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PREDICTING THE EDDY KINEMATIC VISCOSITY IN NATURAL RIVERS

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ABSTRACT

The prediction of the depth-averaged velocity distribution across a natural river is complicated by the interaction which occurs between the slower moving flow in the flood plains and the faster moving water in the main channel. This interaction is the result of the turbulent shear stresses occurring between the streamtubes across the cross-section, which can be modelled by introducing an eddy kinematic viscosity coefficient. This paper describes a numerical study which has been undertaken to assess the suitability of an eddy kinematic viscosity model. The depth-averaged model with eddy viscosity has been applied to a set of published experimental data from both full sized and small scale laboratory flumes, in which the velocity distributions for uniform flow in channels with compound cross-sections is presented.

NOTATION

A	Cross-sectional area of the element.
d	Depth of flow.
g	Acceleration due to gravity.
h	Elevation of the water surface.
n	Manning roughness coefficient.
P	Wetted perimeter of the element.
R	Hydraulic radius of the element.
S_e	Slope of the energy line.
S_f	Component of the energy slope due to the bed friction.
U	Depth-averaged velocity.
U_*	Shear velocity $= \sqrt{gRS_f}$
Γ^*	Dimensionless transverse diffusivity.
ΔX	Element length in streamwise direction.
ΔY	Element width in transverse direction.
ε	Eddy kinematic viscosity.
λ	Dimensionless eddy kinematic viscosity.
ν	Kinematic viscosity coefficient of water.
ρ	Density.
σ_t	Turbulent Prandtl number.
τ_o	Bed shear stress $= \rho gRS_f$

INTRODUCTION

In a natural river the flood plains are generally wider and rougher than the main channel segment of the cross-section. The resulting velocity distribution is generally not uniform across the cross-section, in particular the velocity tends to be higher in the deeper main channel section than in the shallower flood plain sections. The prediction of the depth-averaged velocity distribution in these channels is complicated by the interaction between the different sections of flow, which is the result of the turbulent shear stresses occurring between the streamtubes across the cross-section.

The turbulent shear stresses between the streamtubes can be modelled by introducing the concept of a virtual, or eddy kinematic viscosity, which is analogous to the kinematic viscosity effect in laminar flow, Schlichting (1968). This approach which has been popular with numerical modellers due to its relative simplicity, requires the prediction of a suitable eddy kinematic viscosity which is not a property of the fluid, as is kinematic viscosity, but is a function of the flow conditions.

A number of numerical models have been developed which use depth-averaged parameters to predict the transverse distribution of velocity and bed shear stress in compound channels. An example is the *k-epsilon* model described by Rastogi and Rodi (1978) and Perera and Keller (1987), which is a computationally intensive method.

This study has been undertaken to determine the suitability of a simple model to predict the eddy kinematic viscosity for natural rivers of compound cross-section.

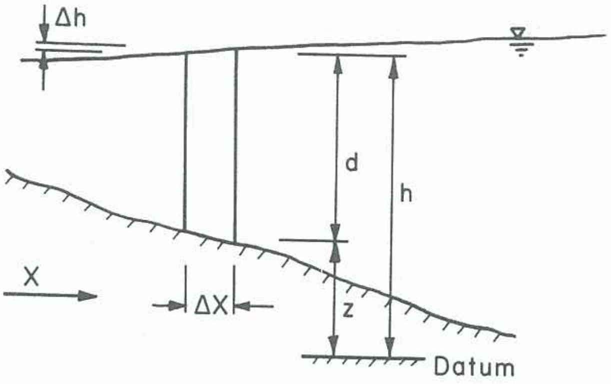
THE NUMERICAL MODEL

For a typical natural channel cross-section, as shown in Figure 1, the equations governing flow can be determined by equating the forces acting in the longitudinal direction, on a small element of steady non-uniform flow which has dimensions d, ΔX and ΔY In this derivation the following assumptions have been made;

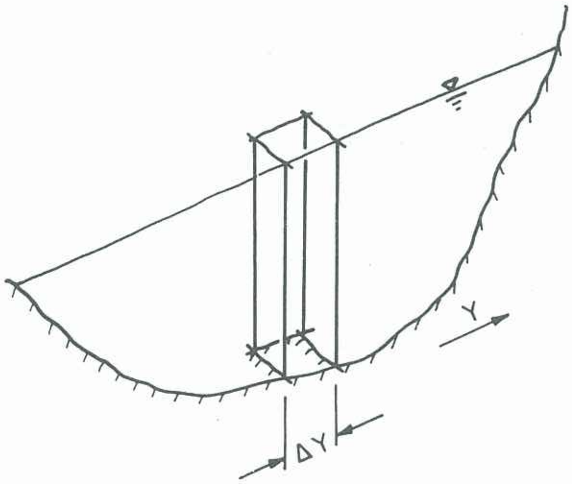
- The channel slope is relatively small,
- The pressure distribution is hydrostatic, and
- The energy slope is constant across the cross-section.

The forces acting on the element are composed of a net hydrostatic thrust, a bed shear force and a turbulent shear force on each side face of the element, each acting in the upstream direction. Because the flow in the channel is accelerating and the flow is steady, the resulting force on the element must equal the mass of the element times the convective acceleration.

$$-\rho g\Delta h - \tau_o P\Delta X - \tau_l d_l \Delta X - \tau_r d_r \Delta X = \rho A U \frac{dU}{dX}\Delta X \qquad (1)$$



a) Longitudinal section



b) Cross-section

Fig. 1. Typical natural channel section.

Noting that the subscripts l and r refer to the left and right faces, this can be simplified to give;

$$S_e = \frac{1}{\rho g A}\left(\tau_o P + \tau_l d_l + \tau_r d_r\right) \tag{2}$$

The energy slope has been defined as being positive when the level drops in the downstream direction. Equation 2 can be rearranged to give;

$$S_e = S_f + \frac{1}{\rho g A}\left(\tau_l d_l + \tau_r d_r\right) \tag{3}$$

The friction slope S_f, can be determined using an empirical bed resistance equation, such as the Manning equation. The turbulent shear stresses between the elements of fluid can be modelled using the concept of virtual or eddy kinematic viscosity.

$$\tau_t = \rho\varepsilon\frac{dU}{dy} \tag{4}$$

Substituting Equation 4 into Equation 3 gives;

$$S_e = S_f + \frac{\varepsilon}{gA}\left[\left(\frac{dU}{dy}d\right)_l + \left(\frac{dU}{dy}d\right)_r\right] \tag{5}$$

Rastogi and Rodi (1978) and Shiono and Knight (1991) show that the eddy kinematic viscosity is related to the local shear velocity by the relationship;

$$\varepsilon = \lambda U_* d \tag{6}$$

Rastogi and Rodi (1978) show that the eddy kinematic viscosity and the dimensionless transverse diffusivity are related by the relationship;

$$\Gamma^* = \frac{\varepsilon}{U_* d \sigma_t} \tag{7}$$

Fischer et al (1979) recommend that the dimensionless transverse diffusivity be adopted as 0.15 ± 50% for laboratory channels, and 0.6 ± 50% for irregular natural channels. Assuming that the turbulent Prandtl number is taken as 0.5, Rastogi and Rodi (1978), this would suggest that a reasonable value for λ is 0.075 ± 50% in laboratory channels, and 0.3 ± 50% in irregular natural channels.

The transverse velocity distribution in the channel can be predicted by the solution of Equation 5 at each element across the cross-section. Most natural channels are irregular in cross-section, so that a general analytical solution cannot be found, and a numerical solution method must be used.

In the numerical model developed as part of this study, Equation 5 is rewritten for each element across the channel, by expressing the partial derivatives $\frac{dU}{dy}$, as finite differences. The resulting set of simultaneous equations are solved using an iterative technique.

THE NUMERICAL STUDY

A numerical study has been undertaken to investigate the use of the simple eddy kinematic viscosity model defined by Equation 6. Two methods of applying the model were used.

1. A constant eddy kinematic viscosity, applied to the whole cross-section, was calculated from Equation 6 taking d as the depth of the main channel and using the energy slope, rather than the friction slope component in the calculation of the shear velocity.
2. An eddy kinematic viscosity, calculated at each element across the cross-section from Equation 6.

The numerical channel flow model was applied to physical model data published by Hadjipanos (1980) and Hegly (1937 a), which included transverse velocity profiles for open channels with compound cross-sections.

Experiments of Hadjipanos (1980)

Hadjipanos (1980) describes a compound tilting flume study. A detailed description of the experimental flume is given by Wormleaton et al (1982). The cross-section of the experimental channel is as shown in Figure 2. The Manning roughness coefficients for the four different flood plain roughness conditions, which were estimated by Hadjipanos (1980) using a roughened rectangular tilting flume, are given in Table I.

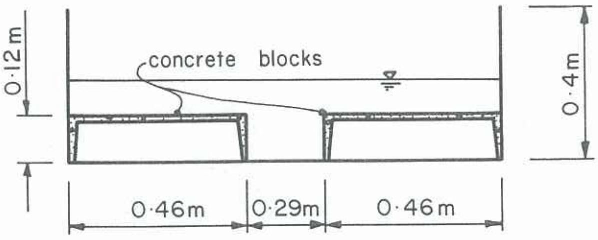


Fig. 2. Cross-section for experiments of Hadjipanos (1980)

Table I: Roughness coefficients for the experiments of Hadjipanos (1980).

Roughness Type	Manning Coefficient n	
	Estimated by Hadjipanos (1980)	Calibrated
Main Channel	Smooth	0.011
A	0.011	0.010
B	0.0145	0.013
C	0.017	0.016
D	0.021	0.020

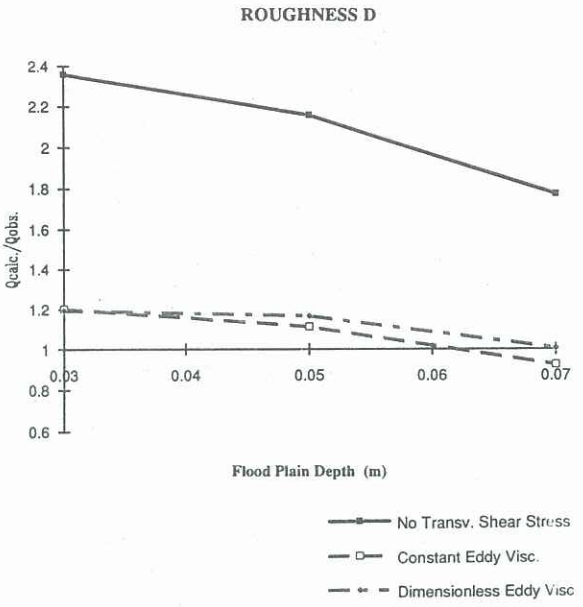


Fig. 4. Comparison of predicted to observed discharge for experiments of Hadjipanos (1980).

Hadjipanos (1980) presented velocity distributions within the main channel, flood plain interaction region for twelve runs using uniform flow. The numerical model was applied to the Hadjipanos (1980) cross-section using a hydraulically smooth main channel and flood plain roughnesses as given in Table I, and assuming that there were no transverse turbulent shear stresses between the elements. For all cases tested, the numerical model over-estimated the discharge through the cross-section. In general the predicted depth-averaged velocities in the main channel were much higher than those observed by Hadjipanos (1980). This demonstrated that the velocity distribution in the main channel is significantly affected by the slower moving water on the flood plains, due to the transverse shear stresses acting between the elements across the channel.

The model was then calibrated using the experimental data by adjusting the Manning roughness coefficients for the flood plains and main channel, as well as the dimensionless eddy viscosity. The resulting predicted velocity profiles and total discharges compared well with the observed values. The calibrated roughnesses, shown in Table I, are within the range of roughnesses determined in the small flume tests of Hadjipanos (1980). The calibrated value for λ was 0.1125, which is the upper limit for the dimensionless parameter recommended by Fischer et al (1979) and Rastogi and Rodi (1978).

The predicted depth-averaged velocity distributions are compared with the observed distributions in Figure 3. A comparison of the ratios of predicted discharge to observed discharge for the calibration runs and the zero transverse turbulent shear stress runs is given in Figure 4. It can be seen from these figures that the numerical model predicted similar results when either of the two methods of applying the eddy kinematic viscosity model was used.

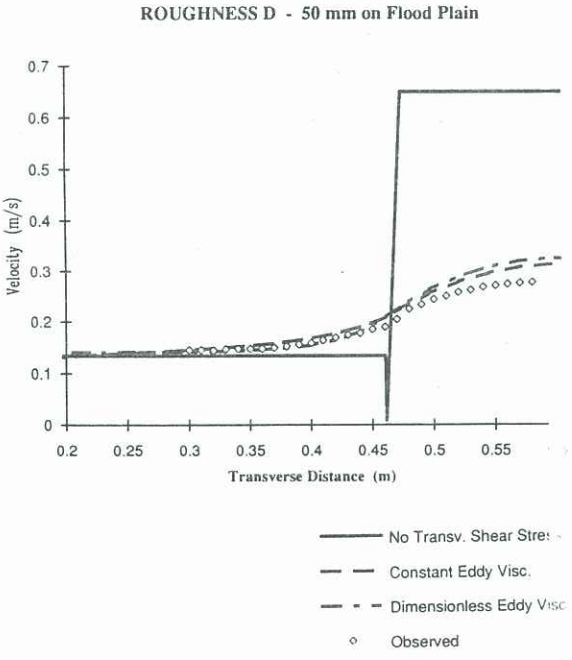
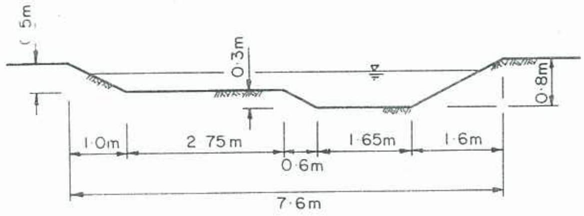


Fig. 3. Comparison of predicted and observed velocity distribution for experiments of Hadjipanos (1980).



1:20 channel cross-section

Fig. 5. Cross-section for experiments of Hegly (1980).

Experiments of Hegly (1937 a)

A series of open channel flow experiments were conducted by Hegly (1937 a), to provide design data for a barge canal to be built in Europe. A description of the experiments undertaken is given in an abridged translation of the original paper, Hegly (1937 b). Details of the experimental channel cross-sections are given in Figure 5.

Hegly (1937 a) describes five experiments in the compound channels in which conditions close to uniform flow were achieved, and velocities were recorded at ten locations across each of the channels. Two experiments are recorded for a 1:20 model, and three for a 1:50 model.

The numerical model was calibrated using the observed velocity profiles for the five experiments described. The bed and banks of both parts of the 1:20 channel were composed of a tamped clay soil, while in the second test the deep section was covered in a tamped clay soil and the shallow part was covered by pebbles. For both tests a roughness of $n = 0.015$ for the tamped clay soil was determined, and for the pebble covered shallow part in the second test, a roughness of $n = 0.024$ was determined. In the calibration runs for the 1:50 channel a Manning roughness of $n = 0.017$ for both the shallow and deep parts of the channel gave the best fit to the observed data. This value appears slightly higher than expected as the bed of the channel was also a tamped clay soil. In general, the calibrated Manning coefficients are within the average range of for these types of surfaces, as set out in Chow (1959).

For the five sets of experimental data, it was found that the numerical model gave the best fit to the observed velocity profile when the dimensionless eddy kinematic viscosity λ was taken as 0.4. This value is above the value suggested for laboratory channels, although it is within the range suggested for irregular natural channels, Fischer et al (1979) and Rastogi and Rodi (1978).

As with the previous calibration runs, the use of either method of applying the eddy kinematic viscosity model appeared to give similar results. Comparisons of the numerical model and the observed velocity profiles, for both the 1:20 and 1:50 models are given in Figure 6.

CONCLUSION

The numerical model runs undertaken in this study suggest that when adopting the eddy kinematic viscosity model described by Equation 6, the dimensionless parameter λ should be considered in the normal calibration phase of the numerical model application. Where observed velocity profiles are not available to help in calibration, then the adoption of $\lambda = 0.075 \pm 50\%$ for laboratory channels and $\lambda = 0.3 \pm 50\%$ for natural irregular channels appears to give a reasonable approximation. For both sets of experiments studied, the effect of the eddy kinematic viscosity was to lower the depth-averaged velocities in the main channel section of the cross-section.

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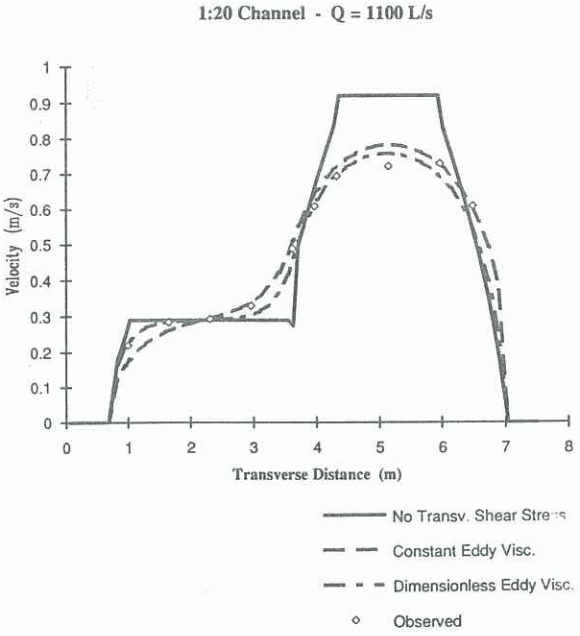

Fig. 6. Comparison of predicted and observed velocity distribution for experiments of Hegly (1937 a).

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