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RESPONSE OF A TRANSITIONAL BOUNDARY LAYER TO WALL VIBRATIONS

S.Y. KIM, X. BONNARDEL, J.P. GUIBERGIA and E. BROCHER

Institut de Mecanique des Fluides
1 Rue Honnorat, 13003 Marseille, FRANCE

ABSTRACT

The effects of wall vibration on the development of Tollmien-Schlichting waves in a transitional flat plate boundary layer are considered.Theoretically, the dynamic interaction between T-S waves and velocity perturbations induced by the vibrating wall is demonstrated through the kinetic energy balance of velocity perturbation.For a compliant wall,the interaction of T-S waves with velocity perturbations induced by the vibrating wall has been studied for various phase angles between the T-S wave velocity and the wall velocity,and for various amplitudes of the wall vibration. Experimentally,T-S waves are induced by a vibrating wire located in the upstream portion of the wall .The downstream portion is vibrated at the same frequency as that of the wire and the phase between the two oscillations can be varied.Depending on the phase angle ,the T-S waves may be either amplified or attenuated,even if the wave lengths do not coincide.

1.INTRODUCTION

Drag reduction is an important area of research in fluid mechanics because it has many practical applications in aerial,terrestrial and underwater transportation.This reduction may be achieved by boundary layer control.In the past,blowing and suction were the popular techniques to influence the boundary layer behaviour and consequently the drag.More recently,riblets and lebus have been used to reduce the wall shear stress of a turbulent boundary layer.Another recent technique consists in developing means of delaying the transition from the laminar to the turbulent state of a boundary layer.

The present paper describes one these means which consists in vibrating the wall in a way so as to influence the development of instability waves (Tollmien-Schlichting waves, in short ,T-S waves)which are at the origin of the laminar-turbulent transition.

2.THEORY

2.1 General

For an incompressible fluid ,the Navier-Stokes equations read (in tensor notation)

Continuity: $\partial u_i/\partial x_i=0$ (1)

Motion: $$Du_i/Dt=-\frac{1\partial P}{\rho\partial x_i}+\upsilon\frac{\partial^2}{\partial x_j\partial x_j}u_i \quad (2)$$

where $D/Dt=\partial/\partial t+u_j\partial/\partial x_j$ represents the convective derivative.

For small perturbations of an otherwise steady, two-dimensional boundary layer, we write the physical quantities as

$$u=\overline{U}+u',v=\overline{V}+v',p=\overline{P}+p' ; \quad (3)$$

For a flat plate boundary layer, we make the parallel flow approximation, that is $\overline{U}=\overline{U}(y),\overline{V}=0$.With the assumptions above, neglecting the quadratic terms of small quantities ,the equations of motion read

$$\partial u'/\partial t+\overline{U}\partial u'/\partial x+v'\partial\overline{U}/\partial y+\partial p'/\partial x=1/Re_{\delta 1}\Delta u' \quad (4)$$

$$\partial v'/\partial t+\overline{U}\partial v'/\partial x+\partial p'/\partial y=1/Re_{\delta 1}\Delta v' \quad (5)$$

$$\partial u'/\partial x+\partial v'/\partial y=0 \quad (6)$$

where $Re_{\delta 1}$ is the Reynolds number based on the displacement thickness δ_1 of the boundary layer.For solving the stability problem with a vibrating wall, it is appropriate to use the velocity rotational of velocity perturbations $\xi=\partial u'/\partial y-\partial v'/\partial x$.

By double cross derivation of Eq (4) and (5),we obtain an equation for ξ:

$$\partial\xi/\partial t+=\overline{U}\partial\xi/\partial x-\partial\psi/\partial x.\partial^2\overline{U}/\partial y^2=(1/Re_{\delta 1})\Delta\xi \qquad (7)$$

where Ψ is the stream function such that $u'=\partial\psi/\partial y$ and $v'=-\partial\psi/\partial x$,$\xi=\partial^2\psi/\partial x^2+\partial^2\psi/\partial y^2$ Use of ξ and Ψ has the following advantages

-it reduces the number of equations

-it eliminates the pressure terms

-it consequently enables an easier numerical computation

-it enables to introduce periodic solutions through the stream function

We now derive the equation of the kinetic energy of the perturbation which gives a clear picture of the physical processes leading to the laminar-turbulent transition.

Multiplying Eq.(4) by u',Eq.(5) by v',adding the results,and rearranging the terms,we arrive to the following equation

$$\underset{<1>}{\partial q/\partial t}+\underset{<2>}{\overline{U}\partial q/\partial x}=\underset{<3>}{-u'v'\partial\overline{U}/\partial y}\underset{<4>}{-\partial u'p'/\partial x-\partial v'p'/\partial y}$$

$$+\frac{1}{Re_{\delta 1}}\left[\underset{<5>}{\Delta q}-\underset{<6>}{\left\{(\partial u'/\partial y)^2-2\partial u'/\partial x\,\partial v'/\partial y+(\partial v'/\partial x)^2\right\}}\right] \qquad (8)$$

where q is the kinetic energy of the perturbation velocity.The terms appearing in Eq.(8) may be interpreted as follows:

<1>:time variation of q

<2>:convective variation of q

<3>:rate of production of q

<4>:energy diffusion due to pressure fluctuations

<5>:energy diffusion due to viscosity

<6>:rate of viscous dissipation

To study the stability of the perturbations,we write the streamfunction as

$$\psi[x,y,t]=[\phi(y)]e^{i(\alpha x-\omega t)} \qquad (9)$$

where ϕ,α and ω are, in general,complex quantities designing amplitude ,wave number and frequency of the perturbation.

The ratio ω/α may be written as $\omega/\alpha=c=c_r+ic_i$ where c_r represent the phase velocity and c_i the amplification coefficient.The velocity perturbations then read

$$u'=[\phi(y)]e^{i\alpha(x-ct)}\ ,v'=-i\alpha[\phi(y)]e^{i\alpha(x-ct)}$$

2.2Interaction fluid-structure

The purpose of the present paper is to characterize the field of velocity perturbations resulting from the interaction between T-S waves and a flexible wall artificially excited.When the hydrodynamic coincidence between the 2 waves is achieved ,T-S waves are either damped or amplified depending on the phasing imposed between the 2 perturbations.The equation for the rotational (Eq. 7) has been numerically solved by a scheme of finite,centered differences leading to a system of discretised equations.It has first been verified that for a rigid plate ,the stability curve,the amplitude distribution of u' and v' across the boudary layer,obtained from the rotational equation are identical to that obtained from the Orr-Sommerfeld equation by Jordinson (1970),Carpenter and Garrad (1985),and Stuhmiller (1978).Then ,the case of the vibrating wall has been computed.The wall displacement may be written as

$$\eta_w=ae^{i\alpha(x-ct)}$$

where a is a constant.With the no-slip condition at the wall (u'w =0),the following boundary conditions are found

$$\psi_w=Ae^{i\alpha(x-ct)}\ ,\xi_w=\left[\partial^2\psi/\partial y^2\right]-\alpha^2Ae^{i\alpha(x-ct)}$$

where A=ca is determined by the maximum amplitude of v' of the T-S waves and $\left[\partial^2\psi/\partial y^2\right]_w$ by development in Taylor series of ψ near the wall .When the wave number and the frequency of the sinusoidal perturbations are the same,the influence of the wall vibration on the T-S waves is governed by the phasing between them.We therefore end up to the following boundary condition for ξ_w:

$$\xi_w=\frac{2}{\Delta y^2}(\psi_{p+1}-\psi_p)-\alpha^2Ae^{i\alpha(x-ct+\theta)}$$

where $\psi_p=Ae^{i\alpha(x-ct+\theta)}$. Δy the mesh size,ψ_{p+1} the value of Ψ at the mesh point nearest to the wall and θ the phase.

The various terms appearing in the kinetic energy balance of the T-S waves (Eq. 8) have been computed with the values obtained from the rotational equation (Eq.7) .The results obtained for a rigid plate in conditions for which the T-S waves are unstable ($Re\delta_1=1000$,$\alpha\delta_1=0,225$) are shown in Fig 1.

It is observed that the time variation of q(<1>in Eq.8) is positive,because the rate of production of q (<3>) is positive and is largely superior in magnitude to the only damping mechanism which in case of a rigid plate, is the energy diffusion due to viscosity (<5>) .It is also observed that energy diffusion due to viscosity (<5>) is slightly positive.These results are in qualitative agreement with those obtained by Domaradzki and Metcalfe(1987).To study the

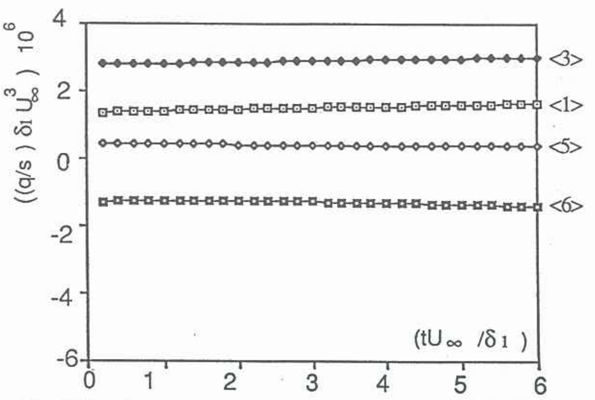

Fig.1 Kinetic energy balance of T-S waves (rigid plate)

response of T-S waves to a vibrating wall ,we have imposed velocity perturbations induced by the wall during a non-dimensional time tU_∞/δ_1 going from 1 to 5 .The amplitude of the velocity perturbation is made non-dimensional with respect to the maximum value of v' of the T-S waves at the instant $tU_\infty/\delta 1$ =0.Several cases have been computed for 3 values of the non-dimensional amplitude (4%,8%,14%) and 2 phasing θ (0°and 180°) .Generally speaking, energy diffusion due to viscosity (<5>) and rate of vicous dissipation (<6>) are not substantially modified by the wall vibration .However,the effect becomes important on time variation of q (<1>) and on rate of production(<3>) of q.

As an illustration,we are showing on Fig.2 the results obtained for a relative amplitude of 14%.On Fig.2a ,the phasing θ is 180° and on

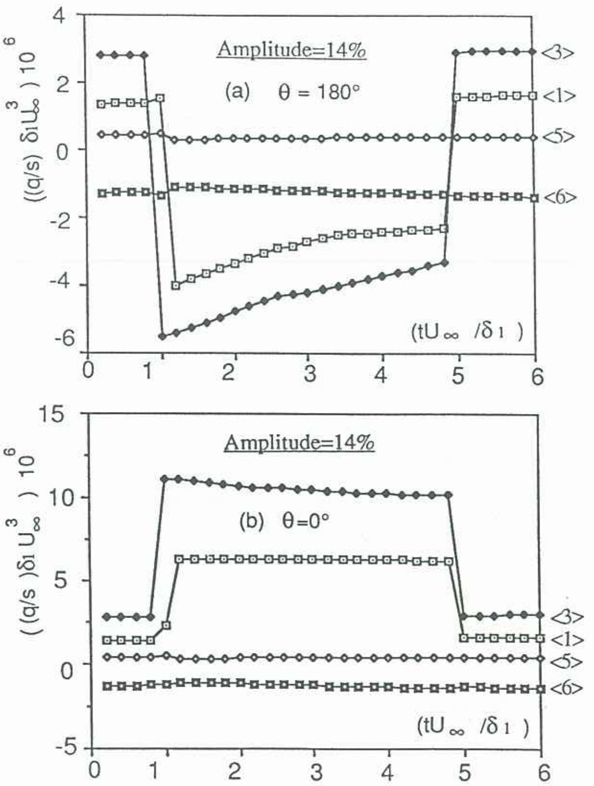

Fig.2 Kinetic energy balance of T-S waves over a vibrating plate

Fig.2b it is 0° .By comparison with the case of the rigid plate (Fig.1) ,it is seen that the influence of the wall vibration is very important .With θ=180 ° , the time variation of q (<1>) is negative because the rate of production of q (<3>) dominates and is largely negative .With θ=0° the opposite is true ,that is ,<1> is positive because <3> is largely positive.To summarize, phasing of 180° damps the T-S waves whereas 0° amplifies them.

3.EXPERIMENTS

3.1Experimental set -up

The experimental setup is shown in Fig.3 .A flat plate is mounted in the test section of a windtunnel.Upstream of the plate leading edge,suction of the boundary layer is applied to ensure laminarity of the flow in the upstream part of the plate.This part is rigid and a first device vibrates a wire (0,5 mm in diameter) above its surface at a frequency ranging from 150-200 Hz and at a distance approximately equal to a third of the boundary layer thickness which corresponds to the location of the maximum amplitude of the T-S waves. Another device vibrates the flexible wall which is located downstream of the rigid part.

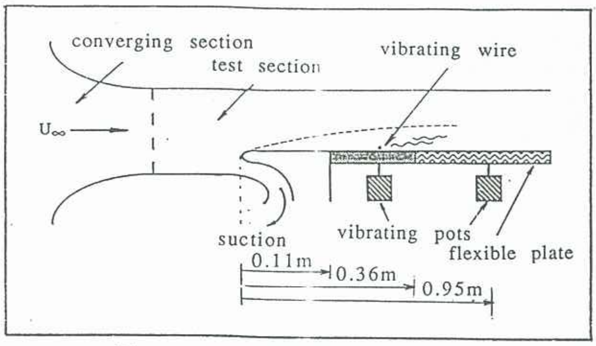

Fig.3 Experimental set-up

3.2 Coincidence condition of flexion modes of the flexible wall with the T-S waves

In order to realise a linear coupling of T-S waves with the flexion waves in the flexible wall ,both frequency and wave lengh of the 2 oscillations should be the same.It has been found that no existing material is able to fulfil both conditions at the same time .The choice of a 2 mm thick plexiglass ,which appears to be the best compromise ,has been made.

3.3 Measurements

The velocity fluctuations were measured with hot wire anemometry (DISA 55 P 11) and the flexible plate vibration with an accelerometer (B and K 4374).The data analysis was made by decomposing the signal in: $u(t)=\bar{u}+u'_p+u'$,where $\bar{u}$,u'_p and u' are respectively the time average, the periodic part and the random part .The frequency spectrum of the velocity has been determined with a Data Precision DATA 6100.

3.3 Time mean flow

Measurements of the boundary layer along the plate have been made for Reynolds numbers ranging from $Re_x = 1.1\ 10^5$ to $1.2\ 10^6$, where x represents the distance from the plate leading edge.At the 1st measuring station (x=0.1 m) ,the profile corresponds to a Blasius profile (boundary layer shape parameter H=2.58).As x increase ,H decreases down to a value of 1.48 at the last measuring station .It has been found that the mean velocity profile is not influenced by the wall vibration.

3.4 Fluid-structure interaction

Globally ,the response of the T-S waves to wall vibration may be characterized by the change in the R.M.S of the velocity fluctuations as a function of the phase difference between the 2 vibrating pots.When the wall is not vibrated at x=0.45 m and $y/\delta = 0.3$ the R.M.S. value is about 1,5% of U_∞ .When the wall is vibrated at 150 Hz , the R.M.S. value depends very much on the phase difference as can be seen on Fig.4 . At $y/\delta = 0.3$ and with 45 °,the non-dimensional value of the R.M.S. has decreased to 1.4% whereas with 225 ° it has increased to about 1.95%.

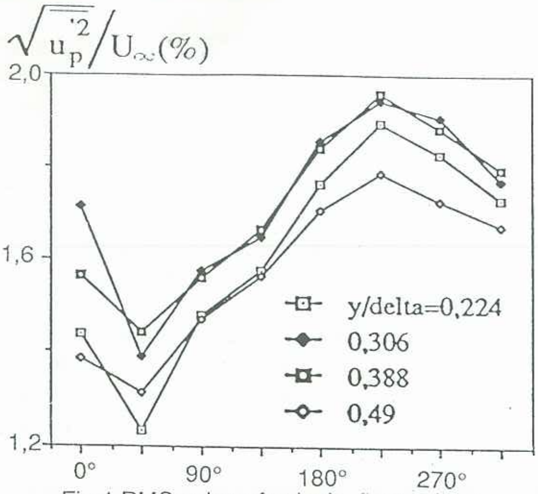

Fig.4 RMS value of velocity fluctuations as a function of the phase between vibrating devices(x=0,45 m ;150 Hz)

A more fundamental way of evaluating the influence of wall vibration on the T-S waves is to look at the change in amplitude of the velocity fluctuations at the excitation frequency.As an exemple,at the same location as above (x=0.45 m and $y/\delta=0,3$) the influence of the phase is very clearly seen on the frequency spectrum .When the plate is not vibrated the R.M.S. value of u' at the frequency of the vibrating wire is about 0,2 m/s.For a phase of 45% ,this value decreases to 0.15 m/s (Fig.5 b)owing to the wall vibration whereas it increases to 0.25 m/s for a phase of 225° (Fig.5 a).

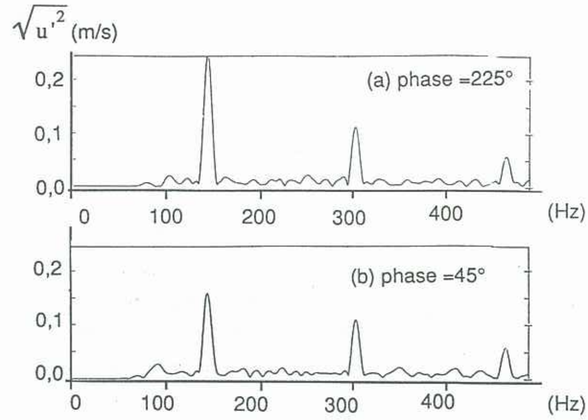

Fig.5 Frequency spectrum of velocity fluctuations

4.CONCLUSIONS

The response of the T-S waves to wall vibrations has been studied both theoretically and experimentally.A numerical solution of the equation of the velocity rotational has been favorably compared with results previously obtained for a rigid plate by authors on the basis of the Orr-Sommerfeld equation .The numerical solution clearly shows the effect of the phase difference between the velocity fluctuation and the wall vibration. On the experimental side ,although it was not possible to have both coincidence of frequency and wave length of T-S waves and wall flexion,the influence of the phase has also been clearly demonstrated .Depending on the phase ,the T-S waves are either damped or amplified.

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