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NUMERICAL STUDIES OF FLUID FLOW THROUGH MULTIPLY-CONSTRICTED TUBES

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ABSTRACT

The flow fields in the neighbourhood of constrictions in a tube were studied numerically. The effects on the streamline, velocity and vorticity distributions as the flow passes through the constrictions in the tube were studied for a Reynolds number range of 5 to 200. Constrictions with constriction spacing ratios of 1, 2, 3 and ∞ were studied for diameter constriction ratios of 0.2 to 0.6.

NOTATIONS

a_o	radius of the tube having a constant cross section.
c_1	upstream constriction, $c_1=(D-d_{c1})/D$
c_2	downstream constriction, $c_2=(D-d_{c2})/D$
d_c	openning of the constiction.
s_1, s_2	distance of first and second constrictions from inlet plane.
S	$S = (s_2-s_1)$
v_r	r-direction velocity component
v_z	z-direction velocity component
v_∞	centreline axial velocity at infinity
z_1, z_2	limits of first constriction
z_3, z_4	limits of second constriction
ψ	stream function
ζ	vorticity
t	time
Re	Reynolds number
p	pressure
ρ	density
ν	kinematic viscosity

INTRODUCTION

The flow field in the neighbourhood of constrictions in tubes has been of great interest to fluid dynamicists because of its many engineering applications. This type of configuration is used in design of heat exchangers to enhance its heat transfer performances. Configuration of tubes with constrictions are also of great interest to biofluid dynamicists because of their relationship to localised stenoses in blood and urinary flow; and the optimal design of artificial organs. Viscous fluid flow past undulating boudaries has also been of great interest to researchers because of the importance which it plays in phemomena such as the generation of wind waves on water; the stability of a liquid film in contact with a gas stream; the transpiration cooling of re-entry vehicles and rocket boosters and film vaporization in combustion etc.

One of the first numerical work on this type of problem was done by Lee & Fung(1970) to study the flow in locally constricted tubes for a Reynolds number range of 0 to 25. In more recent work, Wille(1980) studied the pressure and flow fields in arterial simulated by mathematical models. Sober(1980) studied numerically the flow through furrowed channels to investigate the Reynolds number effects on the separated flow. Prata & Sparrow(1984) obtained numerical solutions for a periodic fully developed regime in an annulus of varying cross section of a double-pipe heat exchanger. Other recent studies of constricted flow includes a study of laminar steady flow in sinusiodal channels by Tsangaris & Leiter(1984).

Most of the above studies, however, are for single constriction flow (ie S/D=∞). Few consider the influence of the upstream constriction on the flow fields near the downstream constriction. In the present work, the flow behaviour in a double constrictions tube is studied numerically.

PROBLEM FORMULATION

The geometrical configuration of the vascular tube with double constrictions and its coordinate system is shown in **Fig. 1**. Constant fluid properties are assumed and the flow is considered axisymmetric and laminar. The dimensionless governing equations representing the fluid flow through the constrictions in their unsteady form are :

$$\frac{\partial\zeta^*}{\partial t^*} + \frac{\partial(v_r^*\zeta^*)}{\partial r^*} + \frac{\partial(v_z^*\zeta^*)}{\partial z^*} = \frac{1}{Re}\left(\frac{\partial^2\zeta^*}{\partial r^{*2}} + \frac{\partial^2\zeta^*}{\partial z^{*2}} + \frac{1}{r^*}\frac{\partial\zeta^*}{\partial r^*} - \frac{\zeta^*}{r^{*2}}\right) \quad (1)$$

and the vorticity-stream function equation

$$\zeta^* = \frac{1}{r^*}\left(\frac{\partial^2\psi^*}{\partial r^{*2}} + \frac{\partial^2\psi^*}{\partial z^{*2}} - \frac{1}{r^*}\frac{\partial\psi^*}{\partial r^*}\right) \quad (2)$$

The velocities are given by

$$v_r^* = \frac{1}{r^*}\frac{\partial\psi^*}{\partial z^*} \quad ; \quad v_z^* = -\frac{1}{r^*}\frac{\partial\psi^*}{\partial r^*} \quad (3)$$

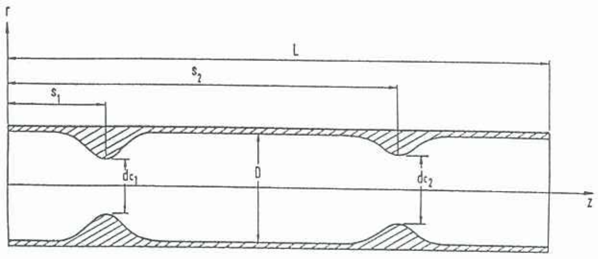

Fig. 1
MODEL OF MULTIPLE CONSTRICTIONS IN TUBES

where the dimensionless variables are defined as : $r^* = r/a_o$; $z^* = z/a_o$; $v_r^* = v_r/v_\infty$; $v_z^* = v_z/v_\infty$; $\psi^* = \psi/\ (v_\infty a_o^2)$; $\zeta^* = \zeta/\ (v_\infty/a_o)$; $t^* = t/(a_o/v_\infty)$ and the Reynolds number, $Re = v_\infty a_o/\nu$.

(For subsequent expressions, the * is dropped for simplicity)

In dimensionless form, the geometry of the constrictions are described by the following bell-shaped Gaussian distribution profile

$$f(z) = 1 - c_i \exp(-c_s(z - s_i)^2) \quad (4)$$

where c_i = constriction ratio $(D - d_{ci})/D$
c_s = a shape constant
s_i = dimensionless distance of constriction from inlet plane.

Refering to **Fig. 1**, for the variable double constrictions considered here, $c_i=0.5$ (ie c_1) for the first constriction and c_i (ie c_2) has values from 0.2 to 0.6 for the second constrictions; the shape factor $c_s=0.4$; the distance $s_i=s_1$ for $z_1< z < z_2$; the distance $s_i= s_2$ for $z_3< z < z_4$ and $c_i=0$ elsewhere. z_1, z_2 are the upper and lower limits of the first constriction; z_3, z_4 are the upper and lower limits of the second constriction. s_1, s_2 are the distance of the first and second constriction from the inlet plane respectively. The spacing between the two constrictions is given by $S=(s_2-s_1)$ and $s_1=2.0$ in this study. When the dimensionless spacing ratio $S/D=\infty$, this is equivalent to a single constriction tube with $c_i=0.5$, $c_s=0.4$, $s_i=s_1$ for $z_1< z < z_2$ and $c_i=0$ elsewhere.

For the present study, the incoming flow is assumed to be Poiseuillean and outflow is assummed unrestictive (ie a weak boundary condition is specified). Non-slip boundary condition is assumed for the tube wall. The flow is assumed symmetry about its axis with $v_r=0$ along the axis. ie.

At the inlet, $\quad v_r = 0 ;\ v_z = (1 - r^2) \quad (5)$

At the outlet, $\quad \dfrac{\partial^2 \zeta}{\partial z^2} = 0 \quad (6)$

Along the tube axis, $v_r = 0 ;\ \dfrac{\partial v_z}{\partial r} = 0 \quad (7)$

Along the tube wall, $v_r = v_z = 0 \quad (8)$

NUMERICAL SOLUTION

The tube with the bell-shaped constrictions are mapped into a rectangular solution domain and the flow fields are solved with a finite difference method. The new coordinates system is defined as follows :

$$\varepsilon = F_1(z) = z$$

$$\eta = F_2(r,z) = r/f(z) \quad (9)$$

The domain in the ε-η coordinate system as defined by Equation (9) is a rectangular region. In order to obtain better resolution of the solution near the wall regions while preserving the second order accuracy of the finite difference scheme, the rectangular solution domain is overlaid with a non-uniform mesh with the grid generator given by

$$F_3 = \frac{2}{\pi}\sin^{-1}\left((\eta + 0.5)^{1/2}\right) \quad (10)$$

At the node points of the domain defined by Equation(10), the finite difference solution to Equation (1) and the other governing and boundary equations in the ε-η co-ordinate, are obtained through an Alternating Direction Implicit (ADI) procedure proposed by Samarskii and Andreev (1963). Successive over-relaxation (SOR) method with a relaxation parameter of ω = 1.1 was used to solve the vorticity-stream function Equation (2).

RESULTS AND DISCUSSIONS

The study presented here is one portion of an overall study of flows in regions of the double constrictions. The analysis is restricted to steady, laminar flow of a Newtonian fluid through a rigid tube which has different sizes of multiple localized axisymmetric constrictions. For the present study, the first constriction (c_1) of the tube is set at 0.5 while the second constriction (c_2) is allowed to vary from 0.2 to 0.6.

For a given approaching Reynolds number, typical streamlines and vorticity fields at different proximity (S/D) of the constrictions in the present study are shown in **Fig. 2** for $c_2>c_1$ at Re=25 and in **Fig. 3** for $c_2<c_1$ at Re=50. Studies of the numerous similar flow fields for $c_1=0.5$ with $c_2<c_1$ show that, a recirculation zone usually fills the valley between the two constrictions for small S/D ratios, with little changes to the separation and reattachment points as the approaching Reynolds number is increased. A separation streamline divides the flow into two parts: a recirculating flow field between the two constrictions and the main flow field near the centre of the vascular tube with relatively straight and parallel streamlines. Streamlines and vorticity fields in similar studies for $c_1=0.5$ and $c_2>c_1$ show that as the Reynolds number increases, the recirculating eddy between the two constrictions spread beyond

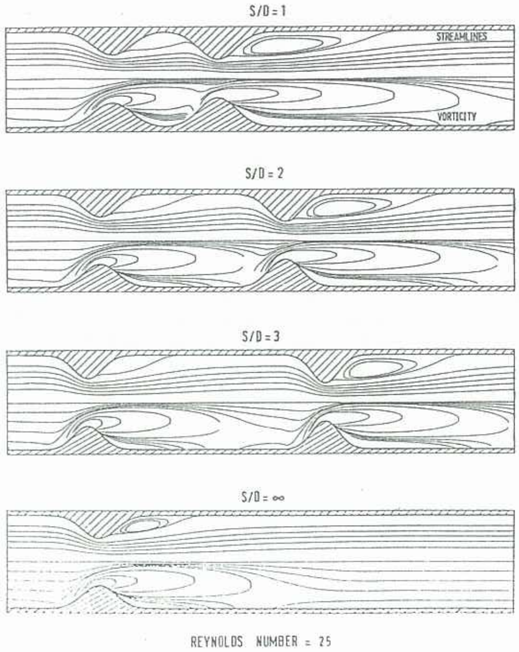

Fig. 2
TYPICAL STREAMLINES AND VORTICITY CONTOURS
c_1=0.5, c_2=0.6, Re=25

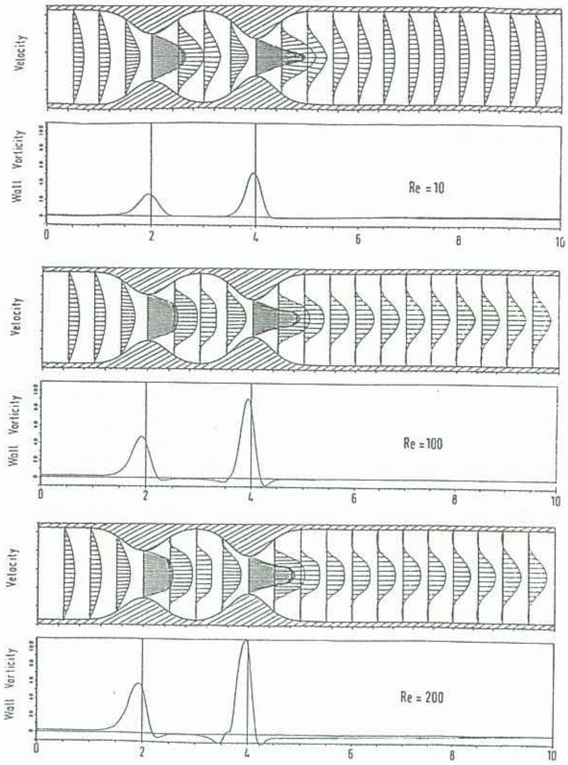

Fig. 4
VELOCITY PROFILES AND WALL VORTICITY DISTRIBUTIONS
c_1=0.5, c_2=0.6, S/D=1

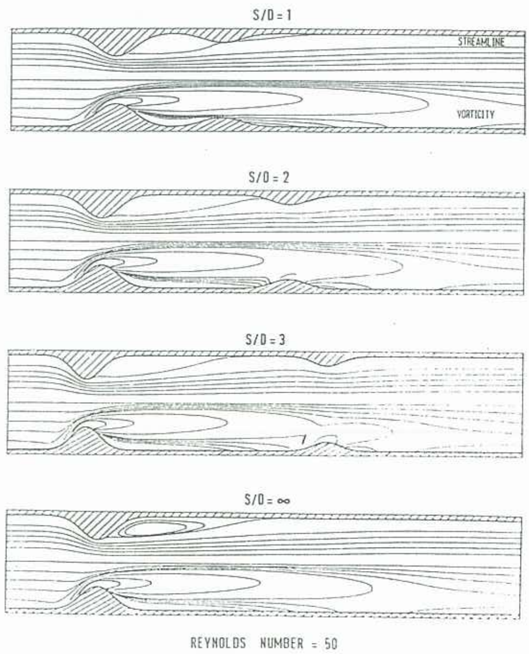

Fig. 3
TYPICAL STREAMLINES AND VORTICITY CONTOURS
c_1=0.5, c_2=0.2, Re=50

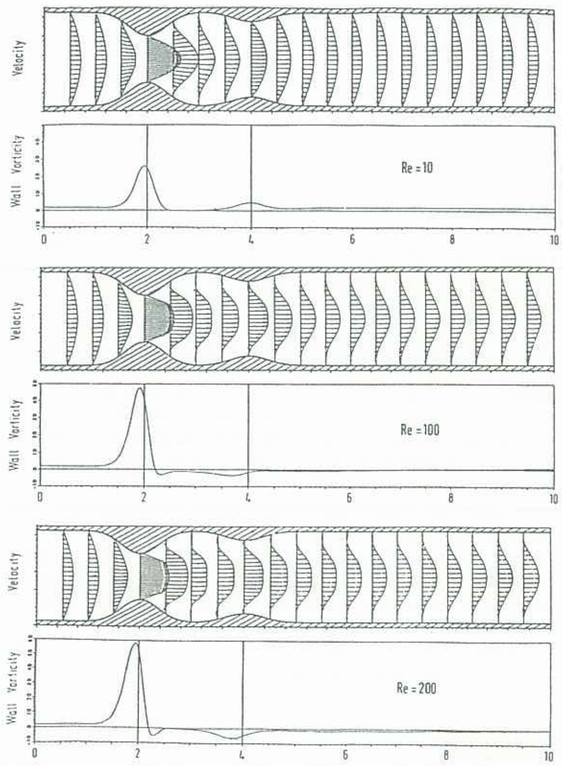

Fig. 5
VELOCITY PROFILES AND WALL VORTICITY DISTRIBUTIONS
c_1=0.5, c_2=0.2, S/D=1

the second constriction, merging with the eddy that formed behind the second constriction. This flow phenomenon significantly influences the downstream vorticity characteristics near the second constriction.

Characteristics of the flow through the variable constrictions with various separation ratio can also be described by the velocity vector fields in the tube. The velocity vectors in **Fig. 4** and **Fig. 5** are for c_2=0.6 and c_2=0.2 respectively with c_1=0.5, S/D=1 and Re=10, 100, 200. The wall vorticity values are shown in the corresponding figures. The magnitude of the wall vorticity values increase rapidly when the flow approaches the constriction and reaching a peak value near the maximum constricted area. At higher Reynolds number, the peak wall vorticity value was found slightly upstream of the maximum constricted area. At a location downstream of this peak value, the wall vorticity decreases rapidly and will reverse to negative values when separation begins at the wall of the tube. For the first constriction, it is observed that the peak wall vorticity value increases with increasing Reynolds number. For c_1=0.5 and c_2=0.2 with increasing Re, the merging of the recirculating eddy from the first constriction with the eddy formed behind the second constriction resulted in a negative wall vorticity peak occurring at the second constriction. However, for c_1=0.5 with c_2=0.6 , the maximum wall vorticity at the second constriction ($\zeta_{max,2}$) in the present investigation is found to be nearly twice that of the first constriction ($\zeta_{max,1}$). It was also observed that the rate of increase of $\zeta_{max,1}$ for the first constriction with Re was higher than for $\zeta_{max,2}$ of the second constriction.

CONCLUSION

The effect on streamlines, vorticity, flow seperation and reattachment, velocity distribution and wall vorticity as the fluid flow passes through two adjacent constrictions (c_1, c_2) are numerically investigated. For c_1=0.5 with c_2>c_1 and small constriction spacings, recirculation tends to fill the valley region between the two constrictions with little changes to the separation and reattachment points as the approaching Reynolds number is increased. For c_1=0.5 with c_2 <c_1 with small spacing ratios, the recirculating eddy between the two constrictions tends to merge with the eddy formed downstream of the second constriction when the approaching Reynolds number is increased. This produces negative wall vorticity peak near the second constriction. For c_1=0.5 and c_2=0.6, the maximum wall vorticity at the second constriction is nearly twice that of the first constriction. However, the rate of increase of the maximum wall vorticity with respect to the Reynolds number is higher for the first constriction than for the second constriction. If no merging or interacting of the recirculation eddies occurred, the maximum wall vorticity near each constriction would increase with Reynolds number (Re) and constriction spacing ratio (S/D).

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