11th Australasian Fluid Mechanics Conference
University of Tasmania, Hobart, Australia
14-18 December 1992

GRAVITY-DRIVEN FLOW OF VISCO-PLASTIC FLUIDS AS THEY COOL AND SOLIDIFY

Ross W. GRIFFITHS[1] and Jonathon H. FINK[2]

[1]Research School of Earth Sciences, Australian National University, Canberra, ACT 2601, AUSTRALIA

[2]Geology Department, Arizona State University, Tempe AZ 85287-1404, USA

ABSTRACT

Visco-plastic materials extruded from a vent onto a horizontal plane and losing heat from their free surface can spread laterally under any of a number of dynamical regimes, depending on extrusion rate, cooling rate and rheology. Cooling and solidification produce a surface layer of greater viscosity, greater yield strength or brittle solid. For flows with small Reynolds number spreading is driven by buoyancy or vent over-pressure, but resisted by basal shear stresses or the tensile strength of a brittle crust. Because the cooled surface layer comes into contact with the base at the flow front the shear stress applied to this crust can dominate over that applied to the basal area of the warmer interior. Similarity solutions and laboratory experiments indicate the behaviour under each dynamical regime, and each regime tends to be associated with different flow structures. The results can be applied to lava flows.

INTRODUCTION

At the previous Fluid Mechanics meeting in this series (Griffiths et al., 1989) we reported the results of a first set of experiments aimed at understanding the flow behaviour that ensues when a viscous liquid is extruded at a temperature above its freezing point from a small source beneath a colder and less dense environment. The conditions of particular interest were those that led to solidification of the surface of the extruded liquid as it spread under gravity over a base that was either horizontal or inclined. The experiments used a polyethylene glycol wax extruded beneath an aqueous sugar solution (Fig.1). In this system heat transfer was by turbulent thermal convection in the water coupled with conduction in the more viscous wax. These experiments revealed a variety of flow behaviours, or morphologies, reflecting different means by which a solidifying crust was deformed or fractured as the extrusion continued and buoyancy forced the wax to spread.

A dimensional analysis (Fink and Griffiths, 1990) indicates that the behaviour should depend primarily on a dimensionless parameter which descibes the rate of cooling (or solidification) relative to the rate of spreading in the absence of cooling. This parameter can be expressed as the ratio $\Psi=t_s/t_a$, where t_s is the time elapsed before the surface (contact) temperature of the extruded material falls to the solidification temperature and t_a is a time scale for horizontal advection of an isothermal, isoviscous fluid through a distance equal to the depth of the flow. The surface solidification time t_s can be calculated assuming one-dimensional heat transfer by turbulent thermal convection and, where appropriate, by radiation. The normalizing time scale is found by dimensional analysis of viscous gravity currents in the chosen geometry. The distance D_s from the vent at which the surface begins to solidify in axisymmetric flow is $D_s = H\Psi^{1/2}$, where H is the depth scale $H{\sim}(\eta Q/g\Delta\rho)^{1/4}$.

Additional experiments have since been carried out, extending the range of conditions for point sources and giving results for line sources (Fink and Griffiths, 1992). These experiments show the same sequence of flow morphologies with only small differences between flows from point and line sources. At large values of Ψ (Ψ>55; slow cooling rates or large extrusion rates) the wax remains nearly isoviscous and no solid crust forms before the nose of the viscous gravity current reaches the side walls of the tank. At small values of Ψ (Ψ<3; rapid cooling and slow extrusion) the wax develops a strong crust even above the source (Fig.1c) and spreads through formation of many small bulbous structures, which we refer to as 'pillows' by virtue of their similarity with pillow lavas formed on the sea floor. At intermediate cooling rates crust forms at varying distances from the source and the dominant features are solidified levees at the leading edge of the flow (at 25<Ψ<55), transverse ridges formed by streamwise buckling of the crust behind the flow front (at 10<Ψ<25, Fig.1a), or large rigid plates of solid crust which diverge at 'rifts' where melt upwells and freezes onto the plates (at 3<Ψ<10, Fig.1b). Implications of some of these morphologic features for the stresses present during the flow of lava had been discussed previously (Hulme, 1974; Fink and Fletcher, 1978; Anderson and Fink, 1992). However, the sequence of morphologic types obtained by varying flow boundary conditions had not been recognised and each structure had not been related to the extensive variables.

The experiments also allowed detailed measurements of the time-dependence of flow length and height which can be compared with theoretical solutions. Here we summarise similarity solutions for these complex flows and draw conclusions about the dynamical regimes operating in the experiments. It is also shown how the results can be used to determine the dynamics of the spreading of some naturally occurring lava domes.

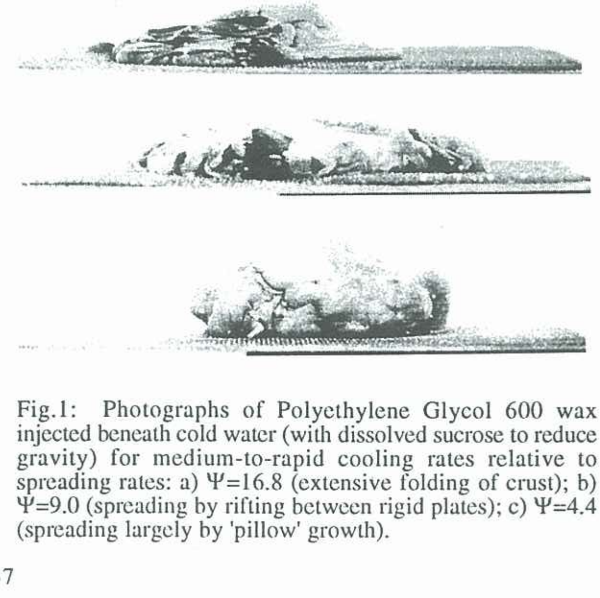

Fig.1: Photographs of Polyethylene Glycol 600 wax injected beneath cold water (with dissolved sucrose to reduce gravity) for medium-to-rapid cooling rates relative to spreading rates: a) Ψ=16.8 (extensive folding of crust); b) Ψ=9.0 (spreading by rifting between rigid plates); c) Ψ=4.4 (spreading largely by 'pillow' growth).

DYNAMICAL REGIMES

A dynamic model for solidifying viscous gravity currents which includes the role of surface cooling can be developed by considering the flow to be made up of two components: an isothermal interior of uniform rheology and a thin, relatively strong or stiff, and generally solidified, crust at the surface. Cooling at the base of the flow is neglected and it is assumed that the thin rheologically distinct surface boundary layer is not continually engulfed into the interior of the flow (thus excluding turbulent and convectively unstable flows). In order to retain the isothermal interior and for the boundary layer to be thin, we restrict attention to flows having large Peclet number.

Depending on flow conditions the solid may form a continuous ductile sheet over the lava surface, but may be torn apart into separated fragments, or may form a layer of closely spaced solid fragments. Some melt may upwell between segments of crust, as seen on some lava flows (Crisp and Baloga, 1990). In some cases where the overall effective strength of the crust is small, the viscous flow will tend to accumulate the cooled surface layer near the flow front. Thus, the effective mechanical properties of the surface layer are uncertain and an abundance of solid on the surface does not necessarily imply that this rheological boundary layer behaves as a brittle crust on the scale of the flow. We will consider the possibilities of a viscous, plastic or brittle effective surface rheology. The effective viscosity or yield strength of the crust is potentially much greater than the viscosity or yield strength of the interior fluid

The interior of the lava flow is modelled as a fluid with either a constant Newtonian viscosity or as a Bingham fluid (Blake, 1990). Our scaling analyses do not include cases in which solidified surfaces are overrun by successive outflows, leaving much of the interior solidified, or cases in which melt flows through isolated internal channels (lava tubes) beneath a stationary crust.

Here we concentrate on the case of a lava extruded from a point source onto a non-slip, horizontal planar surface. By formulating the problem in terms of the forces integrated over the flow, a number of asymptotic dynamical regimes can be identified. For each case consider an extruded volume V of lava which varies as

$$V=Qt^{\alpha}, \tag{1}$$

where t is the time since commencement of the extrusion. In this flexible notation, first suggested by Huppert (1982), the index α may take any value greater than or equal to zero: $\alpha=0$ denotes spreading of a fixed volume and $\alpha=1$ corresponds to a constant effusion rate Q. In the case of lavas the effusion rate often decreases with time, placing $\alpha<1$. However, the flowrate sometimes increases for a time at the start of an eruption, placing $\alpha>1$. If the point source is assumed to give rise to an approximately axisymmetric flow with radius R(t) and the shape remains self-similar, the accumulated volume is given by

$$V=c_1 HR^2, \tag{2}$$

where H(t) is the maximum depth and c_1 is a constant.

Inertial regimes for extrusions having uniform viscosity have been discussed by Huppert (1982), Didden and Maxworthy (1982) and Lister and Kerr (1989). However, for sufficiently viscous flows, including most extrusions of lava and the wax flows in our experiments, the inertial regime occurs only at very small times after the start of the extrusion and will not be discussed further.

The remaining forces available to affect the flow (assuming the rate of extrusion (1) is imposed by external factors) result from the pressure difference between the vent and the environment, and the stresses associated with deformation of the material. The pressure difference consists of both the hydrostatic component $\rho g'H$ (where ρ is the density of the lava, $g'=g\Delta\rho/\rho$ is the reduced gravity, $\Delta\rho$ is the density difference between the lava and its environment and g is the gravitational acceleration), and the over-pressure component p_0 (the difference between the total pressure, p, at the vent and the hydrostatic component, less the pressure applied on the top surface of the extrusion by the environment). The integrated buoyancy force is of order

$$F_B \sim \rho g' H^2 R, \tag{3}$$

and the over-pressure force is

$$F_P \sim p_o R^2. \tag{4}$$

When buoyancy is the dominant driving force, the horizontal scale of the extrusion will tend to grow more rapidly than the depth H. However, in the limit of $F_P >> F_B$ the extrusion must take on an approximately spherical shape, distorted only by effects of the rigid non-slip base and, possibly, by asymmetric outflows from instability (fractures) of the surface. Relations (1)–(4) are not affected by the thermal and rheological structure of an extrusion.

Homogeneous Flows

Stresses retarding the flow may arise from the rate of strain $\partial u/\partial r$, the shear $\partial u/\partial z$, and yield strength. In flows having uniform Newtonian viscosity (η) the forces F_τ resulting from shear stress acting over the basal area of the flow (Huppert, 1982) and those due to the internal viscous stress $\eta\partial u/\partial r$ both scale with the strain rate $\dot{\varepsilon}$:

$$F_\tau \sim \eta \dot{\varepsilon} R^2, \tag{5}$$

Basal shear stress comes to dominate as the flow becomes broader, with $\dot{\varepsilon}\sim U/H$, $U=dR/dt\sim R/t$ and $F_\tau \sim \eta R^3/Ht$.

Alternatively, for the case of a fluid having a finite yield stress there can be a static balance in which the buoyancy force (3) is equated to the yield stress (σ) integrated over the basal area of a lava dome (Blake, 1990). The retarding force in this case is

$$F_\sigma \sim \sigma R^2. \tag{6}$$

Blake, following earlier work on lava flows (Hulme, 1974), modelled the extruded material as a Bingham fluid, in which case the retarding force changes from (5), when $\sigma << \eta\dot{\varepsilon}$ (i.e. at times $t << \eta R/\sigma H$), to (6) when $\sigma >> \eta\dot{\varepsilon}$.

Two-component flows

If surface cooling leads to a thin upper layer that is more viscous or stronger than the interior of the flow, the advance of the flow front is potentially controlled by stresses required to deform this crust. For flows which are broad and shallow, most of the crustal contribution will derive from the flow front, where the crust has cooled most, is thickest, is in contact with the base, and must undergo the greatest deformation in order for the flow to advance. For a crust described by an effective viscosity η_c ($\eta_c >> \eta$), the additional contribution $F_{\tau c}$ to the force (5 or 6) retarding the flow is

$$F_{\tau c} \sim \eta_c \dot{\varepsilon} R \delta_f \quad (\text{or } \eta_c \dot{\varepsilon} \delta_f), \tag{7}$$

where δ_f is the crust thickness near the flow front. Alternatively, if the crust has a finite effective yield stress σ_c, the additional retarding force has a magnitude

$$F_{\sigma c} \sim \sigma_c R \delta_f \quad (\text{or } \sigma_c \delta_f). \tag{8}$$

The yield stress may be relevant if motion is driven by buoyancy and involves deformation of the surface layer near the flow front by shearing or rolling over. However, other flows may advance by intermittent fracturing of a brittle crust which is in tension (as appears to be the case during inflation of small lava domes (Iverson, 1990; Denlinger, 1990)). In these cases a tensile strength γ_c may be more appropriate in (8) than the yield strength σ_c.

We approximate the crust thickness near the flow front in (7) and (8) as proportional to the thickness of a conductive thermal boundary layer which develops as the flow advances: $\delta_f \sim (\kappa t)^{1/2}$, where t is the time since the eruption commenced and κ is the thermal diffusivity. This $t^{1/2}$ dependence of crust thickness (following an element of the crust) assumes that the thermal and rheological boundary layers thicken by one-dimensional conduction as heat is lost from the flow surface. For very flat spreading flows this thickening with time is likely to occur as the surface layer is carried horizontally away from the vent, whereas during endogenous growth of small domes (Fig.1c) it will occur more or less uniformly over the dome surface.

Solutions

Either the buoyancy force (3) or pressure force (4) can be equated to any one of the forces (5-8) arising from the material strength or viscosity. The consequent time-dependence of radius and depth can be found using the volume (1) and geometric relation (2). Results for the eight dynamical regimes are summarized by Griffiths and Fink (1993). There are a number of differences between the regimes which can be used as discriminators in determining which regime best describes a given flow. For example, for homogeneous buoyancy-viscous flow the depth is given by

$$H \sim [(\eta Q/\rho g')t^{\alpha-1}]^{1/4},$$
$$H \sim [(\eta/\rho g')^{\alpha} Q R^{2\alpha-2}]^{1/(3\alpha+1)}, \qquad (9)$$

and is constant when $\alpha=1$, whereas it is predicted to increase with time for flow with a strong crust for any value of α:

$$H \sim (\sigma_c/\rho g')^{1/2}(\kappa t)^{1/4},$$
$$H \sim [(\sigma_c/\rho g')^{2\alpha}\kappa^{\alpha} Q^{-1} R^2]^{1/(4\alpha-1)}. \qquad (10)$$

In addition to the time-dependence of H and the shape H(R), the depth depends on the effusion rate Q in different ways for different dynamical regimes: in the above example the coefficient of H(R) gives $H \sim Q^{1/4}$ for the viscous flow, whereas $H \sim Q^{-1/3}$ when there is a strong crust. Thus for buoyancy-driven flows larger effusion rates give *greater* extrusion depths (at a given flow extent) if the flow is viscous, whereas larger effusion rates give *smaller* depths if the flow is controlled by strong crust that develops with time. In another alternative the primary balance is between over-pressure and stresses in the crust, in which case the depth is directly proportional to radius and independent of flow rate: $R \sim H \sim Qt^{\alpha/3}$.

REGIME TRANSITIONS

There are a number of transitions from one regime to another that are of interest in studies of solidifying gravity currents, and in their application to lava flows in particular. However, the transition of most interest is that from conditions under which the retarding forces are dominated by basal shear stress acting on the interior viscous fluid to those under which the flow is controlled by a crustal yield strength. From (5-8) the crust is dominant when

$$\tau_c/\tau > R/\delta_f. \qquad (11)$$

If an extrusion commences as a uniform viscous flow but cooling and solidification lead to a thickening plastic surface layer, the crust strength becomes dominant (if $\alpha<4.5$) after a time t_c, where

$$t_c \sim (\eta Q^{1/3}/\sigma_c \kappa^{1/2})^{6/(9-2\alpha)}. \qquad (12)$$

EXPERIMENTAL RESULTS

Measurements of the radius and height in solidifying wax extrusions formed by a constant volume flux ($\alpha=1$) show that the radius and height increase with time at rates which depend on the rate of crust development. The height is constant for $\Psi \geq 100$, conditions under which no solid crust formed, but it increases as $H \sim t^{0.3}$ at $\Psi \approx 1$, where solid forms over the whole of the surface area of the extruded wax and spreading takes place by growth of many small 'pillows'. This is most clearly seen by plotting H(R) for each run, fitting a power law curve and plotting the power against Ψ (Fig.2). Again, at large Ψ the behaviour is that for an homogeneous viscous gravity current, at small Ψ the results are consistent with those predicted for a balance between vent over-pressure and a tensile strength of the solid crust, and at intermediate values of the cooling rate ($1<\Psi<50$) there is an intermediate behaviour best descibed as a balance between buoyancy and an effective yield strength of the crust. The predicted dependences of height on the extrusion rate Q (9,10) are also confirmed by the measurements: $H \sim Q^{1/4}$ for a set of runs with $\Psi>50$ and $H \sim Q^{-1/3}$ for a set of eight runs with similar water and wax temperatures but differing Q, and having $3<\Psi<25$.

Expression (10) can be used to estimate the effective yield strength of the wax crust by fitting a $t^{1/4}$-power law to H(t) or a $t^{2/3}$-law to H(R). We find $\tau_c \approx 40$-100 Pa.

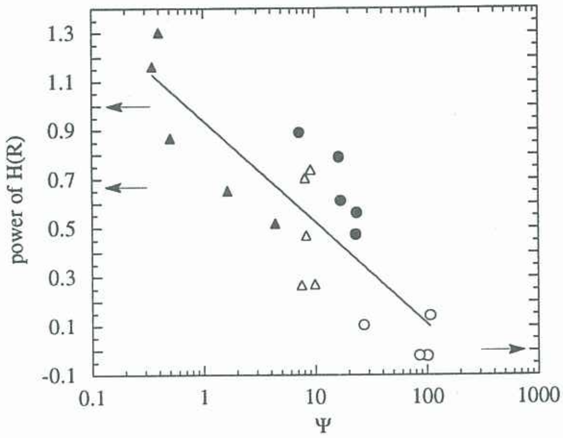

Fig.2: Exponents of the simple power laws which best fit the evolution of extrusion height H as a function of radius R, plotted as against the ratio Ψ of solidification and advection time scales. Symbols indicate 'no crust' (O), folding of crust (O), rifting (Δ), and 'pillows' (Δ).

APPLICATION TO LAVA DOMES

In order to highlight some of the results of the analysis and, at the same time, to illustrate its relevance to lava flows, we consider the evolution of radius and height measured for a lava dome which slowly grew in the crater of *Mount St Helens* from 1980 to 1986 (Swanson and Holcomb, 1990) (Fig.3). During most of this period the volume of the dome can be described by a single power law:

$$V \sim 4.66\, t^{0.88}, \qquad 70<t<2200 \text{ days}, \qquad (13)$$

where V is the volume in m^3 and t is the time in seconds (Fink et al., 1990). Likewise, single power laws provide satisfactory descriptions of dome radius and height as functions of time, and of height as a function of radius. The best fits (in SI units) are:

$$R \sim 10.4\, t^{0.24}, \quad H \sim 1.45\, t^{0.28}, \quad H \sim 0.105\, R^{1.14}. \qquad (14)$$

The data (14) can be compared with predictions of the similarity solutions for $\alpha=0.88$. First, in the isothermal, isoviscous flow model (9) the height decreases with time and radius ($H \sim t^{-0.03} \sim R^{-0.066}$), and the radius increases as $R \sim t^{0.45}$. These predictions are not consistent with the data. The isothermal Bingham model, too, predicts a height increase that is too slow ($H \sim t^{0.176} \sim R^{0.5}$) and a radius increasing too rapidly ($R \sim t^{0.352}$). In closer agreement with the data is the model for flow driven by buoyancy and controlled by a shear strength of the crust (10), which predicts $R \sim t^{0.315}$, $H \sim t^{0.25}$ and $H \sim R^{0.794}$. A regime in which over-pressure opposes the tensile strength of a strong crust ($R \sim H \sim Qt^{\alpha/3} \sim t^{0.293}$) is also in reasonable agreement with the

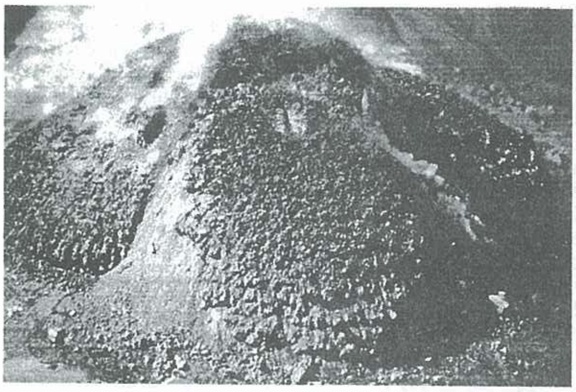

Fig.3: Photograph of the Mount St Helens lava dome on September 11, 1981. The crease structure over the vent suggests 'rifting' behaviour. Photo courtesy of U.S. Geological Survey Cascades Volcano Observatory.

data. However, the measured aspect ratio $H/R \approx 0.27 \pm 0.03$ is smaller than the ratio expected for growth dominated by over-pressure and is similar to those measured in wax experiments where the evolution of radius and height more clearly follow those predicted for a balance between buoyancy and crustal yield strength. Hence we conclude that the evidence supports the contention that the dome grew and spread under a buoyancy-crust strength balance.

Assuming this balance it is possible to estimate the effective yield strength of the crust of the Mount St Helens lava dome. The coefficient of the 1/4-power law fit to the data for $H(t)$ provides the best estimate: using $\Delta\rho = 2.6 \times 10^3$ kg m^{-3}, $g = 9.8$ m s^{-2} and $\kappa = 10^{-6}$ m^2 s^{-1} we find $\sigma_c \sim (1.3 \pm 0.2) \times 10^8$ Pa. By way of comparison we note that the yield strength of some high-viscosity magmas (before chilling at the surface) has been estimated at less than 10^5 Pa (McBirney and Murase, 1984; Blake, 1990). If this upper value is relevant to the interior of the present dome, (11) indicates that stresses in the crust should be dominant over stresses in the hot interior if the thickness of the strong surface layer is greater than 0.1% of of the dome radius. The radius increases to nearly 1000m over 2200 days, implying a thermal boundary layer thickness of ~20 m, or 2% of the final radius. The boundary layer in this case is thick because the extrusion rate, averaged over many incremental growth spurts, was very small. So long as the rheological boundary layer is a major fraction of this thermal boundary layer the crust is more than thick enough to control the lateral spreading of the lava.

CONCLUSIONS

Similarity solutions for spreading and solidification of a viscous or plastic material extruded from a point source predict behaviour that depends on the rheology of the source fluid and the rheology of a chilled surface boundary layer, or crust. Comparison with measurements on extrusions of wax beneath cold water, on which a solid crust develops, reveals some of the dynamical regimes. In cases, where the cooling is very slow and the effusion rate large, solid crust does not form on the flow front until it is a large distance from the source. Before solidification starts in this case there exists a balance between buoyancy and basal shear stress on an effectively homogeneous flow. Once solid forms the crust dominates the dynamics and extrusions enter a regime in which the buoyancy is balanced by crustal yield strength. Vent over-pressure can also be dominant over buoyancy if the development of strong crust is sufficiently rapid while the extrusion is sufficiently small.

Each dynamical regime can be identified with a class of flow morphology: a smooth surface (viscous flow), folded crust (buoyancy-viscous flow giving way to buoyancy-crustal yield strength), rifts separating rigid crustal plates (buoyancy-crustal strength), or a field of bulbous outgrowths (largely dominated by over-pressure and crustal strength). This correspondence suggests that the fossilized morphology might be used as an indicator of the dynamics of a past lava flow. For example, orbiter images of planetary lava flows (Griffiths and Fink, 1992b) and photographs of submarine lavas taken from deep submersibles (Griffiths and Fink, 1992a) may be used to estimate extrusion rates or lava rheology. For many terrestrial lava flows, both our analysis and the observed morphology point to crust strength as the dominant source of the retarding stresses and the primary factor determining the final length of a flow.

REFERENCES

ANDERSON, S W and FINK, J H (1992) Crease structures: Indicators of emplacement rates and surface stress regimes of lava flows. Geol. Soc. America Bull. 104, 615-625.

BLAKE, S (1990) Viscoplastic models of lava domes, in *Lava Flows and Domes: Emplacement Mechanisms and Hazard Implications*, J.H. Fink (ed.). IAVCEI Proc. in Volcanology, 2, 88-128.

CRISP, J A and BALOGA, S M (1990) A model for lava flows with two thermal components. J. Geophys. Res. 95, 1255-1270.

DENLINGER, R P (1990) A model for dome eruptions at Mount St. Helens, Washington based on sub-critical crack growth, in *Lava Flows and Domes: Emplacement Mechanisms and Hazard Implications*, J.H. Fink. (ed.). IAVCEI Proc. in Volcanology, 2, 70-87.

DIDDEN, N and MAXWORTHY, T (1982) The viscous spreading of plane and axiasymmetric gravity currents. J. Fluid. Mech. 121, 27-42.

FINK, J H and FLETCHER, R C (1978) Ropy pahoehoe: Surface folding of a viscous fluid. J. Volcanol. Geotherm. Res. 4, 151-170, 1978.

FINK, J H and GRIFFITHS, R W (1990) Radial spreading of viscous-gravity currents with solidifying crust. J. Fluid. Mech. 221, 485-510.

FINK, J H and GRIFFITHS, R W (1992) A laboratory analog study of the morphology of lava flows extruded from point and line sources. J. Volcanol. Geotherm. Res., in press.

GRIFFITHS, R W and FINK, J H (1992a) Solidification and morphology of submarine lavas: a dependence on extrusion rate. J. Geophys. Res., in press.

GRIFFITHS, R W and FINK, J H (1992b) The morphology of lava flows under planetary environments: predictions from analog experiments. J. Geophys. Res., in press.

GRIFFITHS, R W and FINK, J H (1993) Effects of surface cooling on the advance of lava flows and domes. J. Fluid Mech., submitted.

GRIFFITHS, R W, FINK, J H and CORRIGAN, D (1989) Solidification and deformation of crust on viscous-gravity flows. 10th Australasian Fluid Mechanics Conference, Melbourne.

HULME, G (1974) The interpretation of lava flow morphology. Geophys. J. Roy. Astron. Soc. 39, 361-383.

HUPPERT, H E (1982) The propagation of two-dimensional and axisymmetric viscous gravity currents over a rigid horizontal surface. J. Fluid Mech. 121, 43-58.

IVERSON, R M (1990) Lava domes modeled as brittle shells that enclose pressurized magma, with application to Mount St. Helens, in *Lava Flows and Domes: Emplacement Mechanisms and Hazard Implications*, J.H. Fink (ed.). IAVCEI Proc. in Volcanology, 2, 47-69.

LISTER, J R and KERR, R C (1989) The propagation of two-dimensional and axisymmetric viscous gravity currents at a fluid interface. J. Fluid Mech. 203, 215-249.

McBIRNEY, A R and MURASE, T (1984) Rheological properties of magmas. Ann. Rev. Earth Planet. Sci. 12, 337-357.

SWANSON, D A AND HOLCOMB, R T (1990) Regularities in growth of the Mount St. Helens dacite dome, 1980-1986, in *Lava Flows and Domes: Emplacement Mechanisms and Hazard Implications*, J.H. Fink (ed.). IAVCEI Proc. in Volcanology, 2, 1-24.