$ .

#### RELIABILITY OF BUOYANCY FLUX ESTIMATES

The buoyancy flux  $B$  is the mean value of the quantity  $g/\rho \rho' w'$ . The reliability or confidence of a mean estimate of a large number of points will, by the central limit theorem, be a Gaussian distributed variable with standard deviation  $\sigma$ . The number of points  $N$ , required in a data record for such a distribution can be expressed as (Bendat and Piersol, 1986)

$$N = \left[ \frac{\sigma t_{\infty; A}}{\alpha \bar{B}/100} \right]^2 \quad (9)$$

where  $t$  is the student's  $t$  value and  $A$  is the probability that the measured mean will lie within  $\pm \alpha$  percent of the actual mean  $B$ . If  $\rho' w'$  is auto-correlated the number of statistically independent points will be less than the total number of points. It is reasonable to expect  $\rho' w'$  to be correlated within an overturn or eddy but not at larger length scales. Fig 3 shows the probability density function for the displacement scale  $L_d$  for one of the profiles ( $L_d$  is basically the distance that a parcel of fluid has moved from its stable original position). Since, from Fig 3, there are very few motions larger than 0.02m in extent, an appropriate decorrelation length is  $L_{corr} \approx 0.02$ m. The number of independent samples for an auto-correlated timeseries is given by (Garrett and Petrie, 1981)

$$N = L/2L_{corr} \quad (10)$$

where  $L$  is the length of the data record. Combining (9) and (10) leads to the required length of the data series being

$$L = 2L_{corr} \left[ \frac{\sigma t_{\infty; A}}{\alpha \bar{B}/100} \right]^2 \quad (11)$$

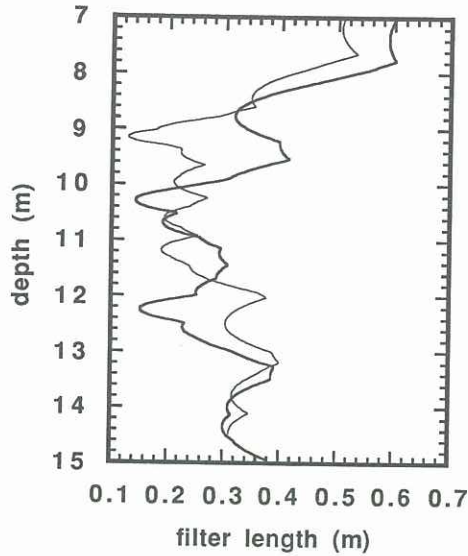


Fig. 2 The filter lengths used for the Reynolds decomposition for the vertical profiles collected at 1104 (thick line) and 1247 (thin line).

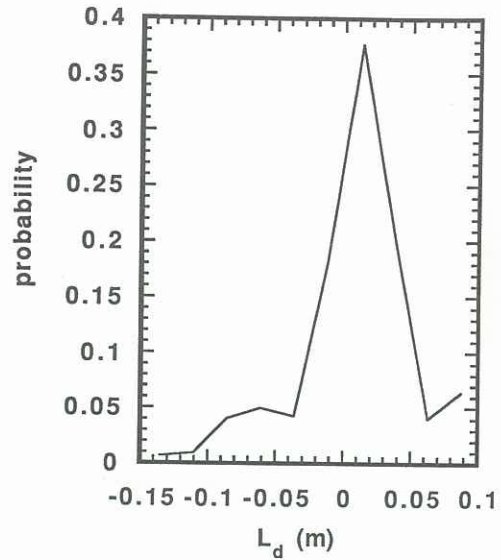


Fig. 3 The probability density function for the Thorpe displacement scale  $L_d$  for profile collected at 1247. Values of  $L_d$  less than 2 mm were not included when determining the probability density function.

The average value of  $\sigma$  for  $g/\rho(\rho' w')$  over the two casts was  $4.7 \times 10^{-8} \text{ m}^2 \text{ s}^{-3}$ .  $B$  was evaluated using the indirect methods of (1) and (5) to give an average value of  $2.3 \times 10^{-9} \text{ m}^2 \text{ s}^{-3}$ . Values of  $L$  derived using equation (11) are shown in Table I.

$\alpha \backslash A$	20%	50%	80%	90%	95%
5%	428	3035	10981	18080	25667
10%	107	759	2745	4520	6417
20%	27	190	686	1130	1604
50%	4	30	110	181	257
100%	1	8	27	45	64
200%	0	2	7	11	16

Table I The length of data (in metres) required for there to be probability  $A$  that an estimate of  $B$  will be within  $\pm \alpha B/100$  of  $B$

For example, a single 8 m profile could be expected to yield an estimate of the mean  $\hat{B}$  so that there is a 50% probability that  $0 < \hat{B} < 2B$  or an 80% probability that  $-B < \hat{B} < 3B$ . Alternately, the table can be used in the opposite manner. For example, if it is desired to estimate, with a confidence of 80 percent, that  $\hat{B}$  will satisfy  $0.5B < \hat{B} < 1.5B$ , then a total of 110 m of data will be required.

## CONCLUSIONS

Depending upon the desired accuracy and confidence for estimates of  $B$ , large amounts of microstructure data can be required for shear driven turbulence. In many cases it will not be possible, with existing technology, to collect sufficient data.

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