11th Australasian Fluid Mechanics Conference
University of Tasmania, Hobart, Australia
14-18 December 1992

FLOW AROUND A LIFTING AEROFOIL : ABSOLUTE INSTABILITIES AND VORTEX SHEDDING

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Abstract

Using a steady state two-dimensional Navier-Stokes solver the flow around an aerofoil is computed. The mean velocity profiles thus obtained are used as the basis of a stability analysis in which the wake is treated as a locally parallel spatially developing flow. By examining the dispersion relation between perturbation wavenumber and frequency, the absolutely and convectively unstable regions of the flow are identified. The frequency selection criteria proposed by Koch (1985), Pierrehumbert (1984) and Monkewitz and Nguyen (1987) are compared with the experimentally observed vortex shedding frequency.

INTRODUCTION

With the availability of modern computers, a number of researchers have studied the stability of fully developed wake profiles by modeling them with analytic functions, for instance Sato and Kuriki (1961) and Mattingly and Criminale (1972). More recently Koch (1985) and Monkewitz and Nguyen (1987) considered velocity profiles characteristic of the near wake behind a bluff body, investigating them for complex wavenumbers - spatially growing disturbances.

The wake of a body is continuously developing in the streamwise direction, so the stability characteristics of the profile at one downstream location will in general be different from those at another. This leads to the question - which of these profiles' stability characteristics determines the vortex shedding frequency in the wake? A number of criteria have been proposed, notably by Koch (1985), Pierrehumbert (1984) and Monkewitz and Nguyen (1987).

The understanding of flow instabilities has been further advanced with the introduction of the concepts of absolute and convective instability by Briggs (1964) and Bers (1975), originally in connection with plasma instabilities. They suggest that the instability of a flow may be characterised by its response to an impulse. In particular, if it can be shown that at least one of the components of the impulse will remain in place and grow in time, then the flow will eventually be dominated everywhere by the instability.

In this paper vortex shedding from an aerofoil at moderate incidence and Reynolds number is observed in experiments, Figure 1. A finite element code is used to solve the steady incompressible Navier-Stokes equations for this case, from which the mean velocity profiles necessary for a stability analysis are obtained. The wake immediately downstream of the aerofoil is treated as a quasi-parallel spatially developing flow. The stability characteristics of the velocity profile at a number of streamwise stations are obtained by treating the velocity profiles as locally parallel. At each of the stations the dispersion relation for instabilities with complex frequency and wavenumber are obtained and the character of the instability determined from the behaviour of the instabilities with group velocity zero. The frequency selection criteria proposed by Koch (1985), Pierrehumbert (1984) and Monkewitz and Nguyen (1987) are then compared with the experimentally obtained velocity spectra from the near wake of the aerofoil.

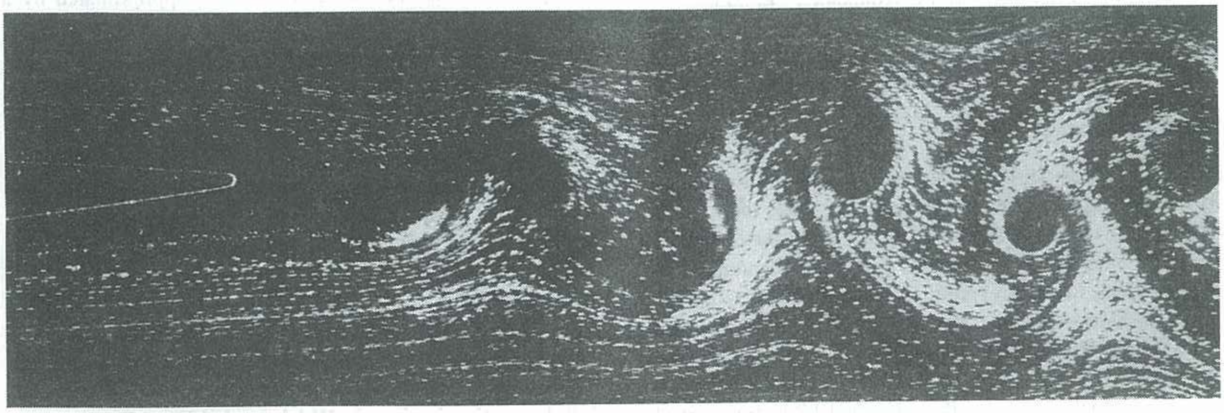

Figure 1. Vortex shedding from a symmetrical Aerofoil, Re_{chord}15000, incidence 4 deg

THEORY

Linear Stability Analysis

Considering the linear stability of an inviscid parallel shear flow, $U(y)$, to small two dimensional disturbances we treat the velocity and pressure as having mean and fluctuating components. Exploring disturbances in the form of normal modes yields the Rayleigh equation

$$\hat{v}'' + \left(\frac{U''}{c-U} - k^2\right)\hat{v} = 0 \qquad (1)$$

Where $\hat{v}$ is the complex modeshape of the fluctuating velocity. k, c and ω represent wavenumber, wave speed and angular frequency respectively. In this work the quantities $U(y)$, k, c and ω have been made dimensionless using aerofoil thickness and freestream velocity as appropriate. In the case of a two dimensional plane wake flow the boundary conditions are those of vanishingly small disturbances far from the wake,

$$\lim_{|y|\to\infty} \hat{v} = 0 \qquad (2)$$

Equation 1 with boundary conditions 2 represents an eigenvalue problem. The method of solution adopted was a shooting scheme using a Runge-Kutta integration, similar to the one described in Betchov and Szewczyk (1963).

Impulse Response Theory

In examining the stability characteristics of a locally parallel velocity profile we adopt the theoretical framework outlined by Huerre and Monkewitz (1990). Of principal importance when considering the stability of a profile is whether it is absolutely or convectively unstable. Physically, when impulsively perturbed an absolutely unstable profile will admit modes with zero group velocity at least one of which grows in time. On the other hand, a convectively (or advectively) unstable profile will admit growing modes all of which will travel away from the source of the disturbance and will eventually leave the original flow undisturbed. More formally, given a velocity profile we can determine which wavenumbers have a group velocity of zero, $\frac{d\omega_r(k)}{dk} = 0$. If any of these are growing modes then an absolute instability is present, but the most rapidly growing mode will be the one for which $\frac{d\omega_i(k)}{dk} = 0$. Thus the process for determining the nature of an instability may be simplified to determining the wavenumber k_0 for which $\frac{d\omega(k_0)}{dk} = 0 + 0i$, since this will be the most amplified mode with a group velocity of zero. If the value of ω associated with this wavenumber has a positive imaginary part then the mode is growing - and an absolute instability exists. Otherwise the instability is convective. In addition it must be verified that an absolute instability is the result of a coalescence of an upstream and a downstream traveling mode.

Global Instability

Having determined the local stability characteristics of a spatially developing flow, we may then want to relate them to the global stability characteristics. In the case of a wake behind an aerofoil, we may ask how does one particular frequency (the vortex shedding frequency) come to dominate the response of the whole region? A number of criteria have been proposed to account for this frequency selection. Pierrehumbert (1984) has suggested that the local mode with the largest absolute growth rate will come to dominate the flow. Koch (1985) has proposed that the profile with a maximum absolute growth rate of zero

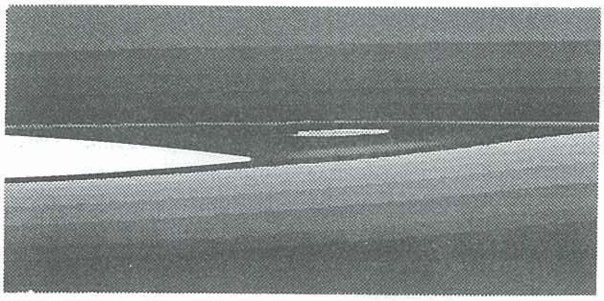

Figure 2: Streamfunction for Re_{chord}15000, 4 deg inc. showing the recirculating flow at the trailing edge.

(the transition profile between absolutely and convectively unstable regions) acts as a reflector of energy and in conjunction with a body will set up a resonance at the neutrally growing frequency of the transition profile. Monkewitz and Nguyen (1987) propose that it is the absolutely unstable profile furthest upstream that comes to dominate the flow.

Flow Modeling

The flow is modelled by a Galerkin finite-element code which solves the steady incompressible Navier-Stokes equations in penalty form. The mesh system is a 320 × 60 node orthogonal 'C-grid' consisting of quadratic quadrilateral elements. The meshlines are compressed towards the surface of the aerofoil in a geometric progression in an effort to properly resolve the thin boundary layers there. There is also mesh compression towards the trailing edge where the flow separates. The far-field velocity is applied at the external boundaries and no-slip at the aerofoil surface. The mesh extends out from the aerofoil by approximately 20 aerofoil thicknesses and the outflow boundary is located about 8 chord lengths downstream.

The resulting nonlinear system of equations is inverted by Newton-Raphson iteration. Velocity fields are computed for a range of Reynolds number using the field at each Reynolds number as an initial guess for the calculations at the next higher Reynolds number. Typically, Reynolds number is incremented in steps of 1000 and four Newton iterations are sufficient to obtain convergence to single-precision machine accuracy. Ten hours of cpu-time on a 5 Mflop workstation is required to calculate the velocity field over the Reynolds number range. Figure 2 shows a small area of the calculated flow field.

For the stability analysis u-velocity profiles were extracted at a series of stations in the near-wake of the aerofoil. Velocities at intermediate points in each profile were obtained by linear interpolation, the second derivative was approximated by a central-difference formula.

EXPERIMENTAL FACILITY

The water tunnel used in all the experiments was of the recirculating type with an entirely enclosed working section. The working section and adjacent walls were fabricated from transparent acrylic to give complete visibility. Water temperature was monitored using a gauge mounted on the inlet pipe. The maximum tunnel speed was 0.4m/s, with a velocity profile uniform in the working section to within 1.0% (outside the wall boundary layers). Turbulence intensity was typically 0.1%. The velocity spectrum was free of sharp spectral peaks. A full description is given in Welsh *et al* (1990). The flow was visualised using the hydrogen bubble technique, the plane of

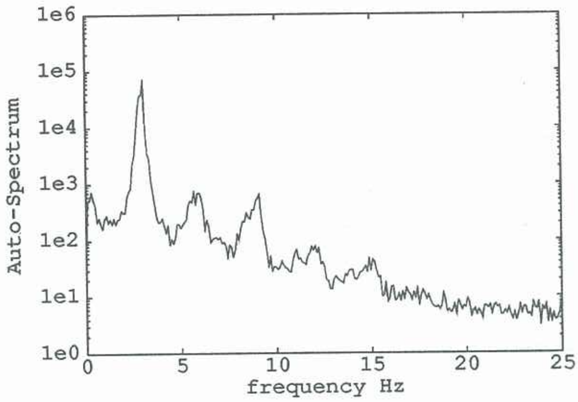

Figure 3: Averaged hot-film spectrum from the near-wake of the aerofoil, showing a peak at the vortex shedding frequency of 3.03Hz

interest being illuminated with a 3 watt argon-ion laser. Velocity spectra were obtained using a hot film anemometer and the data recorded using a PC. Since the purpose of the spectra was only to determine the frequency of the vortex shedding there has been no attempt at linearisation.

The model used in this work was an uncambered C4 aerofoil, of 10% thickness and a chord of 130mm. All experiments were performed at an angle of incidence of 4 deg.

RESULTS AND DISCUSSION

The flow observed behind the aerofoil using the hydrogen bubble technique clearly indicated the regular shedding of vortices over a range of Reynolds numbers. Figure 1 shows the flow for a Reynolds number of 15000. To obtain more quantitative information a hot film probe was placed in the near wake of the aerofoil. The spectra obtained from it clearly shows the vortex shedding frequency (Figure 3).

Treating velocity profiles obtained from the finite-element analysis as locally parallel, a stability analysis was performed on each. From numerical solutions of the Rayleigh equation the dispersion relation between ω and k could be determined for a given local velocity profile. As in previous studies, only the most amplified mode type has been considered. An example of such a dispersion relation is shown in Figure 4. The saddle point clearly visible in the dispersion relation occurs at the point where $\frac{d\omega(k)}{dk} = 0$. It is the value of ω_i corresponding with this saddle point which determines the growth rate of the most rapidly growing mode with a group velocity of zero, and the absolute or convective nature of the instability. Such a dispersion relation was calculated for successive velocity profiles downstream of the trailing edge and from these the nature of the instability determined. The results of this analysis are shown in Figure 5. It can be seen from the graph of $\omega_i(k_0)$ vs. X that a region of absolute instability ($\omega_i > 0$) extends approximately 0.6 chords downstream of the trailing edge of the aerofoil.

As mentioned before, a number of different criteria have been proposed to predict the global instability frequency (the vortex shedding frequency in this case) given the local instability characteristics. A selection of these criteria were applied, the relevant local profiles are marked in Figure 5, and the results are summarised in Table I.

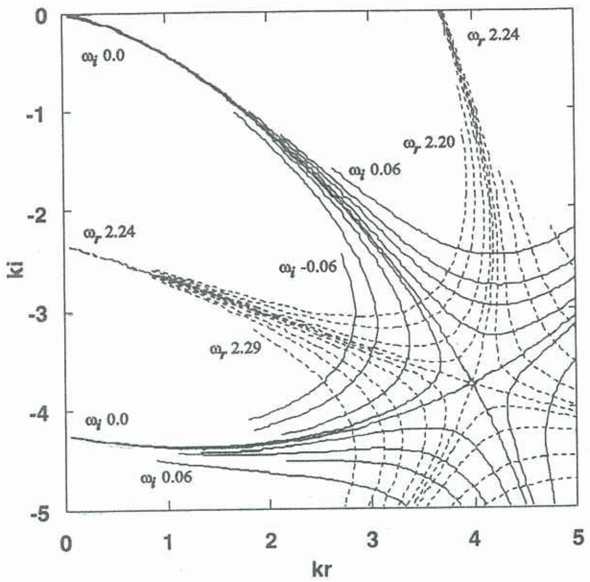

Figure 4: Dispersion relation for a profile, showing a saddle point at $k = 3.98 - 3.76i$ corresponding to $\omega = 2.24 + 0i$

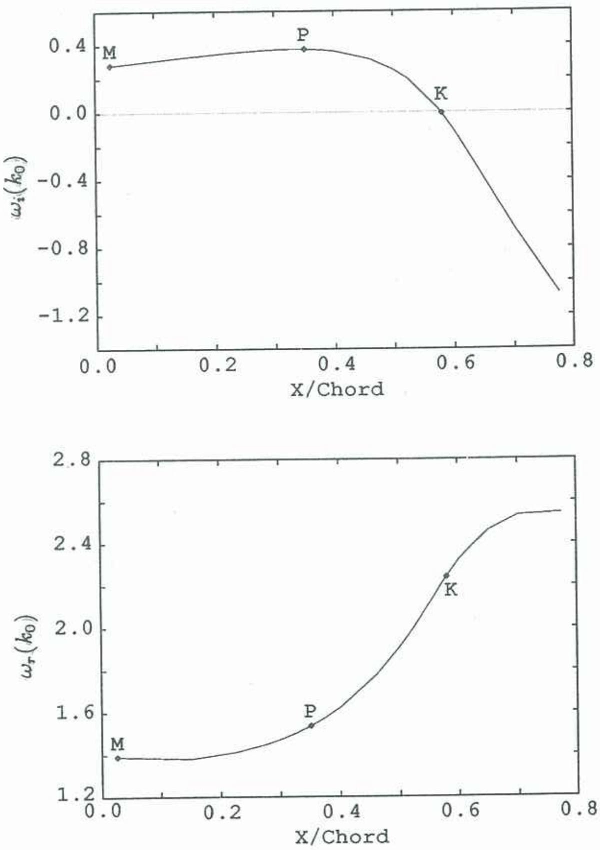

Figure 5: Absolute growth rate and frequency downstream of aerofoil with Re_{chord}15000, incidence 4 deg. Local resonances dominating the global response according to, M: Monkewitz's, P: Pierrehumbert's, K: Koch's Criteria

Table I, Instability Results Re_{chord}15000

Source	Frequency (Hz)	Error
Experiment	3.03	-
Koch	3.43	13%
Pierrehumbert	2.36	22%
Monkewitz *et al*	2.13	30%

From these results it can be seen that Koch's criteria gives fairly close agreement, being 13% in error with the measured frequency. To further examine the usefulness of Koch's criteria the procedure was repeated for two other Reynolds numbers. The results are shown in Table II.

Table II, Comparison of Experiment and Frequency predictions using Koch's criteria.

Re_{chord}	Frequency (Hz)		Error
	Measured	Predicted	
15000	3.03	3.43	13%
10000	1.795	1.94	6%
5000	0.733	0.738	0.6%

At lower Reynolds numbers the predictions of Koch's criteria become, to within the limits of experimental accuracy, identical with the experimentally measured frequency.

It is appropriate now to review some of the assumptions underlying this work to determine the possible causes of the discrepancy in predicted frequency at higher Reynolds numbers.

Aside from the assumption that the mean flow is slowly varying in the streamwise direction and essentially parallel, which is necessary for a simplified analysis to be possible, there are two other simplifications which may have an important bearing on the accuracy of the computed results. The first assumption is that the flow has been accurately modelled at higher Reynolds numbers by the finite-element analysis. In fact, at higher Reynolds numbers convergence is more difficult and that may result in some discrepancy between calculated and observed results. The second assumption was that the inviscid stability analysis (using the Rayleigh equation) would be a sufficiently good approximation at these reasonably high (in terms of stability analysis) Reynolds numbers. A more complete analysis could have been performed by solving the Orr-Sommerfeld equation instead of the Rayleigh equation. At first it may appear paradoxical that we should attempt to account for a discrepancy that increases with Reynolds number by citing the importance of viscous effects at these higher Reynolds numbers. However from figure 5 it can be seen that ω_r is rapidly varying in the vicinity of the Koch frequency - and a small change in the position of the transition profile, such as may occur with the introduction of viscous effects, would shift the predicted frequency by an appreciable amount. On the other hand, for a Reynolds number of 5000 it is found that the gradient $\frac{d\omega_r}{dX}$ is far smaller, so even though the effects of viscosity will in general be more significant at this Reynolds number, the actual frequency shift may be quite small. In this way a viscous analysis may produce results closer to those obtained experimentally.

CONCLUSIONS

It has been shown by experiment that the frequency prediction criterion proposed by Koch produces a very good estimate of the vortex shedding frequency from an aerofoil at low Reynolds numbers. With increasing Reynolds number this estimate becomes less accurate; it is conjectured that these errors result from the inviscid analysis used here.

ACKNOWLEDGEMENTS

Mr J. Gorman wishes to thank the CSIRO Division of Building, Construction and Engineering for the use of their facilities, and acknowledge the financial support provided by Rolls Royce plc. and the S.E.R.C.

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