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FORCE COEFFICIENTS OF DYNAMICALLY RESPONDING VERTICAL CYLINDERS DERIVED FROM "PLUCK" TESTS

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ABSTRACT

Force coefficients relevant to Morison's equation (C_M' and C_D') have been derived from 'pluck' tests in air and still water on four compliant cylinders of varying diameter over a range of water depths and natural frequencies of oscillation. It has also been possible to verify a theoretical model for (wave) radiation damping and C_M' for these cylinders from these experiments.

It was found that estimation of C_M' improved with increasing water depth. C_M' estimations were also more accurate for configurations with a higher stiffness.

On the other hand, C_D' values obtained varied greatly and exhibited a dependence upon the viscosity-frequency parameter β which ranged between 300 and 22000 for the experimental results presented herein. All experiments were conducted outside the regime of viscosity dominated drag with Keulegan-Carpenter numbers ranging between 2 and 18.

NOTATION

C_a'	'Added mass' coefficient(= C_M' - 1)
C_D ,C_D',C_M'	'Normal' drag coefficient, 'Independent' drag coefficient, Inertia coefficient
c'	Coefficient for drag related to damping
D	Diameter of cylinder
f_{0a}	Cylinder natural frequency in air
f_{0w}	Cylinder natural frequency in water
$f(z,t)$	Hydrodynamic force per unit length on cylinder
$F_0(t)$	Total hydrodynamic force on cylinder
H_s	Height of support springs from cylinder base
h	Water depth
k_a	Equivalent stiffness of system at support level in air
k_b	Stiffness at support level due to buoyancy of cylinder in water
k_w	Equivalent stiffness of system at support level in water
m_a	Equivalent mass of system at support level in air
m'	Added mass at support level in water
(S)KC	(Surface) Keulegan-Carpenter number $(= 2\pi (\frac{x}{D}) (\frac{h}{H_s}))$
t	Time
u(z,t)	Horizontal water particle velocity
x	Displacement of cylinder in x-direction at support level
x_0	Displacement of cylinder in x-direction at mean water level (MWL)
z	Vertical direction (z=0 @ MWL, +ve up)
β	$\frac{Re}{SKC} (= \frac{D^2 f_{0w}}{\nu})$
ρ	Density of water
ω_0	Radial natural frequency
$\psi(z)$	Mode shape ($\psi(z)=1+(\frac{z}{h})$ for bottom-pivoted cylinder)
ζ	Damping value (ratio to critical)
ζ_r	Radiation damping value
ζ_s	Structural damping value

INTRODUCTION

In recent years, much attention has been given to the study of the dynamic response of compliant offshore structures (Chakrabarti, 1987). The loading due to hydrodynamic forces is heavily dependant on the Keulegan Carpenter number (KC) and the diameter/wavelength ratio (D/λ). In situations where (D/λ) is less than 0.1, modified forms of Morison's equation have been found to be suitable for predicting the loading upon such slender structures.

In many cases it is observed that a structure is excited in its primary mode of vibration by the loading with its response primarily in the alongwave direction. The one-dimensional equation of motion for a simple model of an offshore structure (a single, vertical compliant cylinder) for a displacement x_0 taken at the mean water level (MWL) is given by:

$$\ddot{x}_0 + 2\omega_0 \zeta \dot{x}_0 + \omega_0^2 x_0 = \frac{F_0(t)}{(m_a+m') (\frac{H_s}{h})^2} \qquad (1)$$

where $F_0(t)$ is the equivalent total hydrodynamic forcing acting at the MWL with other parameters defined as per the Notation section in this paper.

The damping level ζ in equation (1) is observed to principally stem from radiation damping (see later section of this paper) and structural damping. Coupling terms associated with fluid-structure interaction and imbedded in the description of $F_0(t)$ can lead to additional hydrodynamic damping mechanisms (Haritos and Yang, 1991). These mechanisms distinguish between the values realised from still water conditions to those produced by hydrodynamic interaction of waves with the structure and hence the motive behind the conduct of the series of tests being reported in this paper.

'INDEPENDENT FLOW FIELD' FORMULATION

In order to account for a high level of cylinder response when using Morison's equation, the 'Independent Flow Field' formulation has been proposed, in which two independent flow states are assumed (Laya et. al, 1984) viz:

(i) A far field flow state where the wave force acts upon the cylinder assumed to be in an 'as rigid' state.
(ii) A near field state where the cylinder is assumed to oscillate in an otherwise quiescent fluid.

For these conditions Morison's equation then becomes:

$$f(z,t) = \alpha\dot{u}(z,t) + \beta u(z,t)|u(z,t)| - \beta'\psi(z)\dot{x}_0(t)|\psi(z)\dot{x}_0(t)|) \quad (2)$$

where: $\alpha = \frac{\pi}{4}\rho C_m D^2,\ \beta = \frac{1}{2}\rho C_D D,\ \beta' = \frac{1}{2}\rho C_D' D$

The first two terms of equation (2) comprise Morison's equation for a rigid cylinder, the third being the 'near field' modification. The drag coefficient C_D' may differ from C_D. Both drag coefficients are based upon Reynolds number and Keulegan Carpenter number but in different flow fields with different conditions. It has been proposed that the same value may be used for both C_D and C_D' due to difficulty normally encountered for obtaining values for C_D' experimentally. It should however be possible to obtain a value for C_D' by conducting 'pluck' tests in otherwise still water. The form of Morison's equation relevant to this condition is:

$$f(z,t) = -\frac{1}{2}\rho C_D' D|\dot{x}_0|\dot{x}_0\psi^2(z) - \rho C'_m\frac{\pi D^2}{4}\ddot{x}_0\psi(z) \quad (3)$$

which gives, when substituted into equation (1):

$$(m+m')\ddot{x} + (c+c')\dot{x} + kx = 0 \quad (4)$$

where: $m' = \frac{1}{12}\rho C'_a\pi h D^2,\ c' = \frac{1}{8}\rho C'_D Dh|\dot{x}_0|$

Determination of C_D'

The damping ratio ζ, can be expressed as

$$\zeta = \frac{c'}{2k/\omega_0} \quad (5)$$

and letting $|\dot{x}_0| = \frac{2}{\pi}\omega_0 x_0$, for a linear mode shape C_D' can be obtained from:

$$C_D' = \frac{2k_w H_s^3}{\pi\rho D f_0^2 h^4}(\frac{\zeta}{x}) \quad (6)$$

Determination of C_M'

By observing the change in f_0 between air and water tests an estimation of C_a' can be obtained, viz:

$$f_{0a} = \sqrt{\frac{k_a}{m_a}} \quad (7)$$

and: $$f_{0w} = \sqrt{\frac{k_w}{m_a+m'}} \quad (8)$$

Combining equations (7) and (8) and substituting for m' gives the resultant expression for C_M', viz:

$$C_M' = 1 + \frac{3}{\pi^3\rho D^2 h}(\frac{k_w}{f_{0w}^2} - \frac{k_a}{f_{0a}^2})(\frac{H_s}{h})^2 \quad (9)$$

RADIATION DAMPING

As a surface piercing object oscillates in water it generates waves. The waves so generated radiate away from the object removing energy from the dynamic system and thereby reducing its amplitude of response at each successive cycle. This dispersion of energy is known as radiation damping. For a bottom pivoted cylinder, work has been done using diffraction theory (Chakrabarti, 1987, and Dean and Dalrymple, 1984) to predict both added mass and radiation damping. Software applying this theory (RAD_DAMP) has been developed by Haritos (Haritos and Yang, 1991) and made available to this study.

PHYSICAL MODEL AND EXPERIMENTAL PROCEDURE

The physical models used for the experimental work were single aluminium cylinders 2.3m in length and of a range of diameters: 0.15m, 0.10m, 0.05m and 0.025m. Each cylinder was mounted on a low friction 'universal joint' at its base and was supported at the other end by springs connected to support frames. The dynamic properties of the cylinders were changed by different combinations of support spring stiffness and the addition of a range of masses at their top end (see Fig. 1). The natural frequencies of each cylinder were chosen to vary between 0.4Hz and 1.4Hz in air.

Pluck tests were first conducted in air to determine the in air natural frequency of each condition of each cylinder. The cylinders were then individually pluck tested in water with water depths of 1.0m and 1.5m for each condition. To initiate a pluck test, the cylinder was moved from its equilibrium position in the x-direction and released. The cylinder's motion was recorded with force transducers mounted at the support end of the springs by a PC-based data acquisition system recording 4000 points per channel at 20Hz in air and at 50Hz in water.

From the data the primary natural frequency was determined using an FFT method. A plot of ζ versus x was also obtained, the gradient of the line of best fit providing the value of $(\frac{\zeta}{x})$ for the calculation of C_D' using

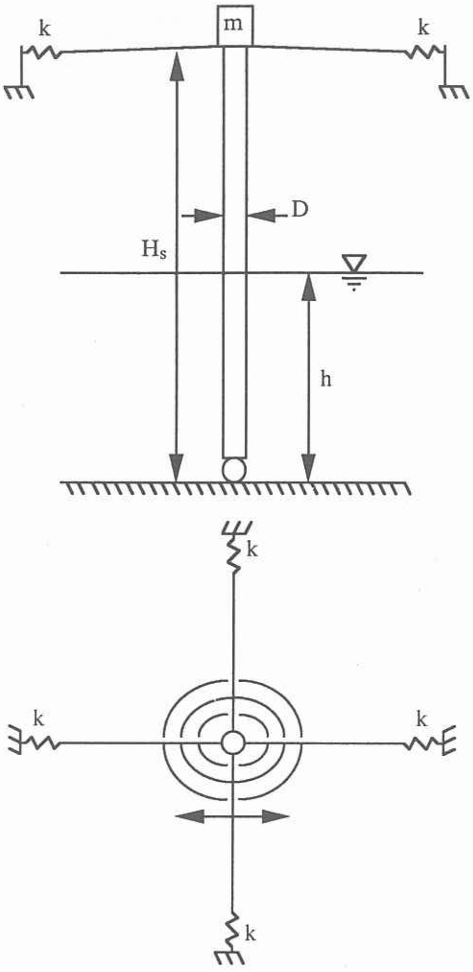

Fig. 1: Schematic of compliant cylinder.

equation (6), while the y (or ζ) intercept yielding the value of structural damping in the system (plus radiation damping in water). The differing natural frequencies between air and water for a condition allowed the estimation of C_M' using equation (9).

RESULTS AND DISCUSSION

Evaluation of C_M'

The C_M' values for both water depths are presented in Figures 2 and 3. As the values corresponded closely for all cylinders each individual cylinder result is not distinguished in these presentations.

A large degree of scatter was present in the C_M' values from the h=1m tests, with considerable improvement noticed when the water depth was increased to 1.5m. A much greater frequency change was observed for the h=1.5m tests meaning that a small error in determination of f_0 had less of an effect than at the lower water depth, thus the improved estimation of C_M' in the deeper water. In both water depths however it can be seen that the scatter of points shows no discernible frequency dependence and values lie about the 'classical' result for C_M' of 2. This fits well with previous work conducted at the University of Melbourne on similar cylinders for their C_M values derived from wave tests (Yang, 1990).

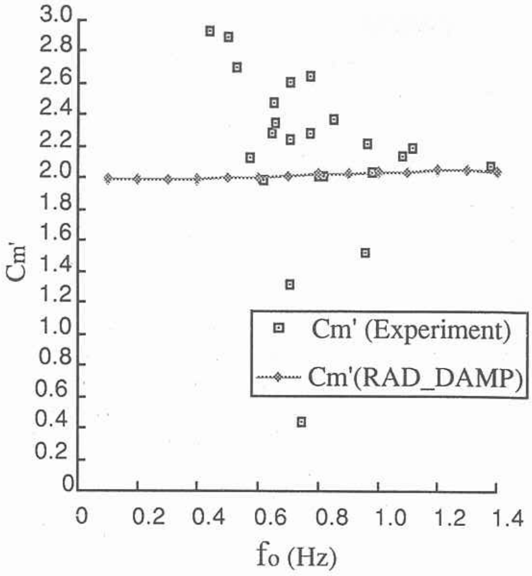

Fig. 2: Comparison of Cm' vs frequency, h=1.0m.

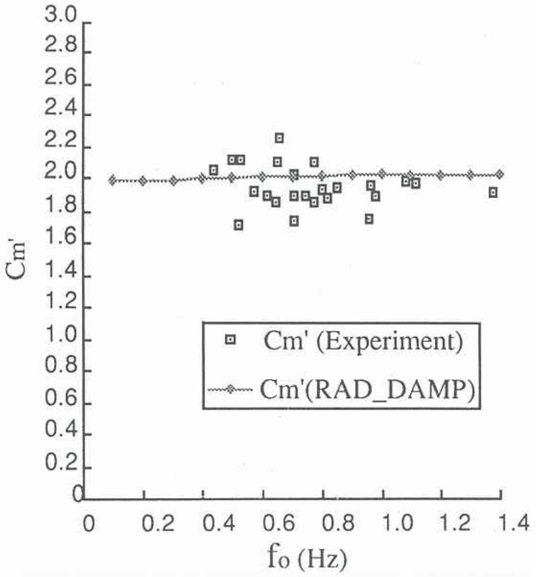

Fig. 3: Comparison of Cm' vs frequency, h=1.5m.

Evaluation of C_D'

Of more importance to the present test program are the results gained for the estimation of C_D'. The drag coefficient for the near field modification of Morison's equation using the 'Independent Flow Fields' assumption is based on the premise that the surrounding fluid is still. Therefore C_D' cannot be obtained from wave tests and pluck tests such as were conducted for this project provide more realistic conditions for the estimation of C_D'. The Keulegan-Carpenter number presented with the current results is determined from the initial offset of the cylinder in question just before release.

Hayashi and Chaplin (1991) have conducted pluck tests on a 19.05mm cylinder in a range of water depths from 0.4m to 0.9m. Most of their data falls within the range of Keulegan-Carpenter numbers where the drag is dominated by viscous effects. Data obtained by Sarpkaya and Bearman et. al is also presented with their work which reaches well into the higher values of SKC where boundary layer separation and vortex shedding is dominant, as is the case for most of the conditions from which the results are presented here (see Fig. 4).

The previous data is related to β and lies mostly between $\beta=400$ and $\beta=550$. Even within this small range, it can be seen that in the higher SKC range that the higher the β value, the lower the value of C_D' for a given SKC. The present results range from β approx. 300 to β approx. 22000 and show a continuation of this behaviour. The results from cylinder number 4 (D=0.025m) share a common range of β values with much of the earlier data and demonstrate a continuation of the levelling out of C_D' at about 2 as SKC rises above 9.

It is clear then that C_D' is heavily dependent upon β and to a lesser degree upon SKC at SKC values greater than 2. When viscous effects dominate the dependence upon SKC increases dramatically. For all tested conditions, C_D' values levelled out at higher SKC's, and generally were in the range 0.2 to 2.0.

CONCLUSIONS

An investigation of the Morison force coefficients for a range of cylinders has been conducted using free vibration ('pluck') tests. Special emphasis has been placed on determination of C_D', the drag coefficient required for the near field term in the Independent Flow Fields modification of Morison's equation.

It was found that the value of C_M' was constant at approximately 2 for all conditions tested. No frequency dependence was observed, nor was any dependency on cylinder diameter. Accuracy in the evaluation of C_M' was enhanced at the greater water depth for the method adopted in these tests.

C_D' was found to vary greatly over the range of conditions tested. All experiments were conducted outside the range of viscosity dominated drag with Keulegan-Carpenter numbers between 2 and 18 in which flow separation is a dominant feature of the structure-fluid interaction. It was observed that C_D' depended greatly upon the viscosity-frequency parameter β which ranged from approximately 300 to 22000. Figure 4 indicates that the higher the value of β, the lower the value of C_D' is for a given SKC.

Experimentally determined values for radiation damping levels and estimates of C_M' were found to match well with the theoretical equivalents determined from wave diffraction theory.

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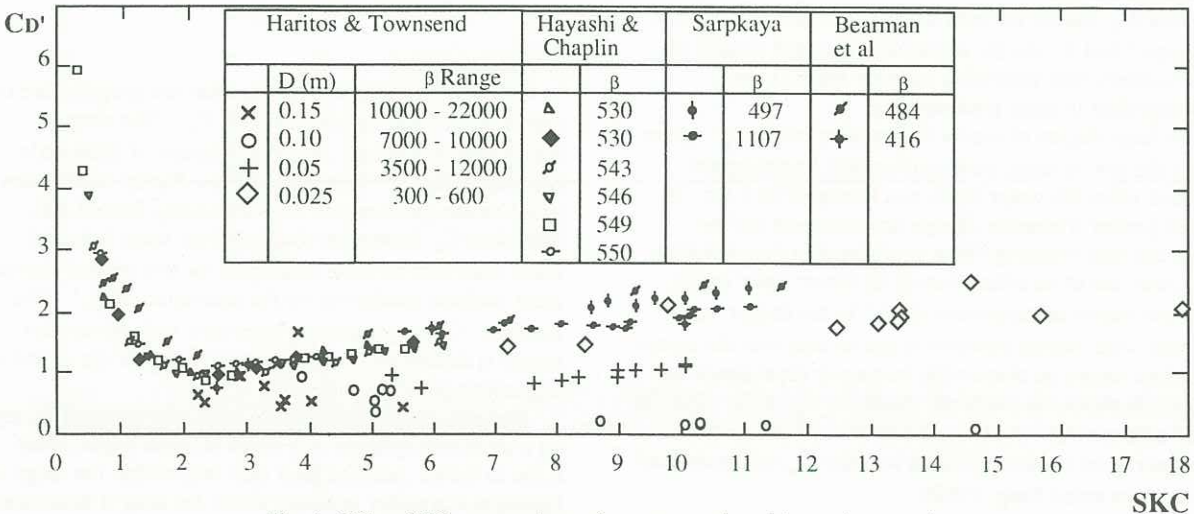

Fig. 4: CD' vs SKC, comparison of present results with previous work.