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VORTEX FORCES AND CYLINDER DYNAMICS

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ABSTRACT

The large-amplitude motion of an elastically mounted cylinder under combinations of oscillatory flow and a mean current shows complex behaviour. This paper parameterises the vortex forcing in a dynamic simulation based on the Morison equation. It is concluded that the motions observed in these experiments can be described by limit cycle rather than chaotic behaviour.

1. INTRODUCTION

This study examines the dynamics of an elastically mounted cylinder subject to oscillation loading. This has widespread practical application in the analysis and design of such systems as offshore structures, undersea pipelines, cables, piles and risers. Present theory can predict only to a very limited extent the response of structures subjected to oscillatory loading.

The main deficiency in the present theory is an understanding of the nature of vortex induced forces on the structure. These forces are often substantial and strongly determinative of the overall dynamics. With a view to gaining further insight into the nature of these vortex forces, and structural response to wave motion in general, an experimental rig has been constructed at Monash University (Reid and Hinwood, 1987).

This rig can subject the elastically mounted cylinder to an harmonic oscillation and a current. The cylinder is aligned perpendicular to the flow, and the flow characteristics are approximately uniform along the axis of the cylinder. Thus, a two dimensional analysis of the motion is appropriate.

Initial experiments performed on this apparatus revealed many interesting and complex behaviours. The path of the cylinder motion was recorded, and it was thought that it may exhibit chaotic behaviour (Thompson et al., 1984; Fenton et al., 1991). Prior to the present investigation, no substantial analysis was performed on the experimental results.

This study seeks to gain an understanding of the dynamics through numerical analysis of the experimental data. The vortex forces shall be isolated, analysed and a phenomenological model for vortex drag proposed. The question of chaotic dynamics shall be considered.

2. THEORETICAL BACKGROUND

The present analysis is based on the Morison equation for wave induced forces. Inclusion of vortex induced forces adds terms to the Morison forces. The resultant equations of motion of the cylinder in the x and y directions are respectively:

$$(M+\rho V(C_m-1))x''+Cx'+Kx=C_m\rho Vu' + \frac{1}{2}C_d\rho A(u-x')\sqrt{(u-x')^2+y'^2}+D_v \qquad (1)$$

$$(M+\rho V(C_m-1))y''+Cy'+Ky= \frac{1}{2}C_d\rho A(-y')\sqrt{(u-x')^2+y'^2}+L_v \qquad (2)$$

where M is the cylinder mass, ρ is the fluid density, V is the cylinder volume, C is the structural damping, K is the structural stiffness, A is the projected area of the cylinder, D_v is the vortex induced drag, L_v is the vortex induced lift, u is the water velocity, x and y are the cylinder position coordinates, and dash denotes differentiation with respect to time. The inertia coefficient C_m, and the drag coefficient C_d, were determined from Hinwood and Chandler (1991).

Vortices shed from a rigid cylinder in a uniform current have a speed of circulation proportional to the current speed and a frequency of shedding given by a Strouhal number:

$$S=\frac{fD}{u}=0.2 \qquad (3)$$

The vortex-induced lift force in general has been found to have a frequency f, while the in-line vortex-induced force has a frequency of $2f$. The vortex-induced force, caused by the circulation around the cylinder following the shedding of a vortex, is proportional to the circulation and to the ambient velocity.

Similar results have been found for vortex shedding in oscillatory flow, with the frequency now found from the Keulegan-Carpenter number (Obasaju et al., 1988). The magnitude of the forces should increase with the current, and lag it, since the circulation is proportional to the velocity during formation of the vortex (Chandler and Hinwood, 1985). Fenton et al. (1991) used three term Fourier series in equations (1) and (2) to represent the vortex forcing and predicted limit cycle behaviour, except for the case of nonlinear stiffness. This technique gave a rough agreement with their experimental observations.

A chaotic system is a deterministic system that becomes unpredictable due to its intrinsic dynamics, rather than outside influences. A system must have coupled nonlinear terms and have at least three independent dynamical variables to exhibit chaos (Baker and Gollub, 1990). This system has four independent variables and contains coupled nonlinearities. It thus has the potential to exhibit chaos.

Dynamic systems exhibiting chaos have fractal phase trajectories (strange attractors). A common route to chaos is via a sequence of period doublings as a system parameter is varied. A chaotic system typically exhibits a broad band power spectrum and sensitive dependence on initial conditions.

3. EXPERIMENTAL APPARATUS

The cylinder used in the experiments is elastically supported by beryllium copper cantilevers connected to the cylinder via teflon sliders and universal joints. The cantilevers are of circular cross-section. This ensures that the cylinder has equal stiffness in all directions. The sliders eliminate any potential geometric nonlinearities. Support stiffness may be altered by changing the length of the cantilevers. The cylinder is smooth, 0.5m long, and has a diameter of 25.4mm and is fitted with endplates of three times this diameter. It is neutrally buoyant, and hence has a mass of 0.25kg.

The apparatus in which the cylinder is submerged consists of a U-tube in which oscillations are generated by a loose fitting plunger. A current may be superposed on the flow by pumping water through a return pipe and the working section of the tube. For this series of experiments, the oscillation frequency was set at 0.44Hz.

4. EXPERIMENTAL WORK

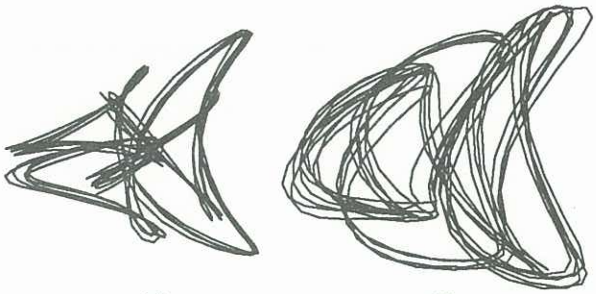
4.1. Scope

The series of experiments previously reported by Fenton et al. were conducted as follows. The cylinder was first excited by oscillation action alone (zero current). The oscillation amplitude was 0.12m/s. The remaining experiments were conducted with both oscillations and a current. The oscillation amplitude was fixed at 0.09m/s. The values of current used were 0.11, 0.12, 0.14, 0.15, 0.16 and 0.17m/s. It is to be noted that for the experiments with current present, there was no flow reversal, ie. the current exceeds the oscillation amplitude.

The data extracted from the experiments were the cartesian position coordinates of the cylinder as time series. These coordinates were obtained by using a video position analyser in conjunction with the video recorded during the experiments. Sampling was conducted at a rate of 25 frames per second. For each case 512 frames were sampled, giving approximately 20 seconds of time series.

4.2. Results

Figure 1 displays the trajectories of the cylinder for currents of 0.11 and 0.17 m/s:



(a) (b)

Figure 1. Trajectory of the cylinder in the U-tube for currents of (a) 0.11m/s (b) 0.17m/s.

This paper shall focus on results from the experiment where the current was 0.11m/s; the data from the other experiments display broadly similar characteristics.

Both position traces exhibit a complex periodic motion, with interior regions in which the cylinder never ventures. This is characteristic of all the experiments, although the shape changes continuously as current is varied. The principal features of these trajectories are symmetrical about a horizontal line.

Figure 2 shows the time series of the cylinder position in each coordinate direction.

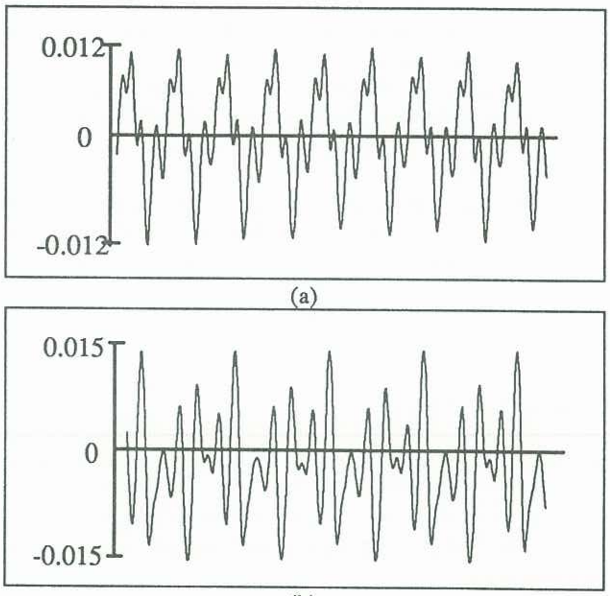


Figure 2. Time series in metres of cylinder motion for current 0.11m/s (a) x direction (b) y direction. (Arbitrary origins)

The phase space is four dimensional since there are two equations of motion, each of second order. Two dimensional cross sections of the phase space, phase planes, are shown in figure 3.

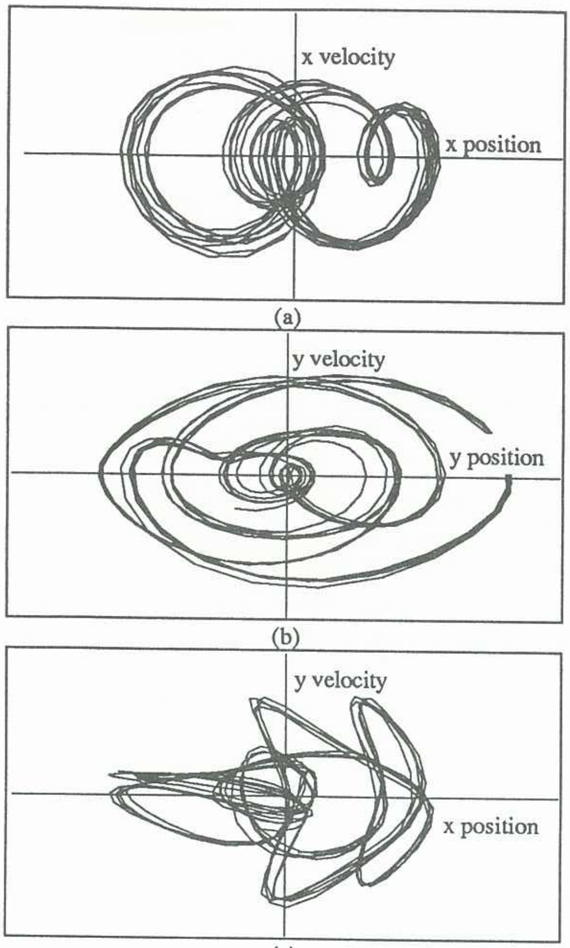


Figure 3. Experimental phase planes. (Arbitrary origins)

4.3. Analysis Of Vortex Forces

Figure 4 shows the vortex forces calculated from equations of motion (1) and (2), superposed upon the water velocity for phase comparison.

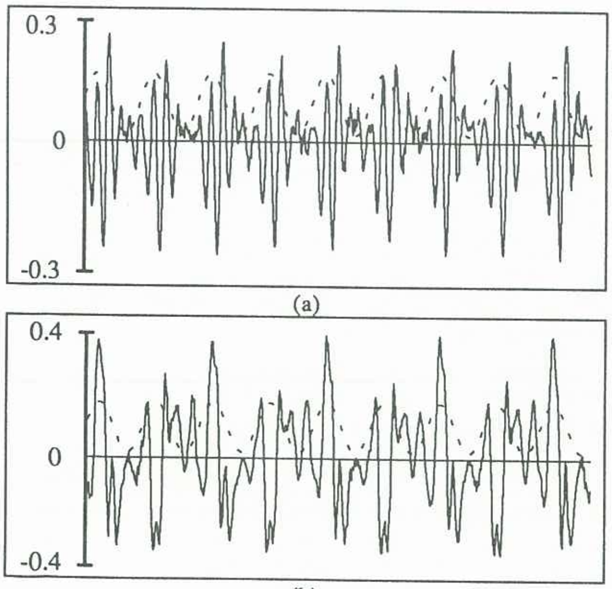


Figure 4. Vortex forces in Newtons and water velocity time series. (a) vortex drag (b) vortex lift. (Arbitrary origins)

Substantiation for the vortex interpretation is provided firstly by the (crude) agreement between the frequency of the vortex force and the criteria of section 2. In particular the *y* force component had twice the frequency of the *x* component.

Secondly, the magnitude variations of the vortex induced forces are approximately in phase with the water velocity, with a slight phase lag.

Figure 5 displays the power spectra of the vortex forces on a linear scale.

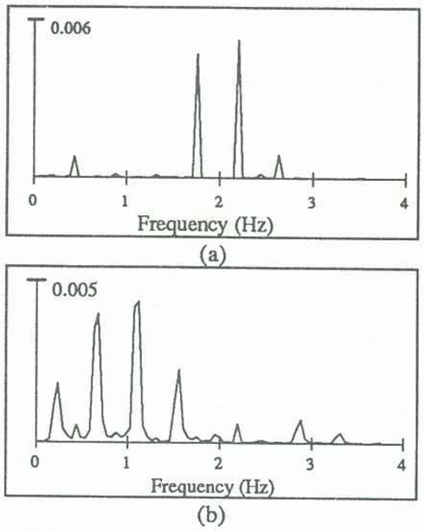


Figure 5. Vortex force power spectra.
(a) vortex drag (b) vortex lift.

All the power spectra are extremely spiked, indicating a narrow band process. This means that the energy is confined to specific frequencies and implies that these frequencies have considerable physical significance.

Consider the forces in the x direction. The spectra for all cases exhibit two dominant spikes. Both spikes are always comparable in height and are separated in frequency by precisely the forcing frequency. This indicates that the vortex induced drag may be described by a beating phenomenon, as in equation (4).

$$\sin(\omega_1 t) + \sin(\omega_2 t) = [2\cos(\frac{\omega_1 - \omega_2}{2} t)]\sin(\frac{\omega_1 + \omega_2}{2} t) \quad (4)$$

The sum of two sinusoids, such as those described by the vortex drag power spectra, may be represented by the product of two sinusoids.

If one sinusoid provides a high frequency of oscillation and the other provides a low frequency variation of the amplitude, then an envelope is formed around the high frequency oscillation. The oscillation amplitude is seen to 'beat' (rise and fall) at ***twice*** this envelope sinusoid frequency, as shown in figure 6. Thus, the vortex drag is exhibiting beating at the frequency of the water oscillation.

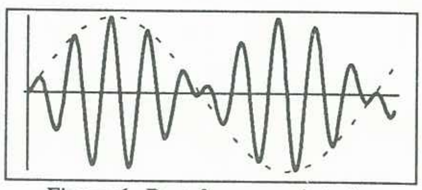


Figure 6. Beat frequency is twice amplitude modulation frequency.

Since the frequency in the centre of the two spikes represents the oscillation frequency of the vortex drag force, this frequency must be closely related to the vortex shedding frequency. An examination of all the power spectra reveals that as the current increases, so too does the vortex shedding frequency. Figure 7 shows the correlation between the vortex shedding frequency and the forcing current. Equation (3) provides a crude estimate of this frequency, with the deviations presumed to be due to the motion of the cylinder.

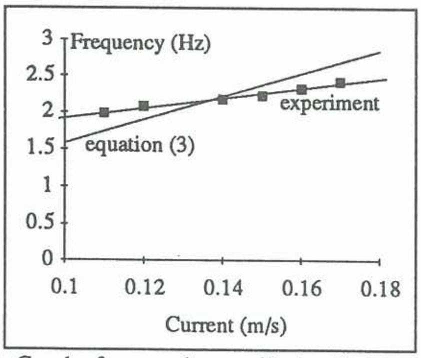


Figure 7. Graph of vortex drag oscillation frequency against forcing current.

5. THEORETICAL WORK

The equations of motion (1) and (2) may be integrated numerically if functions for the vortex forces are specified.

From the experimental analysis above, the vortex induced drag is the product of two sinusoids, as in equation (4). The high frequency is obtained from figure 7, while the low frequency is that of the water oscillation. The lift force has not yet been analytically predicted. The simulations employ an expression with the same spectral properties as found in the experiments. The magnitudes of the vortex forces were set equal to the experimentally determined amplitudes; an analytical expression for the magnitudes is being sought.

Figure 8 illustrates the simulated x position time series and x phase plane.

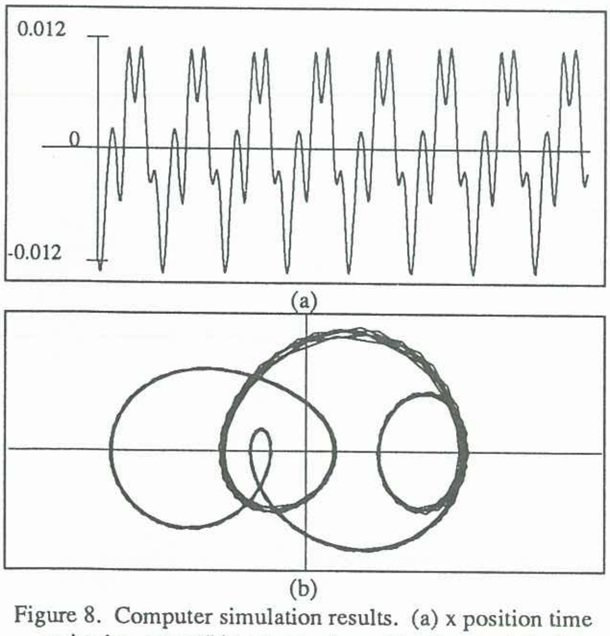


Figure 8. Computer simulation results. (a) x position time series in metres (b) x phase plane. (Arbitrary origins)

Comparison of figures 2(a) with 8(a), and 3(a) with 8(b) show that the Morison equation when coupled with a suitable expression for the vortex forces yields a topologically accurate solution.

6. IS THE MOTION CHAOTIC?

The experimentally determined phase trajectories and the computer simulations converge towards a limit cycle attractor, not a strange attractor. The requirement of sensitive dependence on initial conditions is not met as this convergence occurs for any starting conditions, both in experiments and simulations.

Although the period of the response may increase as the current is increased, it appears to do so continuously, not through period doubling bifurcations. The power spectra of the motion are very narrow band as shown in figure 5.

Thus this dynamic system is within the regime of predictability, not chaos, for the range of parameters used.

7. CONCLUSIONS

The phase and amplitude of the vortex induced drag force are given by the beating of two components, with an envelope frequency equal to the frequency of the water oscillation and an oscillation frequency given as an empirical function of the forcing current.

The composite force expression consisting of vortex induced forces and the Morison equation has been shown to describe the dynamics of an elastically mounted cylinder in oscillatory flow with current.

For the parameter range used, the system is not in a chaotic regime.

8. REFERENCES

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