11th Australasian Fluid Mechanics Conference
University of Tasmania, Hobart, Australia
14-18 December 1992

FRICTION FACTOR AND FLOW CHARACTERISATION OF NON-NEWTONIAN FLUIDS

M.M.K. KHAN

Department of Mechanical Engineering, University of Central Queensland
Rockhampton Mail Centre, QLD 4702, AUSTRALIA

ABSTRACT

Estimation of head loss for non–Newtonian fluids through straight and curved pipes is discussed. In particular, a power law fluid and its flow characteristics have been considered. Flow of polymeric solutions through these pipes and its frictional factor and Reynolds number relationship are examined.

INTRODUCTION

Head loss or pressure drop is an inherent characteristic of fluid flow through pipes. It is caused by friction between the fluid and the fitting wall and by turbulence caused by abrupt changes in direction or cross section area. Straight or curved pipes with valves and fittings are an essential component of any transporting or processing plants. In the design of process, transporting and power plants it is often necessary to estimate the head loss in a flow system before all the required information is available.

The estimation of head loss for laminar and most turbulent flows in straight pipes, bends and fittings are well defined for Newtonian fluids. However, some degree of uncertainty exists in the estimation of head losses in certain flows for non–Newtonian fluids and data for non–Newtonian fluid flows are generally limited. Many industrial materials of commercial and technological importance are non–Newtonian fluids. It is therefore felt that we discuss and study the flow characteristics of these fluids in straight and curved pipes.

THEORY

All non–Newtonian fluids show shear rate dependent viscosity. They can be classed as time–independent if stress is constant with time at a given rate, and time–dependent if stress changes with the duration of deformation. The rheological measurements of these materials are needed for the estimation of head loss.

A flow curve diagram of shear stress vs shear rate is generally used to characterise a non–Newtonian flow. In some cases, knowledge about the type of fluid is also desirable. Shear stress, τ_w, at the wall of a pipe of diameter D and length L is related to pressure drop Δp by

$$\tau_w = \frac{\Delta pD}{4L} \qquad (1)$$

The shear stress for a Newtonian fluid is given by

$$\tau = \mu \left[\frac{dv}{dr}\right] \qquad (2)$$

where μ is the fluid density and $\frac{dv}{dr}$ is the velocity gradient which at the wall is

$$\left[\frac{dv}{dr}\right]_w = \frac{8V}{D} \qquad (3)$$

where V is the mean linear velocity. Then,

$$\tau_w = \frac{\Delta pD}{4L} = \mu \left[\frac{8V}{D}\right] \qquad (4)$$

$$\mu = \left[\frac{\Delta pD}{4L}\right] / \left[\frac{8V}{D}\right] \qquad (5)$$

$$\Delta p = \frac{32\mu LV}{D^2} \qquad (6)$$

For a Newtonian fluid, μ is constant for all values of $\frac{8V}{D}$ in the laminar flow regime.

In the case of a non–Newtonian fluid, μ is not constant and is dependent on the shear rate. An appropriate viscosity form derived from the plot of $\frac{\Delta pD}{4L}$ vs $\frac{8V}{D}$ flow diagram is required.

For many non–Newtonian fluids, a log–log plot of shear stress vs shear rate is a straight line. The shear stress then follows a power law behaviour as in Figure 1.

$$\tau = K\dot{\gamma}^n = K\left[\frac{dv}{dr}\right]^n \qquad (7)$$

where K is consistency constant, $\dot{\gamma}$ is shear rate and n is flow index. The values of n lie between zero and unity and for Newtonian fluids n equals 1.

The velocity profile of power law fluid for laminar flow in a pipe is

$$\frac{v}{V} = \frac{3 \quad + 1}{n + 1}\left[1 - \left[\frac{r}{r_w}\right]^{\frac{n+1}{n}}\right] \qquad (8)$$

The velocity gradient at the wall

$$\left[\frac{dv}{dr}\right]_w = \frac{3n + 1}{4n}\left[\frac{8V}{D}\right] \qquad (9)$$

Then the generalised power law at the wall of the pipe

$$\tau_w = \frac{\Delta pD}{4L} = K'\left[\frac{8V}{D}\right]^{n'} \qquad (10)$$

where $$K' = K\left[\frac{3n+1}{4n}\right]^{n}, \quad n = n' \qquad (11)$$

Here n' is the slope of line on a logarithmic plot of $\frac{\Delta pD}{4L}$ vs $\frac{8V}{D}$, and K' a consistency index equals the τ_w intercept at $\frac{8V}{D} = 1$.

For Newtonian fluids, the friction losses in straight or curved pipes are expressed in terms of friction factor, f and Reynolds number, Re defined as

$$f = \frac{\Delta pD}{2\rho LV^2} = \frac{2\tau_w}{\rho V^2} \qquad (12)$$

$$Re = \frac{\rho Vd}{\mu} \qquad (13)$$

where ρ is the fluid density. The same general procedure is also followed for non–Newtonian fluids except that the definition of Reynolds number involves a viscosity term which has various possible rheological definition. A modified Reynolds number is used

$$N_{Rem} = \frac{D^{n'}V^{2-n'}\rho}{K8^{n'-2}} \qquad (14)$$

DISCUSSION

Estimation of head loss in pipes is of practical importance since they are extensively used in industries for heating and cooling of both Newtonian and non–Newtonian fluids.

For a fluid flowing through a curved pipe, for instance in helical and spiral coils, there is a variation of centrifugal force across the tube. This results in a pressure gradient between a maximum pressure at the outer wall and a minimum pressure at the inner wall. A secondary flow develops, thereby increasing the frictional energy loss near the pipe walls and the pressure drop, Δp is greater than for the corresponding flow in a straight pipe.

Figure 2 represents the flow characteristics of non–Newtonian fluids through straight and curved pipes. This figure is based on various experimental data of Newtonian and non–Newtonian fluids flowing through straight and curved pipes.

ACB is the flow curve of capillary tube viscometer. The entire curve represents the laminar flow regime and its slope gives the flow behaviour index n'. ACD is the flow curve through a straight pipe of diameter D with the portion AC representing the laminar flow and CD representing the turbulent flow. AEF is the flow curve through a curved pipe in the form of helical and spiral coils having the same diameter D and a radius of curvature R_e.

Many polymeric additives act as drag reducer in turbulent flows through straight pipes. Whether this is true in a curved pipe needs to be tested. An experimental study of flows of polymeric solutions through helical and spiral coils is now being conducted. The results would be presented as they become available.

REFERENCES

KHAN, M.M.K. and BROADFOOT, R. (1992) Rheology and flow characterisation of massecuites and molasses. XI Int Congress on Rheology, Brussels, 720.

MISHRA, P. and GUPTA, S.N. (1979) Momentum transfer in curved pipes. I. Newtonian Fluids, II. Non–Newtonian fluids Ind Eng Chem Process Des Dev, Vol. 18 No. 1, 130–142.

SULTAN, A.A. (1988) Sizing pipe for non–Newtonian flow. Chem Eng December 19, 140–146.

TANNER, R.I. (1988) Engineering rheology. Clarendon Press, Oxford.

WAGSTAFF, D, (1992) Friction factor in coil flow. BEng final year project thesis.

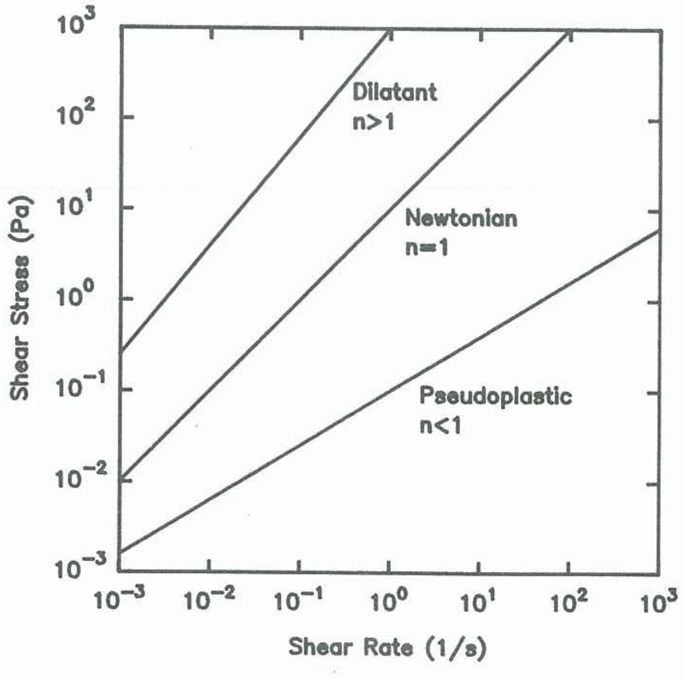

Figure 1 Fluid Shear Diagram

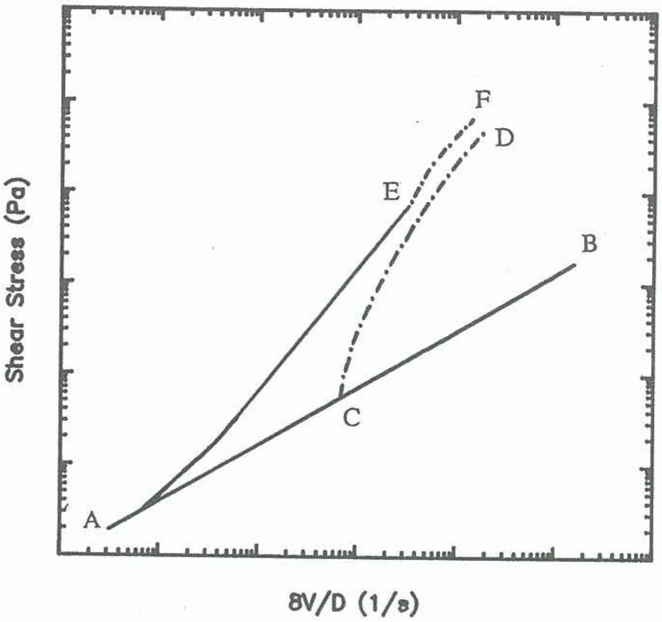

Figure 2 Laminar and turbulent flow through curved and straight pipes for non–Newtonian fluids.