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REGULAR REFLECTION OF A SHOCK WAVE OVER A POROUS LAYER : PROPOSITION OF A TRIVIAL SOLUTION HYPOTHESIS

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ABSTRACT

The regular reflection of a shock wave over porous layer was investigated both theoretically and experimentally. Two types of model were considered. One may be called a simple sink theory, in which a deflection angle is the only parameter taking the sink effect of the porous layer into consideration and the detail of the flow in the porous layer is not taken into account. The other model is a more realistic one, in which the pressure wave and the gas flow in the porous phase are considered as well as in the gas phase. In the former, by assuming the deflection angle, the results were compared with the experiment. In the latter, however, since the coupled problem for both phases was too complicated to analyze theoretically, we analyzed two limiting cases and showed on a physical plane the domain where the solution of the coupled problem was possible.

NOTATION

I	incident shock
M	incident shock Mach number: $M = M_1 \sin\gamma_i$
M_j	Mach number of the flow in region (j) in the reference frame attached to the reflection point (for regular reflection) or the triple point (for Mach reflection)
MS	shock propagating in the porous layer
p_j	pressure in region (j)
P	reflection point
R	reflected wave
S	slipstream
γ_i	angle of incidence in the gas phase
γ_m	angle of incidence in the porous layer
γ_r	angle of reflection
δ	parameter which takes the sink effect into consideration
ϕ	porosity of porous medium
θ_i	deflection angle across the incident shock
θ_m	deflection angle across the Mach stem
θ_r	deflection angle across the reflected shock
θ_w	reflecting wedge angle

INTRODUCTION

When a plane shock wave is incident on a slope, a reflected wave develops. The overall wave configuration is roughly classified into two categories (von Neumann, 1943), i.e., regular reflection and Mach reflection. In regular reflection, ends of both incident and reflected waves coincide on the surface of slope. However, in Mach reflection, an intersection of the incident and reflected waves (which is called a *triple point*) is above the slope surface, and a third shock wave connecting the triple point and the surface appears. This shock characteristic of Mach reflection is called a *Mach stem*. The well-known von Neumann's theory is a simple application of shock relation to such wave configuration by approximating curved shocks with plane shocks. For regular reflection, his theory is often termed as *two-shock theory* because two shocks (incident and reflected shocks) are involved (see, for example, Ikui and Matsuo, 1983; Ben-Dor, 1992). Due to similar reason, his theory for Mach reflection is called as *three-shock theory*, where the Mach stem is also involved. If the slope is smooth and flat, the wave configuration is determined by the incident shock Mach number and the inclination angle of slope, as well as a thermodynamic property of fluid (say, the ratio of specific heats). In ordinary (smooth plane surface) situations, the behavior of the triple point can be regarded pseudosteady, and not only an overall wave geometry but also a flow field is self-similar. Such a problem as an oblique shock reflection over smooth plane surface has long been studied by many investigators (Hornung, 1986; Ben-Dor, 1992), and generally well understood in both theory and experiment.

On the other hand, shock reflection problem over surfaces other than smooth one has attracted, considering its practical importance, less attention so far. When the incident shock is reflected on the inclined porous layer, the velocity component perpendicular to the solid boundary is possible and as a result the boundary acts as a sink (i.e., *sink effect*). Since von Neumann's theory considers uniform flow fields bounded by plane discontinuities such as shocks and slipstream, the effect of the solid boundary on which the incident shock is reflected is not considered. Therefore it is not surprising that the theory and the experimental result are not in good agreement when the effect of boundary is prominent. In the early stage of investigation, the authors had reported (Adachi et al., 1990; Suzuki et al., 1991; Adachi et al., 1992) that the structure of a group of waves reflected from a two-dimensional rough surface ("multi-guttered wedge") and pointed out the importance of a role played by a compression wave formed by accumulation of reflected waves. Moreover, comparison of reflected wave structures for multi-guttered wedge and porous layer had revealed that they were quite similar in spite of the irregularity of particle disposition (Adachi et al., 1991). In the present paper, experiment has been performed with deep porous layer in order to neglect the influence of waves reflected from the bed. Results are compared with numerical calculation for two limiting cases.

EXPERIMENT

The experimental setup is the same as that shown in our previous paper (Adachi et al, 1992) and its schematic diagram is omitted here. The principal part of the shock tube used was composed of a 79-mm-diameter driver section 1200 mm long and a 65 x 30 mm rectangular driven section 3900 mm long. At the end of the driven section was a test section with a pair of 62 x 94 mm optical viewing windows on either side. A dump tank, 155 mm in inner diameter and 450 mm in length, was connected to the test section to moderate shock pressure.

The incident shock Mach number M was calculated by measuring the time interval of passage of an incident shock between two pressure gauges ahead of the test section. Waves were visualized through shadowgraphy using a xenon flash lamp with a pulse width of 180 nsec. The flash lamp was triggered by the pressure gauge nearer to the test section through a delay circuit. Through regulation of the delay circuit, the wave structure at any desired instant was either recorded with a TV camera through a monochromatic image analyzer (SVA-1, Sugawara Laboratories, Inc.) or photographed.

Two kinds of porous layer were used. One was a dusty layer composed of spherical glass particles whose mean diameter is 60 μm and its porosity ϕ was 44%. The other was made of foam rubber whose porosity was 98%.

The working gas was air (κ =1.4). The experimental parameters were the reflecting wedge angle θ_w and the incident shock Mach number M, i.e., θ_w ranges from 30 to 50 at intervals of 5 deg. for dusty layer; from 25 to 47.5 at intervals of 2.5 deg. for foam rubber, M = 1.20 and 1.41. The initial conditions of the driven section were room temperature and atmospheric pressure.

ANALYSIS

Models for Analysis

Figure 1 shows the flow field relative to the reflection point **P** in the case of regular reflection over solid smooth boundary. The sign of the deflection angle of the flow across a discontinuity is consistently defined; i.e., counterclockwise deflection is positive, clockwise negative. The incoming flow parallel to the porous layer is deflected clockwise by θ_i across the incident shock **I** and it is again deflected by θ_r across the reflected shock **R**. The boundary condition in region (3) requires $\theta_i + \theta_r = 0$. On the other hand, in the case of regular reflection over porous layer, the velocity component perpendicular to its surface is possible because of the permeability of the porous layer. As a result, the boundary condition is modified as $\theta_i + \theta_r = \delta$, where δ is a deflection angle, induced by the sink effect, of the flow behind the reflected wave (see Fig. 1 (b)). The value of δ can be a function of the incident shock Mach number M and the angle of incidence γ_i as well as the porosity ϕ of the porous medium. It is easily inferred that the sink effect is marked for regular reflection rather than Mach reflection whose triple point is above the surface of porous layer. We may refer to this model as *simple sink model*, since only the flow into the porous layer is considered while the propagation of shock wave is disregarded. In reality, however, the air-borne porous layer as well as the upstream gas phase (1) goes on in the reference frame fixed to the reflection point, and the incident shock propagates into the porous layer. Such flow field illustrated in Fig. 1 (c) may be called as *realistic model*. Realistic as it is, it is not easy to solve the whole flow field since we must deal with the coupled problem between the pure gas phase and the porous phase.

Numerical Results Based on the Simple Sink Model

By assuming the value of δ in the two-shock theory (Fig. 1 (b)), we can easily obtain the relation between angles of incidence and reflection. The results are shown by dotted curves in Fig. 2 for M = 1.20 and 1.41 when the fluid is air. The dashed curve represents the two-shock theory, i.e., $\delta = 0$, while the solid curve stands for the three-shock theory.

Two Limiting Models

In order to analyze the phenomenon, two limiting modes of regular reflection are introduced. One is an ordinary regular reflection over a solid surface whose porosity is zero (see Fig. 1 (a)). The other is a regular reflection over a porous medium whose porosity is almost unity. The latter is illustrated in Fig. 1 (d) and requires explanation. As the porosity is almost unity (i.e., the layer is substantially an imaginary one), the incident shock propagates into the "porous" layer under least interference and so the angle of incidence in the porous layer γ_m is identical with that in the gas phase γ_i. The reflected wave is acoustic and issues from the interface. Therefore, the flow in region (2) is not deflected by the reflected wave. The flow in porous layer is deflected by θ_m, which is also identical with θ_i, across the incident shock (which is a degenerated Mach stem in the three-shock theory) and the boundary condition on the slipstream **S** is satisfied. This is the trivial case for three-shock theory and the corresponding solution may be termed as *trivial solution*. It is inferred that the real situation lies between these both limiting cases according as its porosity (trivial solution hypothesis).

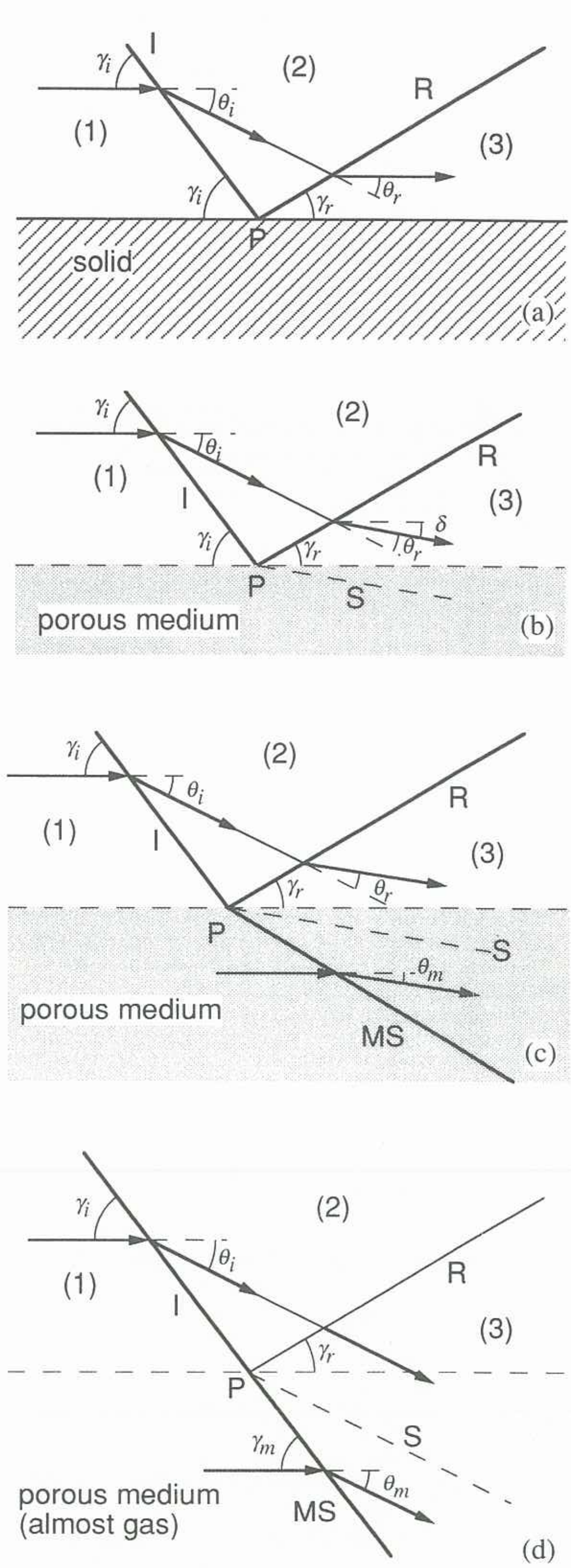

Fig. 1. Flow field relative to the reflection point, (a) ordinary regular reflection over solid surface, (b) regular reflection over porous layer; simple sink model, (c) realistic model, (d) limiting model when the porosity is almost unity and the reflected wave is acoustically weak.

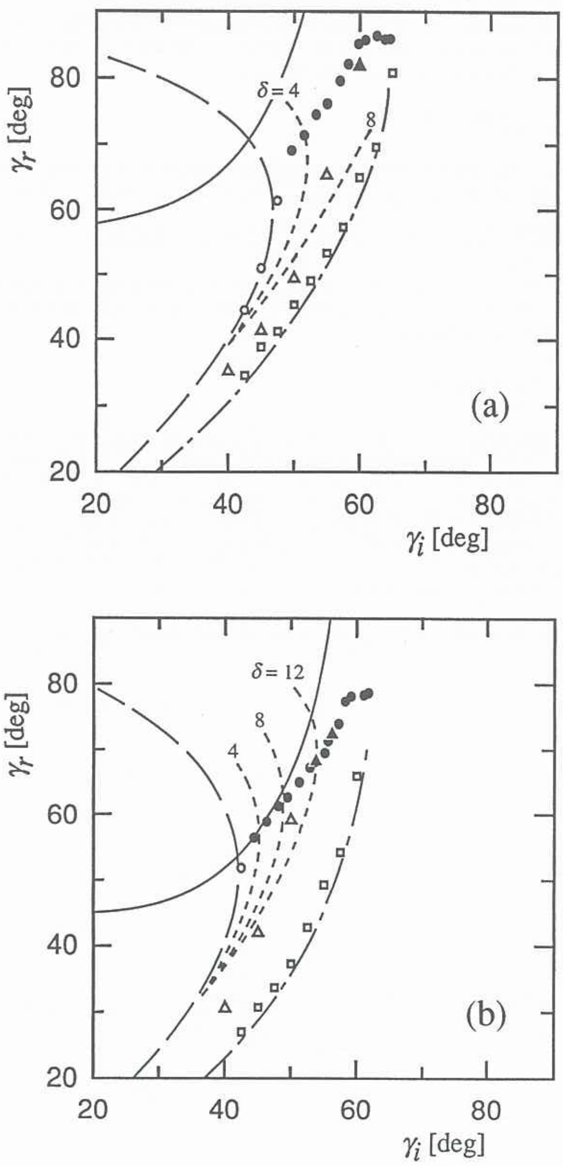

Fig. 2. Relation between angles of incidence and reflection, (a) $M = 1.20$, (b) $M = 1.41$. Experiment: smooth plane surface ○ ●, dusty layer △ ▲, foam rubber □ (open symbols denote regular reflection, solid ones Mach reflection). Theory: ——— ——— the two-shock theory, ———— the three-shock theory, — — — — simple sink model, ——— - - ——— trivial solution

Trivial Solution of Three-Shock Theory

The trivial solution for Fig. 1 (d) is easily obtained by setting $\theta_r = 0$, $\gamma_m = \gamma_i$, $\theta_m = \theta_i$, $p_2 = p_3 = p_4$ in the three-shock theory:

$$\tan\theta_i = \frac{2\cot\gamma_i\,(M_1^2\sin^2\gamma_i - 1)}{M_1^2(\kappa + \cos 2\gamma_i) + 2}, \quad (1)$$

$$M_2^2 = \frac{(\kappa-1)M_1^2\sin^2\gamma_i + 2}{\sin^2(\gamma_i - \theta_i)\{2\kappa M_1^2\sin^2\gamma_i - (\kappa-1)\}}, \quad (2)$$

$$\frac{2\kappa M_2^2\sin^2(\gamma_r + \theta_i) - (\kappa-1)}{\kappa+1} = 1. \quad (3)$$

The last equation reduces to

$$\gamma_r = \sin^{-1}\left(\frac{1}{M_2}\right) - \theta_i\,. \quad (4)$$

When the Mach number M_1 of the flow in region (1) and the ratio of specific heats κ are specified, the deflection angle θ_i across the incident shock and thus the Mach number M_2 of the flow in region (2) are functions of γ_i, so Eq. (4) gives the analytical relation between angles of incidence and reflection. In Fig. 2, the dash-and-dotted curve represents the trivial solution thus obtained.

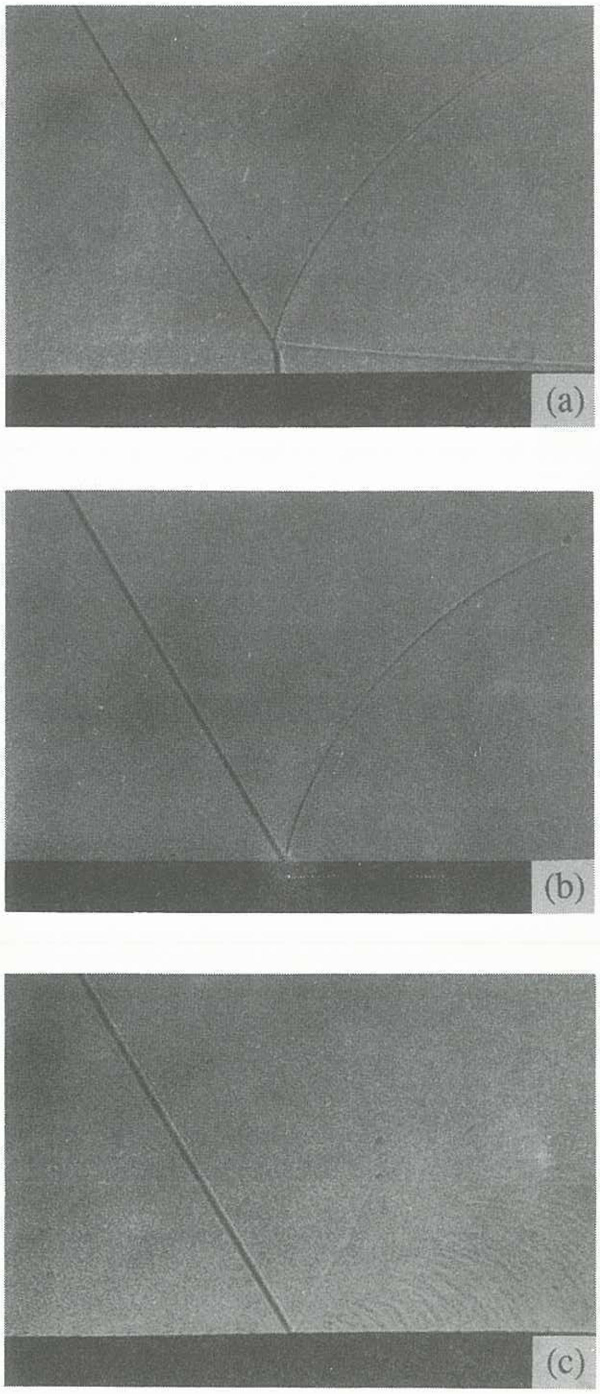

Fig. 3. Shadowgraphs of oblique shock reflection over various wedges, (a) smooth plane surface, (b) dusty layer, (c) foam rubber for $M = 1.20$ and $\theta_w = 30$ deg.

RESULTS AND DISCUSSION

The experimental data are shown in Fig. 2 together with theoretical and numerical results. Circles, triangles and squares correspond to the results for smooth plane surface, dusty layer and foam rubber, respectively. Open symbols stand for regular reflection, while solid symbols Mach reflection. Note that for regular reflection the relation $\gamma_i = 90-\theta_w$ holds. Concerning porous layer data, the distance from the wedge tip to the root of the incident shock (or the Mach stem) is around 30 mm. For M = 1.20 (Fig. 2 (a)), the results of dusty layer (porosity $\phi = 0.44$) do not agree with the two-shock theory. This is more evident for foam rubber ($\phi = 0.98$).

Although the discrepancy between theory and experiment is explained to some extent by considering the sink effect, it still remains especially for M = 1.41 since however the value of δ is large there exists a region where the dotted curves cannot cover.

It is remarkable that the foam rubber results are in good agreement with the trivial solution. In either case, the experimental results lie in the limited region between the solid (two-shock theory) and the dash-and-dotted (trivial solution) curves as mentioned in the previous section. It should be noted that the curve of the trivial solution comes to an end where $M_2 = 1$. This is because the derivative of Eq. (4):

$$\frac{d\gamma_r}{d\gamma_i} = \frac{1}{2\sqrt{M_2^2-1}}\frac{dM_2^2}{d\gamma_i} - \frac{d\theta_i}{d\gamma_i}, \qquad (5)$$

increases indefinitely as M_2 approaches unity.

It is interesting that the trivial solution hypothesis on the basis of the three-shock theory explains the phenomenon better than the simple sink model which is in itself the two-shock theory. Physically this means that the effect of porous material is not restricted to the sink effect but it is essential to take the porous phase into consideration. In this respect, the present result is in accordance with Clarke (1984) who insisted the effect other than the air flow within porosities (i.e., sink effect) should be considered.

Figures 3 (a) to (c) show shadowgraphs of oblique shock reflection over three different wedges, i.e., plane smooth wedge, dusty layer and foam rubber, for M = 1.2 and θ_w = 30 deg. The Mach reflection takes place for plane smooth wedge (Fig. 3 (a)). Although the Mach stem is quite short (Fig. 3 (b)), it still persists in the case of dusty layer. For larger porosity, the reflection type is regular and it is characteristic that the reflected wave is quite weak (Fig. 3 (c)). This fact supports the trivial solution hypothesis in which the reflected wave was assumed acoustic.

CONCLUSION

The regular reflection of a shock wave over porous layer was investigated both theoretically and experimentally. The following conclusions are made:

(1) The simple sink theory explains the experiment to some extent.
(2) The trivial solution hypothesis proposed here is quite effective. It is essential to take the flow in porous layer into consideration.

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