11th Australasian Fluid Mechanics Conference
University of Tasmania, Hobart, Australia
14-18 December 1992

REYNOLDS STRESS EXPRESSION IN A SUPERLAMINAR LUBRICATION FILM

P.B. KOSASIH, A.K. TIEU and M.R. MACKENZIE

Department of Mechanical Engineering
University of Wollongong
PO Box 1144, Wollongong, NSW 2500, AUSTRALIA

ABSTRACT

The paper describes a model of Reynolds stress for application in superlaminar lubrication analysis. The approach relies on Prandtl's mixing length theory which is based on a modified Van Driest Mixing Formula (Granville, 1989). However, unlike past theories, the proposed equation is capable of accounting for the effect of shear stress gradient on the mixing length. Thus it is well suited to superlaminar flow analysis in bearings where the presence of shear stress gradients due to the effect of pressure gradient should be considered. A series of velocity measurements in thin channels in the low Reynolds number superlaminar flow range were performed using a Laser Doppler Anemometer (LDA) system operated in the back scattered mode. The experimental data were analysed using the above theory and they show a significant effect of shear stress gradient in the viscous sublayer. As a result, a new model of mixing length applicable to the superlaminar lubrication analysis is presented. Using the new theory, planar velocity profiles in various film geometries are calculated and compared with experimental results.

NOMENCLATURE

a_x, a_z	$= \frac{Bx\,Re}{\eta_{xh}^2}, \frac{Bz\,Re}{\eta_{zh}^2}$
Bx,Bz	= dimensionless pressure gradient along the x and z axis $-\frac{h^2}{\mu V}\frac{\partial p}{\partial x}, -\frac{h^2}{\mu V}\frac{\partial p}{\partial z}$
C	= intercept of the logarithmic profile
D	= journal diameter
h	= local film thickness
l	= mixing length
L	= journal bearing length
l^+	$= l\,u_\tau / \nu$
p	= pressure
p^+	= pressure gradient parameter $= \frac{\nu}{\rho u_\tau^3}\sqrt{\frac{\partial p}{\partial x}^2 + \frac{\partial p}{\partial z}^2} = \frac{\nabla p^*}{\eta_h^3}$
Re	= Reynolds number ($h\,u_m / \nu$ for Poiseuille flow and $h\,V / \nu$ for bearing flow)
u,w	= velocity in x-z direction, $\bar{u}=\frac{u}{V}, \bar{w}=\frac{w}{V}$
u_m	= mean velocity of the flow
u',v'	= fluctuating velocity components
u^+	$= u / u_\tau$
u_τ	= friction velocity $= \sqrt{\frac{\tau_w}{\rho}}$
V	= relative velocity between two planes
y	= distance from the wall, $\bar{y}=\frac{y}{h}$, $y^+=\frac{y\,u_\tau}{\nu} = \bar{y}\,\eta_h$
ε	= eddy viscosity or eccentricity ratio of the journal bearing
ρ	= density
κ	= constant in the mixing length formula
λ	= Van Driest factor
τ	= shear stress
η_h	$= (\frac{h}{\nu})\sqrt{\frac{\lvert\tau_o\rvert}{\rho}}$, $\eta_{xh}=(\frac{h}{\nu})\sqrt{\frac{\lvert\tau_{yxo}\rvert}{\rho}}$, $\eta_{zh}=(\frac{h}{\nu})\sqrt{\frac{\lvert\tau_{yzo}\rvert}{\rho}}$
τ_o	$= \sqrt{\lvert\tau_{oyx}\rvert^2 + \lvert\tau_{oyz}\rvert^2}$
τ_{yx}, τ_{yz}	= shear stress in yx-plane, shear stress in yz-plane, $\tau^+ = \frac{\tau}{\tau_o}$
μ	= dynamic viscosity
ν	= kinematic viscosity
∇p^*	$= \frac{h^3}{\mu\nu}\sqrt{(\frac{\partial p}{\partial x})^2 + (\frac{\partial p}{\partial z})^2} = \sqrt{Bx^2+Bz^2}\,Re$

INTRODUCTION

A number of approaches for treating turbulent lubrication has evolved in the past decades. This is due to the ever increasing number of bearings operated with low kinematic viscosity fluid and at high operating speeds. As a result of this trend, the bearings tend to operate beyond the laminar regime. The existing approaches have been proven to be satisfactory for designing bearings operating in fully developed turbulent flow. However there is a lack of information or design aids concerning bearing design in the transition region between laminar and fully developed turbulent regimes. Moreover since the extent of this region is still unknown, a general theory is needed to cover transition-turbulent flow.

The approaches to the problem of turbulent lubrication have been presented by Constantinescu(1959), Ng and Pan(1965), Elrod and Ng(1967), and Hirs(1973). All the above theories except Hirs attempted to describe the turbulence effect in the form of Reynolds stresses. Constantinecu applied Prandtl's mixing length theory, while Ng and Pan, and Elrod and Ng used Reichardt's formula for eddy viscosity to model the Reynolds stresses. These theories generally fail to produce an accurate model for transition flow and low Re superlaminar flow. This is due to the fact that these theories applied curve fitting techniques to experimental data of Couette and Poiseuille fully turbulent flows in large channel and pipe flows

* Superlaminar flow means flow beyond the laminar regime including the transition regime.

at high Re numbers. The adopted empirical constants in the expression of Reynolds stress are often used in lubrication analysis which is well beyond the range of experimental conditions of the original data.

In turbulent boundary layer the law of wall calculated by these theories takes the form of Eq.(1)

$$u^+ = \frac{1}{\kappa} \log y^+ + C \qquad (1)$$

with κ=0.4 and C=5.25

In the transition or low Reynolds number superlaminar flow, the law of wall is shown by Huffman and Bradshaw(1972) to be affected by the dimensionless shear stress gradient ($\frac{\partial \tau^+}{\partial y^+}$). It was shown that the logarithmic velocity profile according to Eq.(1) is valid if the dimensionless shear stress gradient ($\frac{\partial \tau^+}{\partial y^+}$) is not much larger than 10^{-3}. At larger values of ($\frac{\partial \tau^+}{\partial y^+}$) the value of constant C in Eq.(1) departs from its basic value of 5.25. The analysis of law of wall profile can also be carried out using the mixing length concept. In the mixing length formula

$$l^+ = \kappa y^+ \sqrt{\tau^+} \, (1 - e^{-y^+/\lambda}) \qquad (2)$$

the Van Driest factor (λ) determines the value of C. Therefore if C departs from its basic value of 5.25 so does λ from 26.

The data presented in this paper show a strong effect of $\frac{\partial \tau^+}{\partial y^+}$ on the viscous sublayer in the transition and low Re number superlaminar flow, and from these results a new model of Reynolds stress for superlaminar lubrication theory applicable of low or high Re and low or high shear stress gradient is presented. Finally velocity profiles in typical lubrication film are presented along with their experimental counterparts.

REYNOLDS STRESSES AND VELOCITY PROFILES ANALYSIS IN LOW REYNOLDS NUMBER SUPERLAMINAR FLOWS

The total shear stress in turbulent flows is given by

$$\tau = \mu \frac{du}{dy} - \rho \overline{u'v'} = \mu \left(1 + \frac{\varepsilon}{\nu}\right) \frac{du}{dy} \qquad (3)$$

with ε expressed by Prandtl's mixing length concept as

$$\varepsilon = l^2 \frac{du}{dy} \qquad (4)$$

Non-dimensionalizing Eq.(3) using the following terms,

$$u_\tau = \frac{u}{u_\tau}, \; y^+ = \frac{y \, u_\tau}{\nu}, \; \tau^+ = \frac{\tau}{\tau_0}, \; l^+ = \frac{u_\tau \, l}{\nu}$$

the total shear stress expression becomes

$$\tau^+ = \frac{du^+}{dy^+} + l^{+2} \left(\frac{du^+}{dy^+}\right)^2 \qquad (5)$$

Integrating Eq.(5) following Granville(1989) the velocity equation is obtained as follows

$$\frac{du^+}{dy^+} = \frac{2 \, \tau^+}{1 + \sqrt{1 + (2 \, l^+)^2 \, \tau^+}} \qquad (6)$$

The data from thin channels presented were fitted by Eq.(6) with λ varied to optimise the fitting. In this paper $\frac{\partial \tau^+}{\partial y^+}$ is calculated with the effect of the inertia term neglected, therefore $\frac{\partial \tau^+}{\partial y^+}$ is equal to the pressure gradient parameter p^+. The results are shown in Figures 1 to 3. It can be seen from these figures that for 3 and 5mm channels the universal law of wall applies at Re>4000, whereas for 2 and 1mm it applies at Re>6500. Thus for flows in thin films the universal law could be used if Re is well above 6500. For lower Re the universal law does not apply and it can be shown that at $\frac{\partial \tau^+}{\partial y^+}$ >0.009 the modified Van Driest mixing length formula has to be used with modified λ or κ values. The change of κ has been considered but the curve fitted profiles were not accurate. So in this paper κ was assumed constant at 0.4 and λ was allowed to vary. The data analysis of the present results also shows a strong effect of $\frac{\partial \tau^+}{\partial y^+}$ on the viscous sublayer. For $\frac{\partial \tau^+}{\partial y^+}$ larger than 10^{-3}, the effect of $\frac{\partial \tau^+}{\partial y^+}$ on the viscous sublayer is more pronounced as shown by the velocity profiles. From the curve fitting analysis of the present data and those given in Huffman and Bradshaw's paper(1972) The variation of optimum λ with $\frac{\partial \tau^+}{\partial y^+}$ is shown in Figure 4. It is clear that $\lambda = f\left(\frac{\partial \tau^+}{\partial y^+}\right)$ and in the turbulent lubrication analysis an approximate numerical relation between $\frac{\partial \tau^+}{\partial y^+}$ and λ will be used.

This approximate relation is given by

$$\left(\frac{A}{\lambda}\right)^2 = 1 + B \frac{\partial \tau^+}{\partial y^+} \qquad (7)$$

with A=26 and B=55

The present analysis shows the sensitivity of the viscous sublayer to $\frac{\partial \tau^+}{\partial y^+}$. Initially, one would think that the influence of channel gap is more prominent especially when the gap is of the order of boundary layer thickness. However the present analysis and those by Huffman and Bradshaw(1972) suggest that this external influence will be felt strongly via the parameter $\frac{\partial \tau^+}{\partial y^+}$.

CORRELATION BETWEEN THEORY AND EXPERIMENTS

Applying Eq. 6 the validity of the above theory in predicting planar velocity profiles (Bz=0) in laminar-turbulent regime was investigated using the available experimental data obtained for various film shapes, namely,

(i) between a circular surface and a plane (Galetuse,1974) in Figure 5a.

(ii) in a tilting pad (Innes and Leutheuser, 1991) in Figure 5b.

(iii) and in a journal bearing by the authors in Figure 5c.

Other experimental works(i and ii)

For the first case, data have been presented by Galetuse(1974). He conducted an experiment to measure the pressure distribution and velocity distribution in the air film between a rotating cylinder and a fixed plane surface. To obtain a two-dimensional flow, the lateral flow was restricted with the aid of thin plates fixed to the fixed plane surface. The numerical values for his experiment were V=40m/s, L=65, 150, 285mm, h_{min}=0.3-3.5mm, D=1004mm. Comparison between the velocity profiles obtained from the theoretical analysis outlined above and Galetuse's results are shown in Figure 6. The velocity profiles are along the y-axis and normal to the air film on the middle line (x=0). It should be noted that since the variation in film thickness in the flow direction is large, the inertia force is expected to have an influence on the velocity profiles. However the results of the comparison in Figures 6 show good agreements between theory and experiment. Figure 6a shows the velocity profile of fully turbulent flow with large adverse pressure gradient (i.e. pressure gradient against the actual flow direction). The shape of the profile is not like a typical turbulent Couette S-shape. This is because of the effect of the pressure gradient which opposes the direction of flow. Good agreements both qualitatively and quantitatively were obtained. The case where the value of the opposing pressure gradient is large and reverse flow occurs is shown in Figure 6b. Both velocity profiles generally show a similar shape, however the reverse flow predicted by the model is smaller. Figure 6c shows the case of transitional flow with a small favourable pressure gradient (i.e.the pressure gradient is codirectional with the actual flow). In this case an excellent agreement has been obtained.

For the second case, the theoretical velocity profiles were compared with Innes and Leutheusser's(1991) experimental velocity profiles in a finite slider bearing. The selected profiles were obtained from a tilting pad with the following dimension h1=12.5mm, h2=6.25mm, giving a taper ratio of 2. The pad length is 2.5m and width is 0.95m. Velocity profiles at three locations, i.e. x/l=0, 0.5, 1 were measured . Since pressure gradients were not measured with the original data. the pressure gradients were estimated by solving the pressure field of a compressible slider bearing using the modified turbulent Reynolds equation in conjunction with the turbulent lubrication theory as presented by Kosasih and Tieu(1992). The three cases investigated correspond to the laminar-transition-turbulent regime. Figure 7 shows the comparison of the experimental velocity profiles in the laminar regime (Re=200, 300, 400) with the analytical results obtained using the procedure described above. Excellent matchings at the three locations measured have been attained. This confirms the accuracy of the numerical process. Figure 8 shows results for Re=500, 750, 1000. In Figure 8a measurements were taken at the inlet section. At this position, excellent agreements have been shown, whereas at the other two locations the theoretical profile are shifted slightly to the left of the experimental curve. Data for Re=800, 1200, 1600 are shown in Figure 9. Excellent agreement is obtained for x/l=0, while at the other two locations the theoretical profiles are slightly shifted to the left again. The discrepancies in Figures 8b and 8c, and 9b and 9c could be due to difference between the inputted Bx in the analysis and the actual Bx which was not given in Innes and Leutheusser's paper.

<u>Velocity profiles in thin lubrication film (a journal bearing case)</u>

For the last case data was obtained from experiments performed by the authors on a journal bearing. The schematic diagram of the experimental facility is shown in Figure 10. The fluid velocity measurement was achieved by using a Laser Doppler Anemometer system (Mackenzie et al.,1992).

Figure 11 show the set of experimental circumferential velocity profiles obtained in the journal bearing. To our knowledge, this is the first time experimental velocity distributions for superlaminar flow in hydrodynamic bearing gap of 0.49mm have been obtained by an LDA system. Observations of these figures show excellent agreem[illegible] in the [illegible] outlined in the section of theoretical analysis.

CONCLUSION

- A model for Reynolds stress in superlaminar lubrication theory using a modified Van Driest mixing length formula has been presented. It is shown that the Van Driest factor (λ) needs to be modified when the flow is in the transition regime.

- Presented in this paper are velocity characteristics of lubricant film in transition-turbulent regime obtained both experimentally and theoretically.

- The velocity distributions in lubricating thin films have been successfully measured with a Laser Doppler Anemometer and they correlate well with the proposed mixing length theory for superlaminar theory.

- Analysis of the theory and their excellent correlation with the experimental work confirmed the suitability of the mixing length formula for transition-turbulent analysis [illegible] [illegible]laminar regime.

ACKNOWLEDGMENT

This project has been funded by an ARC grant.

REFERENCES

CONSTANTINESCU, V.N.(1959)," On The Turbulent Lubrication ", <u>Proc. Instn. Mech. Engrs.</u>, 881-900.

ELROD, H.G., NG, C.W., (1967)," A Theory for Turbulent Fluid Films and Its Application to Bearings ", <u>ASME Journal of Lubrication Technology,</u> 89 , 381-191.

GALETUSE, S., (1974)," Experimental Study on The Interference of Inertia and Friction Forces in Turbulent Lubrication ", <u>ASME Journal of Lubrication Technology,</u>96, 164-167.

GRANVILLE, P. S., (1989)," A Modified Van Driest Formula for the Mixing Length of Turbulent Boundary Layers in Pressure Gradients", <u>ASME Journal of Fluids Engineering,</u> 111, 94-97.

HARADA, M., AOKI, H., (1988)," Analysis of Thrust Bearings Operating in Turbulent Regime ", <u>ASME Journal of Tribology,</u> 110, 555-562.

HIRS, G.G., (1973)," A Bulk Flow Theory for Turbulence in Lubricant Films ", <u>ASME Journal of Lubrication Technology,</u> 95, 137-146.

HUFFMAN. G.D., BRADSHAW, P., (1972)," A Note on Von Karman's Constant in Low Reynolds number turbulent flows ", <u>Journal of Fluid Mechanics,</u> 53, 45-60.

INNES, G.E., LEUTHEUSSER, H.J., (1991)," An Investigation Into Laminar-to-Turbulent Transition in Tilting-Pad Bearings",<u>ASME Journal of Tribology,</u> 113, 303-307.

KOSASIH, P.B., TIEU, A.K., (1992)," A Transition-Turbulent Lubrication Theory Using Mixing Length Concept ", submitted to the ASME-STLE Tribology Conference 1992.

MACKENZIE, M.R, TIEU, A.K., KOSASIH, P.B., BINH, L.N., (1992)," An Application of a Visible Wavelength Solid-State LDA in Thin Channel flow", accepted for publication in Measurement Science and Technology.

[illegible] C.W., PAN, C.H.T., (1965)," A Line[illegible] Turbu[illegible]nt Lubrication Theory ", <u>ASME Journal of L[illegible] [illegible]logy,</u> 87 675-688

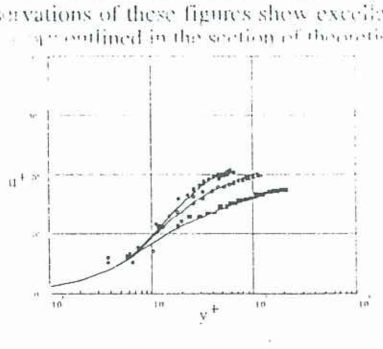

Figure 1. Law of Wall in 5mm channel
• $\partial\tau^+/\partial y^+$ = 0.01,Re=1990, λ=67
○ $\partial\tau^+/\partial y^+$ = 0.00899,Re=4756, λ=45
■ $\partial\tau^+/\partial y^+$ = 0.00559,Re=6324,

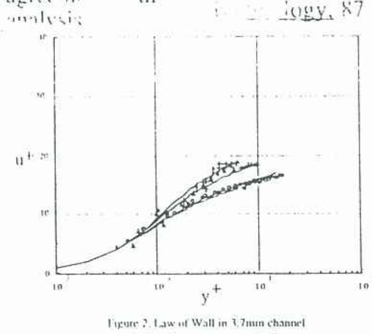

Figure 2. Law of Wall in 3.7mm channel
+ $\partial\tau^+/\partial y^+$ =0.011,Re=1997, λ=45
▲ $\partial\tau^+/\partial y^+$ = 0.010,Re=3219, λ=35
○ $\partial\tau^+/\partial y^+$ = 0.0089,Re=4000, λ=27
• $\partial\tau^+/\partial y^+$ =0.0064,Re=5749, λ=26

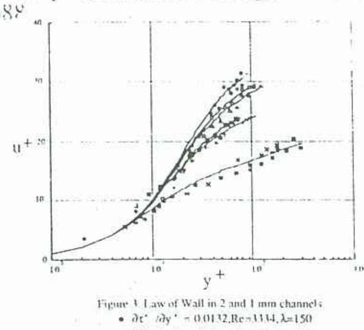

Figure 3. Law of Wall in 2 and 1 mm channels
• $\partial\tau^+/\partial y^+$ = 0.0132,Re=3334, λ=150
○ $\partial\tau^+/\partial y^+$ =0.0122,Re=4445, λ=120
▲ $\partial\tau^+/\partial y^+$ =0.0119,Re=3330, λ=110
■ $\partial\tau^+/\partial y^+$ =0.0113,Re=50[illegible], λ=80
+ $\partial\tau^+/\partial y^+$ = 0.0105,Re=5946, λ=72
× $\partial\tau^+/\partial y^+$ = 0.0084,Re=6641, λ=26
[illegible] $\partial\tau^+/\partial y^+$ =0.0065,Re=7830, λ=2[illegible]

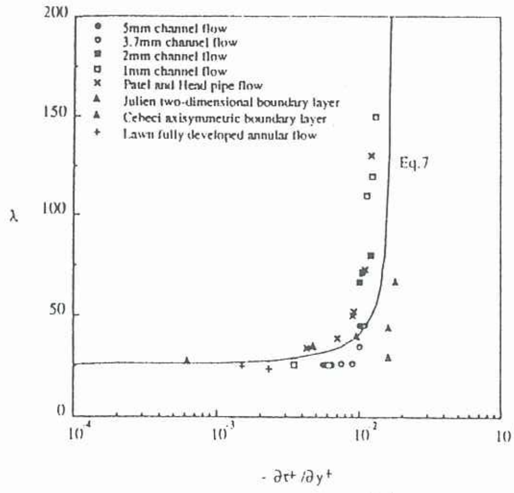

Figure 4. Variation of λ with $\frac{\partial\tau^+}{\partial y^+}$

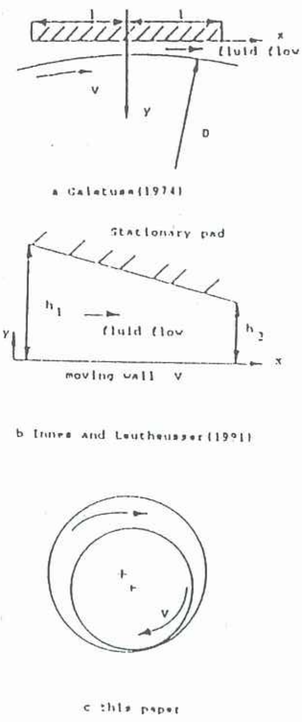

Figure 5. Configurations of film shapes investigated

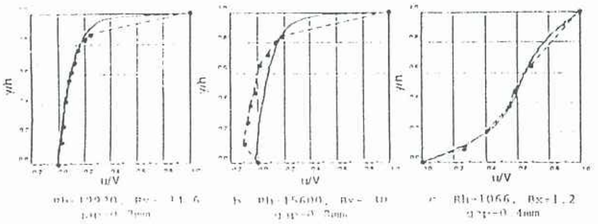

Figure 6. Velocity profiles between a rotating cylinder and a flat plate. — present theory, -•- Caletuse [illegible]

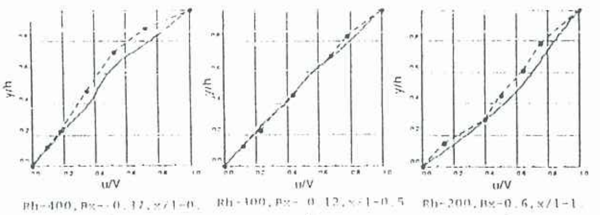

Figure 7. Velocity profiles in slider bearing with mean Rh=300. — present theory, -•- Innes and Leutheusser exp.

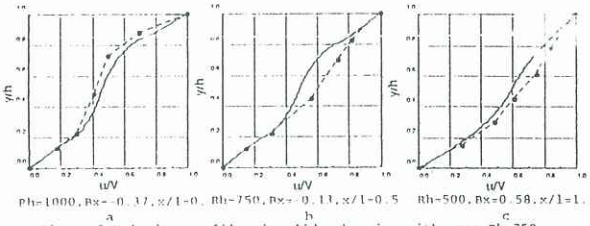

Figure 8. Velocity profiles in slider bearing with mean Rh=750
— present theory, -•- Innes and Leutheusser exp

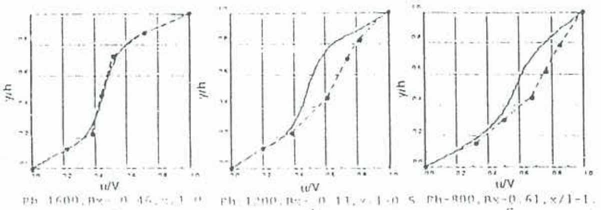

Figure 9. Velocity profiles in slider bearing with mean Rh=1200. — present theory, -•- Innes and Leutheusser exp.

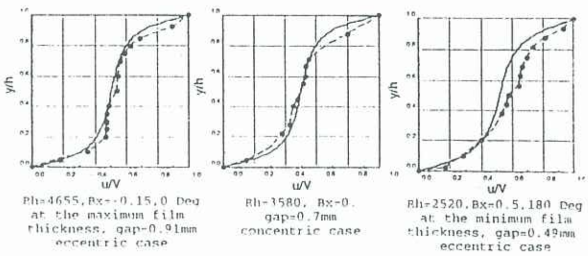

Figure 11 Velocity profiles in journal bearing,
— present theory,
-•- present experiment

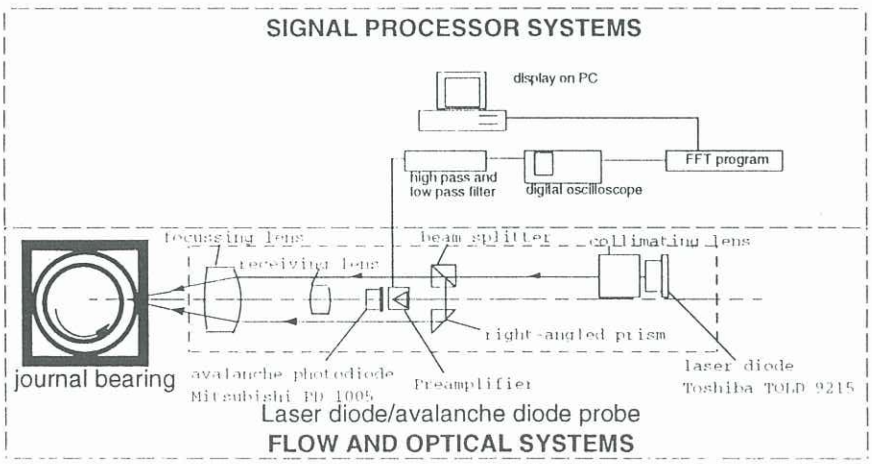

Figure 10. Schematic diagram of the experimental facility (The probe size is exaggerated)