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THE EFFECT OF SWIRL ON THE STABILITY OF A COMPRESSIBLE AXISYMMETRIC JET

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ABSTRACT

The effect of swirl on the linear stability of an axisymmetric compressible jet is studied. It is found that the growth rates of the linearly unstable modes increase substantially with the addition of rotation to the jet. More important, with the introduction of swirl, the stabilizing effect of higher Mach numbers is greatly diminished.

INTRODUCTION

The prospect of sustained hypersonic flight has focused attentions on the required propulsion system. One of the critical technology involved is the design of the combustion chamber where fuel and air must be mixed very efficiently. Owing to the high speeds involved, fast mixing must be accomplished in the free shear layers that are formed during the injection process. However, it is well known that the growth rate of a shear layer is reduced significantly as Mach number increases (Birch and Eggers 1972, Chinzei et al. 1986). Thus, the goal of much current supersonic combustion research is to enhance fuel/air mixing by altering shear layer spreading rates in someway. A common strategy is to employ streamwise vortices or swirl.

Recent experiments by Taghavi et al. (1989) indicate that the superposition of swirl on an axisymmetric jet at low Mach number tends to increase the jet spreading rate and hence may provide a means to enhance mixing. Unfortunately no data are currently available on the effect of swirl on compressible jets. Therefore as a model flow, the stability of a compressible swirling jet is considered. The analysis reported here is restricted to small rates of swirl relative to the jet velocity, which is appropriate for supersonic combustion chambers.

FORMULATION

The natural coordinates for rotating flows are cylindrical-polar (r,θ,z). In this coordinate system $v_r, v_\theta, v_z, \tilde{p}, \tilde{T}$, and $\tilde{\rho}$ represent the three components of velocity, pressure, temperature and density, respectively. The flow variables are decomposed into a mean part and infinitesimally small perturbations. Assuming three-dimensional wave forms for the disturbances, the flow variables then are written

$$\begin{Bmatrix} v_r \\ v_\theta \\ v_z \\ \tilde{p} \\ \tilde{T} \\ \tilde{\rho} \end{Bmatrix} = \begin{Bmatrix} U(r) \\ V(r) \\ W(r) \\ \Pi(r) \\ \eta(r) \\ \rho(r) \end{Bmatrix} + \begin{Bmatrix} F(r) \\ G(r) \\ H(r) \\ P(r) \\ T(r) \\ \delta(r) \end{Bmatrix} e^{i(\alpha z + n\theta - \omega t)}, \qquad (1)$$

where α and n are the axial and azimuthal wavenumbers respectively and $\omega = \omega_r + i\omega_i$ is the complex angular frequency. The first bracket on the right-hand side of Eq. (1) represents the mean quantities and the variables F, G, H, P, T and δ are the eigenfunction components. Substituting Eq. (1) into the Navier-Stokes equations and neglecting higher order terms in the perturbations, the linearized governing equations are obtained. The general form of the linearized equations for parallel and variable properties compressible flows are given by Khorrami (1991) and due to space limitations are omitted here. It is noted that the equation of state for an ideal gas is used and perturbations in the flow properties are related to temperature perturbations via the assumed functional form of dynamic viscosity, μ, and thermal conductivity, κ.

The governing equations, presented in a laboratory frame of reference, are normalized with respect to the jet centerline values and a characteristic length scale, r_0, to be defined later. Moreover, Re, Pr and M represent flow Reynolds, Prandtl, and Mach numbers respectively.

The far-field boundary conditions imposed on the disturbances are

$$\text{as } r \to \infty \qquad F(\infty) = G(\infty) = H(\infty) = T(\infty) = 0. \qquad (2)$$

For single valued and smooth solutions, the boundary conditions required on the centerline, $r = 0$, are:

$$\begin{aligned}
&\text{if } n = 0, && F(0) = G(0) = 0 \\
& && \frac{dH}{dr}(0) = \frac{dT}{dr}(0) = 0 \\
&\text{if } n = \pm 1, && F(0) \pm iG(0) = 0 \\
& && \frac{dF}{dr}(0) = 0 \\
& && H(0) = T(0) = 0 \\
&\text{if } |n| > 1, && F(0) = G(0) = 0 \\
& && H(0) = T(0) = 0.
\end{aligned} \qquad (3)$$

The governing equations plus Eqs. (2)-(3) constitute a generalized eigenvalue problem for which temporal formulation is arranged in the form

$$A\bar{y} = \omega B\bar{y} \ . \tag{4}$$

The coefficient matrices A and B, are discretized employing a staggered Chebyshev spectral collocation technique (Khorrami, 1991, Macaraeg et al., 1988). The resulting matrices are of the O(5N) where N is the number of Chebyshev polynomials employed. The eigenvector $\bar{y}$ is represented as

$$\bar{y} = \{F, G, H, P, T\}^T \ . \tag{5}$$

The far-field boundary conditions are enforced at a radial position $50 \le r_{\max} \le 100$. Preliminary studies revealed that for a $r_{\max} \ge 50$, the eigenvalues are virtually independent of the truncated domain. The complex frequency ω is obtained using the standard IMSL QZ routine. The results presented in this paper are computed using $80 \le N \le 110$ to ensure five or six significant figure accuracy.

BASE FLOW FIELD

The similarity solution for an axisymmetric jet with weak swirl obtained by Görtler (1954) is utilized to represent the mean flow. For large Reynolds numbers, Görtler's zeroth-order solution of a swirling jet can be represented as

$$V = \frac{q}{0.32475} \frac{r}{\left(1+r^2\right)^2} \ , \tag{6a}$$

$$W = \frac{1}{\left(1+r^2\right)^2} \ , \tag{6b}$$

where q is the swirl ratio and the flow is assumed to be locally parallel which results in

$$U \approx 0 \ . \tag{7}$$

The length scale $r_0 = z\tan\phi_0$ is a measure of jet spread and $\phi_0 \approx \tan^{-1} 2/K$ is the spreading angle for K large (ϕ_0 small). K is proportional to the streamwise momentum flux $2\pi\int_0^\infty \rho^* W^{*2} r^* dr^*$ where star demotes dimensional quantities. The normalizing centerline velocity is given by

$$W_0 = \frac{2K^2}{z} \ , \tag{8}$$

and the flow Reynolds number is then $W_0 r_0 / \nu = 4K$ for large K where ν is the kinematic viscosity.

For a weakly rotating jet ($q < 0.1$), the mean radial pressure gradient can be neglected. Hence

$$\frac{\partial \Pi}{\partial r} \approx 0 \ . \tag{9}$$

For such weak swirl, temperature is assumed to be function of axial velocity only. Taking Pr = 1, the radial profile of temperature is obtained via Crocco's relation. The mean temperature profile is given by

$$\eta(r) = \beta + (1-\beta)W + (\gamma - 1)M^2 \frac{W(1-W)}{2} \ , \tag{10}$$

where γ is the ratio of specific heats and the density is obtained via

$$\rho(r) = \frac{1}{\eta(r)} \ . \tag{11}$$

The constant, β, represents the ratio η_∞ / η_0 with η_0 being the centerline temperature. β is taken to be one since computations showed the results to be almost independent of this parameter for $0.7 < \beta < 1.3$.

DISCUSSION

Before presenting the results, it is noted that for the current study the flow is assumed to have constant properties. Furthermore, the Mach number is taken to be in the range $0 \le M \le 1.4$ while q is confined to < 0.1. Since the jet instabilities are inviscid in nature, it was found that for Re > 900 the growth rate of disturbances approach their inviscid values very rapidly. Therefore, the computations discussed in this paper are for Re = 10^3.

Previous research by Batchelor and Gill (1962) and Lessen and Singh (1973) showed that a fully-developed axisymmetric non-rotating jet is only unstable to three-dimensional disturbances having azimuthal wave numbers $n = \pm 1$. It was found early on that the addition of swirl stabilizes the disturbances with positive azimuthal wave number. However, the situation for $n < 0$ modes proved to be quite different. Hence, we focus our attention on this particular class of perturbations.

Figure 1 represents the effect of increasing Mach number on the growth rate of n = -1 modes for a non-rotating jet (q = 0). The significant damping effect of higher Mach numbers is apparent. This particular figure is used as a reference point to compare the rotating jet results.

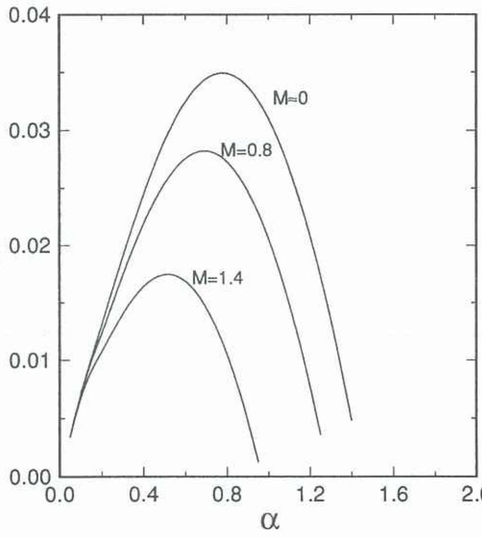

Fig. 1. Effect of increasing Mach number on the growth of n = -1 disturbances for a non-rotating jet (q = 0) with Re = 10^3.

Figure 2 displays the effect of adding 4% swirl (q = 0.04) on the n = -1 disturbances. The growth rate curves show a three to six-fold increase over the no-swirl results shown in Figure 1. At the same time, there is a broadening of the region of instability in the axial wave number space (α). The general damping trend of higher M is also present for a swirling jet, while the effect on the axial phase speed of disturbances, ω_r / α, is negligible. However, it is clear that rotation tends to decrease the percentage of drop in the maximum value of ω_i due to the increase in Mach number. Figure 3 displays the effect of increasing swirl on the n = -1 mode for α = 0.8, M = 0.8, and Re = 10^3. From this figure as an example, it was determined that with the addition of only 4% swirl, the growth rate of the n = -1 disturbances is increased by more than 300%. Also note the presence of a second mode of instability as q is increased beyond 0.07. The computations were stopped at q = 0.09 since, for larger swirl, the validity of the mean flow field employed becomes increasingly questionable. The effect of Mach number on this mode for α = 0.8, q = 0.04, and Re = 10^3 is presented in Figure 4. As mentioned earlier, for moderate values of M, the reduction in the growth rate is slight.

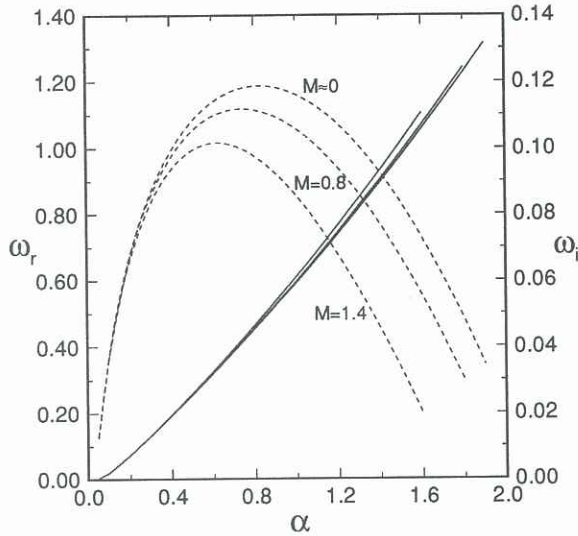

Fig. 2. Effect of increasing Mach number on the angular frequency of the n = -1 disturbances for a swirling jet (q = 0.04) with Re = 10^3. ——ω_r, -----ω_i.

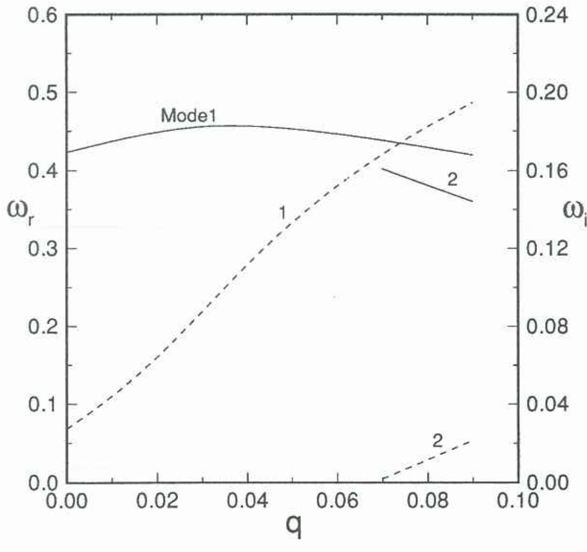

Fig. 3. Variation of angular frequency vs swirl ratio for n = -1 perturbations with α = 0.8, M = 0.8, and Re = 10^3. ——ω_r, -----ω_i.

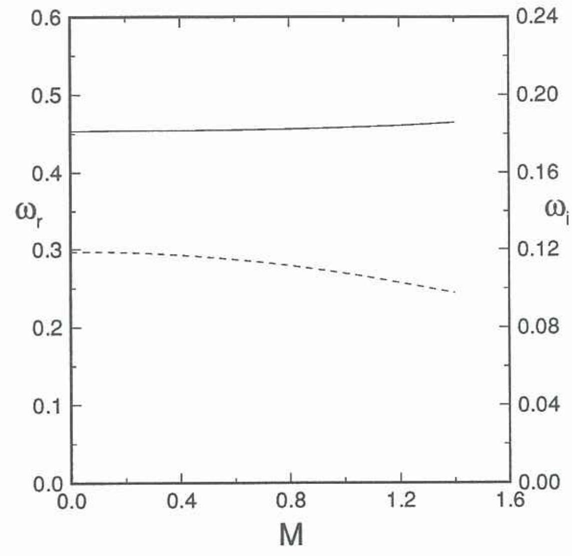

Fig. 4. Frequency vs Mach number for n = -1 perturbations with α = 0.8, q = 0.04, and M = 0.8. ——ω_r, -----ω_i.

The variation of complex frequency, ω, with α for n = -2 modes is presented in Figure 5. The computations were performed for q = 0.06, M = 0.8, and Re = 10^3. Recall that a fully-developed non-swirling axisymmetric jet is stable to such disturbances or any other modes with higher azimuthal wave numbers. Figure 5 depicts an instability with comparable growth rate to the n = -1 modes (Figure 2). Similarly, there are higher modes present. The effect of Mach number on n = -2 modes is shown in Figure 6 for α = 0.5, q = 0.04, and Re = 10^3. For the range of M considered, the stabilizing effect due to compressibility is almost non-existent for this disturbance.

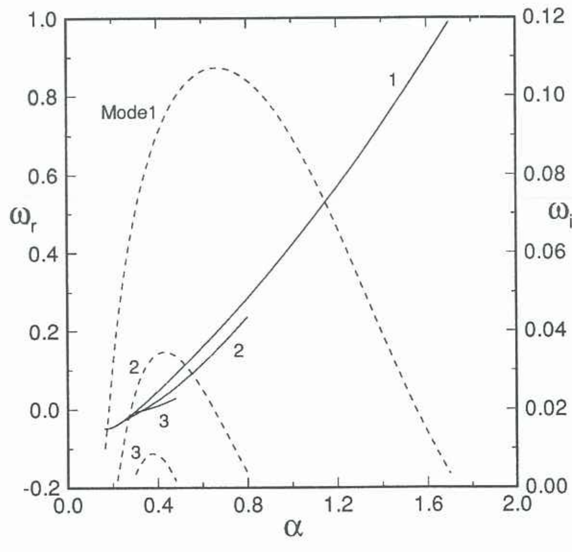

Fig. 5. Variation of the frequency vs axial wavenumber for n = -2 disturbances with q = 0.06, M = 0.8, and Re = 10^3. ——ω_r, -----ω_i.

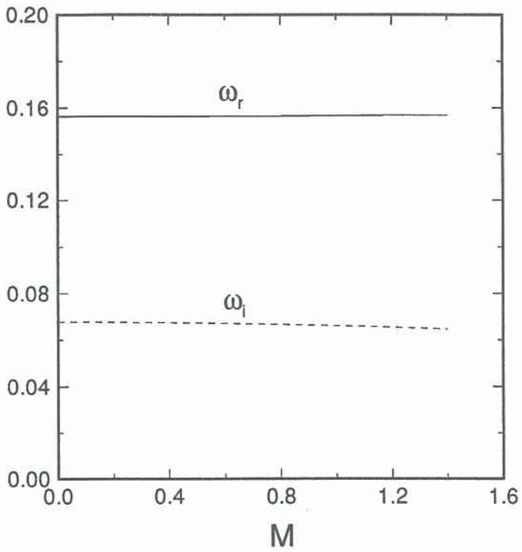

Fig. 6. Frequency vs Mach number for n = -2 disturbances with $\alpha = 0.5$, $q = 0.04$, and Re = 10^3.

Further computations revealed that n = -3 modes behave similarly to the n = -2 disturbances and are therefore not reported here. The disappearance of Mach number effect for $n < -1$ modes is a puzzling behavior which is contrary to the trends obtained for disturbances of two-dimensional as well as axisymmetric free shear layers. While addition of swirl does show promise for mixing enhancement, more extensive studies are needed to fully understand the effect of rotation on the stability characteristics of a compressible shear layer.

CONCLUSIONS

The effect of rotation on the stability characteristics of a compressible axisymmetric jet was considered. Based on linear analysis, the growth rate of n = -1 disturbances is significantly increased with the addition of swirl. Furthermore, disturbances with higher negative azimuthal wave numbers than n = -1 become highly unstable. Such perturbations are stable for a fully-developed non-swirling jet. More important, the stabilizing influence of higher Mach numbers is greatly diminished with the introduction of swirl to the jet.

ACKNOWLEDGMENT

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