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# WATER DISTRIBUTION PATTERNS FROM TRICKLE IRRIGATION SYSTEMS

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**Abstract**

The aim of this study was to investigate the movement of water in a trickle irrigation system. A plane numerical model of soil water movement was developed for this study. The numerical model was developed using the alternating direction implicit (ADI) finite difference method. Results obtained experimentally under laboratory conditions were compared to those predicted by the model. The soil media used in the laboratory experiments was a light loam. The experimental results were found to agree closely with those from the numerical model.

## 1 Numerical Model

The water movement under irrigation is usually linked with imposed conditions at known levels in the soil profile. Suppose the soil profile is a homogeneous and uniform porous media; there is no air resistance to water movement in soil; there is no water movement at the begining because the initial moisture content in soil is negligible; Darcy's law is valid in both saturated and unsaturated soils; the changes in the hydraulic conductivity and diffusion coefficient are only dependent upon the moisture content of the soil; and root suction is assumed to have a negligible effect on the water movement.

Consider a number of trickle points on a grid with spacings $2x$ and $2y$ in the $x$ and $y$ directions respectively. Due to symmetry, half the region of water movement may be considered. The large domain of water movement is divided into a large number of small elements which have the same properties. Thus, water movement over the whole domain may be determined by studying the water movement in one small element. The regions saturated by single trickle will overlap if the distance between trickles in $y$−direction ($y \ll x$, or $y \to 0$) is small enough. Thus, the very long wetting bank with $2\rho(t)$ width is formed (suppose the wetting bank is infinitely long).

We will only discuss the water movement in $XOZ$ section with unit width in the $y$−direction.

The differential equation of soil water movement was obtained from mass conservation and Darcy's law (Gury and Earl, 1977).

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[D(\theta)\frac{\partial \theta}{\partial x}\right] + \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial \theta}{\partial z}\right] - \frac{\partial K(\theta)}{\partial z} \qquad (1)$$

where,

$\theta$ is volumetric soil water content,

$t$ is time,

$D(\theta)$ is soil water diffusivity,

$K(\theta)$ is unsaturated hydraulic conductivity,

$x$, $z$ are Cartesian coordinates, $z$ vertically downward is the positive direction.

The initial and boundary conditions of Equation 1 are set out as below:

$$t = 0: \quad \theta = \theta_0, \quad 0 \le x \le X, \quad 0 \le z \le Z; \qquad (2)$$

$$x = 0: \quad \frac{\partial \theta}{\partial x} = 0, \quad 0 \le z \le Z, \quad 0 \le t \le T; \qquad (3)$$

$$x = X: \quad \frac{\partial \theta}{\partial x} = 0, \quad 0 \le z \le Z, \quad 0 \le t \le T; \qquad (4)$$

$$z = 0: \quad \theta = \theta_s, \quad 0 \le x \le \rho(t), \quad 0 < t \le T; \qquad (5)$$

$$z = 0: \quad \int_0^{\rho(t)} \left[K(\theta) + E - D(\theta)\tfrac{\partial \theta}{\partial z}\right] dx = \tfrac{1}{2}q, \quad 0 \le x \le \rho(t), \quad 0 < t \le T; \qquad (6)$$

$$z = 0: \quad K(\theta) + E - D(\theta)\tfrac{\partial \theta}{\partial z} = 0, \quad \rho(t) < x \le X, \quad 0 \le t \le T; \qquad (7)$$

$$z = Z: \quad \frac{\partial \theta}{\partial z} = 0, \quad 0 \le x \le X, \quad 0 \le t \le T \qquad (8)$$

where,

$\theta_0$ is initial volumetric soil water content,

$\theta_s$ is saturated volumetric soil water content,

$E$ is evaporation rate,

$T$ is total infiltration time.

$q$ is unit trickle discharge (per unit width and unit time,)

Equation 5, 6 and 7 are the soil surface boundary conditions. The difference between them and the other boundary conditions are that they are dynamic boundary conditions expressed by $\rho(t)$.

Equation 1 – 8 are the plane numerical model of the soil water movement under the trickle irrigation.

## 2 Numerical solution

When using finite difference formulation approximation, the calculation region is initially divided into many small regions, namely, the large domain is represented by finite difference mesh. There are many kinds of meshes, and the mesh may be non-uniform. Here the rectangular mesh is discussed only. The numbers of nodes are $i = 0, 1, 2, \cdots, n$, in $x-$direction, the mesh width is $\Delta x$. The numbers of nodes are $j = 0, 1, 2, \cdots, m$, in $z-$direction, the mesh width is $\Delta z$. Time is divided into small intervals, the numbers of time nodes are $k = 0, 1, 2, 3, \cdots,$ the interval is $\Delta t$.

Equation 1 can be written as the finite difference equation by the alternating direction implicit formulation method in every node $(i, j)$. The alternating direction implicit formulation method, normal called the ADI method, was first introduced by Peaceman and Rachford (1955). Its attractive feature is that it is an implicit method which does not involve the costly solution of a large number of simultaneous equations.

It should be pointed out that there is a saturated region near the trickle nozzle at every interval. The saturated region gets larger and larger with the increase in intervals. There are different methods to calculate for unsaturated and saturated soil water problems.

For the above-mentioned calculation method, a computer program was developed for the ADI method. The space distances used in calculation were $2cm$, that is, $\Delta x = 2cm$ and $\Delta z = 2cm$ in $x-$direction and in the $z-$direction. In order to speed the calculation, the time interval was changed with the infiltra-tion time. The trickle discharge rates used in the numerical method were the same as those used in the experimental model, which were 0.72, 1.20 and 1.68 $l/hr$, and 9, 15 and 21 $l/hr/m$ in unit discharge rate respectively. The control precision was 0.001 in the repeated calculation.

# 3 Experiment and results

The soil media used in the laboratory was light loam in the experimental model. The loam contained 56.7% mortar and 24.1% powder. The control unit dry weight was 12.9 $kN/m^3$, the initial moisture content was 0.09, the saturated moisture content was 0.478; The hydraulic conductivity and diffusion coefficient of loam were meassured by the "horizontal soil column method" and "instantaneous section method" respectivelly. The following formulas were given from the meassured results:

$$D(\theta) = \begin{cases} 144.83(\theta/\theta_s)^{9.86} & \theta > 0.289 \\ 11.07(\theta/\theta_s)^{4.75} & \theta \leq 0.289 \end{cases}$$

$$K(\theta) = \begin{cases} 0.083(\theta/\theta_s)^{10.29} & \theta > 0.317 \\ 0.008(\theta/\theta_s)^{4.59} & \theta \leq 0.317 \end{cases}$$

The channel of experimental appratus was made of plexiglass with dimensions $1.50 \times 0.80 \times 0.08m$, they were respectively $x$, $y$ and $z$ direction in turn.

In order to control the discharge rate of the trickle, the hair trickle nozzle was connected to a constant head water tank and the Mariotte's bottle connected to the pipe. The nozzle was located at the center of the surface soil. The surface of soil was covered by plastic film, so, the evaporation was zero at the surface boundary.

The position of the wetting front can be obtained from the moisture content. For comparison between experiment and calculation, the distribution of moisture content for one of the trickle discharge rates (9 $l/hr/m$) are plotted in Figure 1.

The experimental results and theoretical calculation agree closely. The agreement of the wetting fronts in $z-$direction are closer than that in $x-$direction. In addition, the smaller the discharge rate, the closer the agreement between the numerical and experimental results.

# 4 Discussion

## 4.1 The effect of discharge rate on the soil water movement

The properties of the soil, the trickle discharge rate and the total water quantity of irrigation significantly influence the soil water movement in trickle irrigation.

**1. The effect on the distribution of moisture content**

From the calculated results, the final wetted volume of soil does not increase with an increase in the trickle discharge rate for a constent quantity of irrigation water. However, the moisture content around the nozzle increases with an increase in the trickle discharge rate.

**2. The effect on the saturated bank of the soil surface**

The saturated bank refers to the area where the moisture content reached the saturated moisture content at the soil surface. The relationship between the radii of the saturated region and infiltration time are plotted in Figure 2. As shown in Figure 2, the saturated bank gets

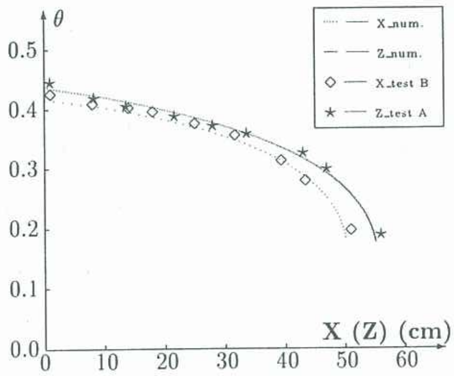


Figure 1: The distribution of the moisture content; trickle discharge: 9 $l/hr/m$

wider as the infiltration time increases, irrespective of the rate. The rate of increase of the saturated bank width decreases with time. Both the width and rate of expansion of the saturated bank are proportional to the trickle discharge rate.

**3. The effect on the contour of the wetting front**

As shown in Figure 3, the greater the trickle discharge rate, the greater the extent of the wetting front $F_x$ and $F_z$ at any given time. Also, for a fixed quantity of irrigation water, the higher the discharge rate, the more rapid the rate of water movement in the $x-$direction, caused by a ponding effect. This result has been verified by field experiment (Fu, 1983, Goldbery and Shmuel, 1970, Hachum et al., 1976). Obviously, for smaller trickle discharge rates, a greater time is required to supply same quantity of water, however, the wetting front is more extensive than for larger discharge rates.

It is important to understand the effect of the trickle discharge on the water movement for the design of trickle irrigation system. Although the water movement in horizontal direction is more rapid with higher discharge rates, the saturated ponding region also increases. The water movement is similar with that in furrow irrigation, which is undesirable in the design of a trickle irrigation system. To avoid this, the larger trickle discharge rate should be chosen only where the soil porosity allow. Prevention of ponding will assist in reducing the likelihood of the trickles becoming blocked, and also will help to reduce salt

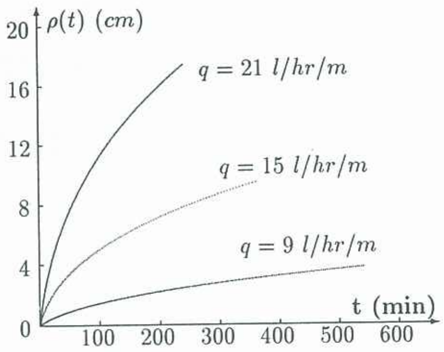


Figure 2: The saturated region radius vs time

accumulation in the wetted soil under brackish water irrigation.

## 4.2 The contour of wetting front and its movement

The relationship between the wetting front and square root of infiltration time are plotted in Figure 3. It is apparent that a linear relationship between the square root of infiltration time and the position of the wetting front in both $x-$direction and $z-$direction, exists as follows,

$$F_x(t) = a_x t^{1/2} + b_x \quad (9)$$

$$F_z(t) = a_z t^{1/2} + b_z \quad (10)$$

where, $a$ and $b$ are constants, dependent on the soil media and the trickle discharge rate.

For the three trickle discharges used in the experiment, $a$ and $b$ are calculated by a regression method. The formulas are as follows,

$q = 9 \quad F_x = 1.926t^{1/2} + 2.556 \quad F_z = 2.061t^{1/2} + 1.893$

$q = 15 \quad F_x = 2.388t^{1/2} + 0.933 \quad F_z = 2.567t^{1/2} - 0.004$

$q = 21 \quad F_x = 2.449t^{1/2} + 1.675 \quad F_z = 3.052t^{1/2} - 2.804$

The relationships between $F_x$ ($F_z$) and $t^{1/2}$ are similar to Philip's equation for horizontal unsaturated infiltration in one dimension.

It is obvious from above equations that: The rate of moisture movement in the $x-$direction is greater than that in $z-$direction at the beginning of trickle irrigation because of the diffusion of the saturated region; The moisture movement are the same in both $x-$ and $z-$direction at the moderate infiltration times; The wetting front in the $z-$direction extends further than that in the $x-$ direction with the increase of infiltration time, this may be because of the increase in hydrostatic head with the larger saturated region.

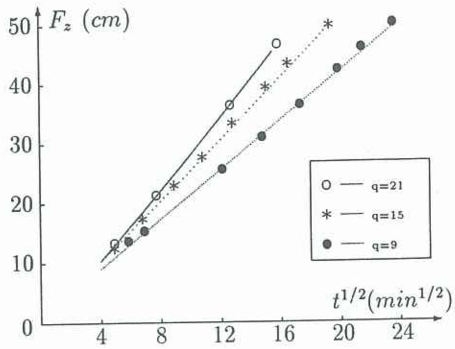


Figure 3: The wetting front vs $t^{1/2}$ in $z-$direction

The contour of wetting front under trickle irrigation can be expressed by semielliptical equation, that is,

$$\frac{x^2}{F_X^2} + \frac{z^2}{F_Z^2} = 1 \quad (11)$$

where, $F_X$ is the maximun wetting front in horizontal direction, $F_Z$ is the maximun wetting front in vertical direction, $x$ and $z$ are the coordinates of wetting front contour.

The semielliptical equation and numerical calculation results agree closely. Hachum et al. (1976) conducted similar experiments with different soil media, they obtained the same result as Equation 11 for the contour of the wetting front. Therefore, the contour of wetting front can be calculated by Equation 11, then the wetting volume of soil can be calculated if required to that under trickle irrigation.

# 5 Conclusions

A comparison of the experimental and the numerical results indicates the value of the numerical method when applied to trickle irrigation. Following verification of the numerical model, soil water movement is analysed to determine the influence of trickle irrigation discharge rates on the contour of the soil wetting front. The results suggest that the discharge rates will have a significant effect on soil water movement. An increase in the trickle discharge rate will result in a corresponding increase in the horizontal ponded area surrounding the nozzle. However, a reduction in the soil wetted depth will occur. It was also found that the maximum distance to the wetting front in both horizontal and vertical directions is proportional to the square root of the infiltration time. The contour of the wetting front at any infiltration time can be expressed by semielliptical equation. All of these factors, are important in the design of trickle irrigation systems.

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