11th Australasian Fluid Mechanics Conference
University of Tasmania, Hobart, Australia
14-18 December 1992

A NUMERICAL STUDY OF NATURAL CONVECTION IN AN INCLINED THERMOSYPHON WITH AN OPEN TOP

F.O. GAA, M. BEHNIA, S.S. LEONG and G.L. MORRISON

School of Mechanical and Manufacturing Engineering
University of New South Wales
PO Box 1, Kensington, NSW 2033, AUSTRALIA

ABSTRACT

The problem of flow into an evacuated tubular solar collector is solved numerically. The geometry is cylindrical with the top open to a constant temperature reservoir. Due to the heating of the tube a natural circulation flow is established and cold fluid from the reservoir enters the thermosyphon and upon heating, warmer fluid leaves the open top. The fluid is assumed to be bounded by solid walls everywhere except at the open top. The heating of the tube has been simplified so that only a uniform temperature is assumed on the upper surface of the curved wall with a different uniform temperature on the lower surface. Fluid entering the thermosyphon from the reservoir is assumed to have the temperature of the reservoir. The closed end of the cylinder is assumed to be adiabatic.

The solution domain is restricted to the cylinder with specially modelled boundary conditions at the open top. The vorticity-vector potential formulation of the conservation equations in cylindrical coordinates is approximated by finite differences and solved by the ADI method. Solutions have been obtained for water (Pr=6.5) and Rayleigh numbers in the range of 1000 to 500000. Cylinder aspect ratios (length/radius) of 4 and 10 have been considered.

I. INTRODUCTION

Thermosyphons have found wide application in the solar heating industry. The usual design is very simple, consisting only of a vacuum-insulated glass tube sealed on the bottom end and open on top. The open top is connected to a reservoir.

Lighthill(1953) laid the groundwork in the investigation of thermosyphons. From his experimental observations he made analytical formulations on the occurrence of laminar and turbulent flows, critical Rayleigh numbers and rates of heat transfer. Other workers have continued doing analytical studies based on experiment (Leslie(1960), Martin(1954), Hasegawa et.al.(1963)). Much later thermosyphons were modelled numerically using high speed computers (Mallinson and De Vahl Davis(1979)).

There have been in fact many works published on mathematical modelling of natural convection inside enclosed cavities dealing with various combinations of geometry, boundary conditions and methods of solution. However very few works have been published to date on partly open cavities or geometries with one or more flow-through boundaries. Values of vector potential, vorticity, velocity and temperature are easy enough to determine at solid boundaries. More often than not they are simply imposed as known boundary conditions. However this is not the case with flow-through boundaries. The vector potential in particular is not very well known.

Hirasaki and Hellums(1967) have derived a general formulation for the specification of boundary conditions for the vector potential. Their formulation can be applied to solid boundaries and it has been widely used as such, but it can be applied to flow-through boundaries as well. Wong and Reizes(1986) and Yang and Camarero(1986) have successfully modelled flow into and out of a square duct using the formulation. This involved the solution of another vector **B**, whose surface curl gives the tangential components of the Ψ on the inlet plane.

There are other problems arising from simulating a flow-through surface. The velocity is no longer zero at the inlet plane as it would be on a stationary solid wall. Wong and Reizes(1986) have assumed a uniform inlet velocity consisting only of the normal component, the transverse ones being zero. Yang and Camarero(1986) have gone further and experimented with a parabolic profile. In both cases however the velocity profile has been imposed rather than solved. For the case of natural convection where motion arises from a temperature difference, velocity cannot simply be imposed on the flow-through surface.

Abib and Jaluria(1988) simulated buoyancy induced flow in a partially open cavity by using a Neumann condition for the velocity at the inlet plane. Velocity was therefore solved there being previously unknown. The resulting velocity profile showed both inflow and outflow occurring stably side by side on the same inlet plane. Their study was conducted in a two-dimensional framework.

The aim of this paper is to extend the study to three dimensions while using cylindrical polar coordinates to accommodate the geometry of real solar absorbers.

II. MATHEMATICAL FORMULATION

A. Governing Equations

The mathematical model is based on the solution of the energy and vorticity transport equations, using the Boussinesq approximation.

$$\frac{\partial \theta}{\partial t} = -\nabla \bullet (\bar{u}\theta) + \nabla^2 \theta \quad (1)$$

$$\frac{\partial \bar{\zeta}}{\partial t} = \nabla \times (\bar{u} \times \bar{\zeta}) - Ra\, Pr\, \nabla \times (\theta \bar{g}) - Pr\, \nabla \times (\nabla \times \bar{\zeta}) \quad (2)$$

where Ra is the Rayleigh number ($g\beta R^3(T_H-T_C)/k\nu$) and Pr is the Prandtl number (ν/k). Figure (1) shows the notation used in the equations.

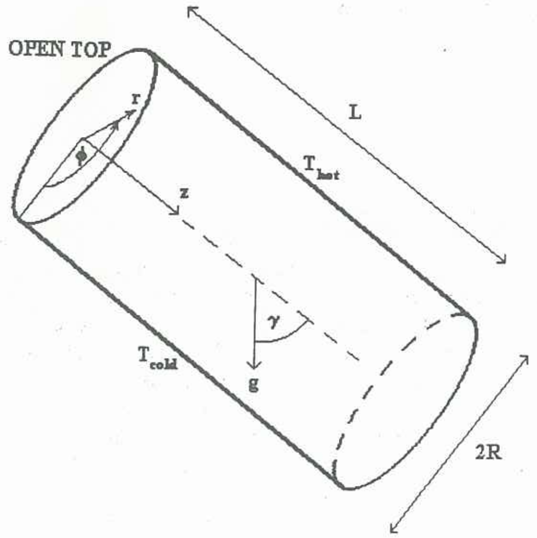

Figure 1: Schematic Diagram of the Cylinder

Equations (1) and (2) are non-dimensionalized using scaling factors R and κ/R for length and velocity respectively. Non-dimensional temperature is defined as $\theta = 2(T-T_{MEAN})/(T_H-T_C)$, T_{MEAN} being $(T_H+T_C)/2$. The thermal diffusivity is κ, kinematic viscosity ν, coefficient of thermal expansion β and the gravity vector **g**. The aspect ratio is defined as L/R.

The vorticity is defined as

$$\bar{\zeta} = \nabla \times \bar{u} \quad (3)$$

and the vector potential as

$$\bar{u} = \nabla \times \bar{\Psi} \quad (4)$$

The vector potential is assumed to be solenoidal, i.e.

$$\nabla \bullet \bar{\Psi} = 0 \quad (5)$$

Combining (3) and (4), we get

$$\bar{\zeta} = \nabla \times \nabla \times \bar{\Psi} \quad (6)$$

The mathematical model consists in the solution of equations (1), (2) and (6) in cylindrical coordinates. The boundary conditions only need to be specified and the model is complete.

The continuity equation $\nabla \bullet \mathbf{u} = 0$ is automatically satisfied in the vorticity-vector potential formulation and need not be solved explicitly.

B. Boundary Conditions (θ, u, ζ)

The temperature is specified at the walls of the cylinder, $\theta = \theta_H$ on the upper surface and $\theta = \theta_C$ on the lower. The closed end on the bottom is adiabatic, hence $\partial\theta/\partial z = 0$. On the open top,fluid outgoing is assumed to be of uniform temperature in the z-direction or $\partial\theta/\partial z = 0$. Fluid in-coming is assumed to have the reservoir temperature, θ_{RES}. All results shown in this paper are based on $\theta_{RES} = \theta_C$.

The velocity is zero at all solid walls, i.e. sides of the cylinder and bottom end. At the open top, velocity is assumed to have only the axial component, i.e. u,v=0. The profile of **w** is not specified, but if continuity is to be satisfied, then

$$\frac{\partial w}{\partial z} = 0 \quad (7)$$

The vorticity boundary condition is obtained from (6) at all boundaries.

C. Boundary Condition of Ψ

The boundary condition for Ψ on solid walls comes directly from Hirasaki and Hellums(1967). The tangential components of Ψ on any solid wall are zero which leaves

$$\frac{\partial \Psi_n}{\partial n} = 0 \quad (8)$$

since vector potential is solenoidal.

For the open top, the existence of a vector **B** is assumed, whose surface curl gives the tangential components of Ψ, i.e.

$$\Psi_{r,\phi} = \nabla_s \times \bar{B} \quad (9)$$

The tangential components of **B** are zero or $\mathbf{B} = (0,0,B_Z)$ therefore

$$\Psi_r = \frac{1}{r}\frac{\partial B_z}{\partial \phi} \quad (10a)$$

$$\Psi_\phi = -\frac{\partial B_z}{\partial r} \quad (10b)$$

Since vector potential is solenoidal, then

$$\frac{\partial \Psi_z}{\partial z} = 0 \quad (11)$$

Equations (10a) and (10b) require the solution of **B** beforehand. By combining (4) and (9)

$$w = \frac{\Psi_\phi}{r} + \frac{\partial \Psi_\phi}{\partial r} - \frac{1}{r}\frac{\partial \Psi_r}{\partial \phi} \quad (12)$$

or

$$w = -\frac{1}{r}\frac{\partial B_z}{\partial r} - \frac{\partial^2 B_z}{\partial r^2} - \frac{1}{r^2}\frac{\partial^2 B_z}{\partial \phi^2} \quad (13)$$

The vector **B** can be solved from the velocity at the inlet plane. There is only one restriction to the solution of **B**. It is necessary for Ψ to be continuous at the edges.

For Ψ_ϕ to be continuous at the intersection of the inlet plane and the side walls, we let $\Psi_\phi = 0$ at $r = R$, or

$$-\frac{\partial B_r}{\partial r} = 0 \qquad \text{there.} \qquad (14)$$

For Ψ_r to be continuous at the edge, the following condition must be applied at $z = 0$:

$$\frac{\partial (r\Psi_r)}{\partial r} = 0 \qquad \text{or} \qquad \frac{\partial^2 B_z}{\partial r\, \partial \phi} = 0$$

But from (14)

$$\frac{\partial^2 B_z}{\partial r\, \partial \phi} = \frac{\partial}{\partial \phi}\left[\frac{\partial B_z}{\partial r}\right] = 0$$

Therefore in order to satisfy the condition of continuity for Ψ_r and Ψ_ϕ, it is sufficient to assume (14) as a boundary condition for the solution of B_z. The third component Ψ_z is already continuous at the edge.

The vector **B** is assumed here to exist only on the inlet plane for the sake of the solution of Ψ. However a similar vector can be assumed to exist on any of the other boundaries and a similar derivation performed. The simplest solution for those other **B**'s would be the null vector in which case the vector potential would end up as specified in (8).

III. THE SOLUTION

In finite difference format, time derivatives are replaced with forward differences and spatial derivatives with central second order differences. The energy and vorticity transport equations are solved using the Samarskii-Andreyev ADI method. The vector potential and the **B** vector equations are solved using SOR. The solution is iterative and begins with the following initial conditions: All variables except temperature are set to zero; temperature at the upper wall is set to θ_H and on the lower wall θ_C.

The iterative process begins with the solution of temperature from equation (1). Then the three components of vorticity are solved using equation (2). The vorticity boundary conditions are solved from current values of vector potential. New values of vector potential are solved using (6). Boundary values of Ψ are solved from current values of B_z. New velocities are solved using equation (4). Then finally new values of **B** are generated from **w** using equation (13). The cycle is repeated until all values become steady or when the change after each iteration is less than a specified value.

The method is based on a modified version of the C3D program used by Leong and de Vahl Davis(1979).

IV. RESULTS

All results presented were generated from a grid of 31 ×32×33 in the r, ϕ and z directions respectively. Solutions for two aspect ratios were obtained, 4 and 10. The fluid was water with Pr of 6.5 at room temperature. Rayleigh number ranged from 10^3 to 5×10^5. The inclination γ of the cylinder was 45° from the horizontal in all results. Computations were performed on an IBM RISC/6000 workstation. It took 12000 iterations at 4.7 cpu seconds per iteration to get convergence at $Ra=10^3$ and $L/R=4$. At $Ra=5\times10^5$ 200,000 iterations were required for convergence.

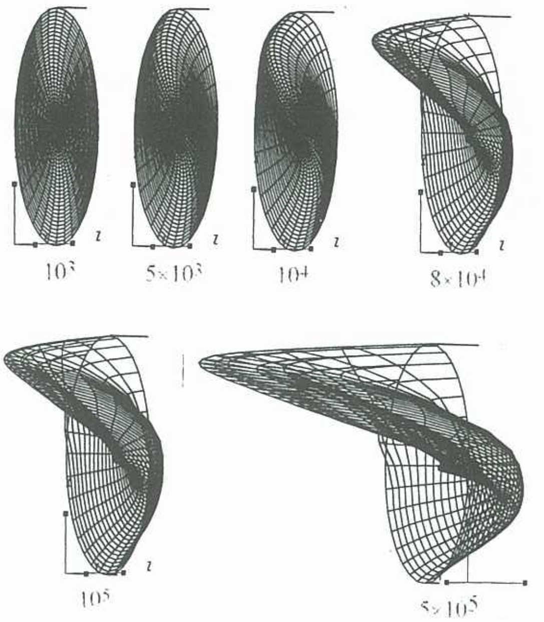

Figure 2: Velocity Profile at Open Top

V. DISCUSSION

Figure (2) shows the development of the velocity profile at the open top with increasing Rayleigh number. At low Rayleigh numbers the area occupied by fluid coming in is roughly equal to the area occupied by fluid going out. The maximum velocities in either direction are also roughly the same, around 35 in non-dimensional terms. At higher Ra the incoming fluid tends to take up a greater area, in the shape of a crescent on the lower half of the cross-section. The fluid going out takes up the centre and the top of the cross-section. Outgoing fluid also travels at a higher velocity than the incoming fluid.

The path of the fluid is shown in Figure (3). There is basically one major loop. Cold fluid creeps down the lower wall of the cylinder, reaches the bottom, turns around and creeps along the upper wall until it leaves the cylinder. There is however a tendency to spread out from the vertical plane of symmetry on the way down. Fluid flows along the walls going up and converges to the middle on the way out. This shows why the area occupied by outgoing fluid in Fig.(2) is smaller than that coming in. There is also an accompanying increase in velocity as a greater volume of hot fluid converges in the middle.

Fluid entering the cylinder at the vertical plane of symmetry remains in this plane throughout the entire

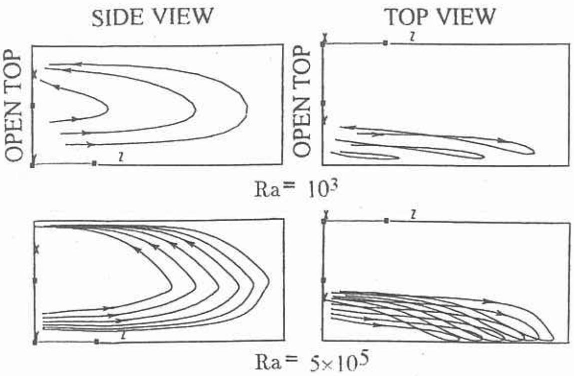

Figure 3: Particle tracks (L/R = 4)

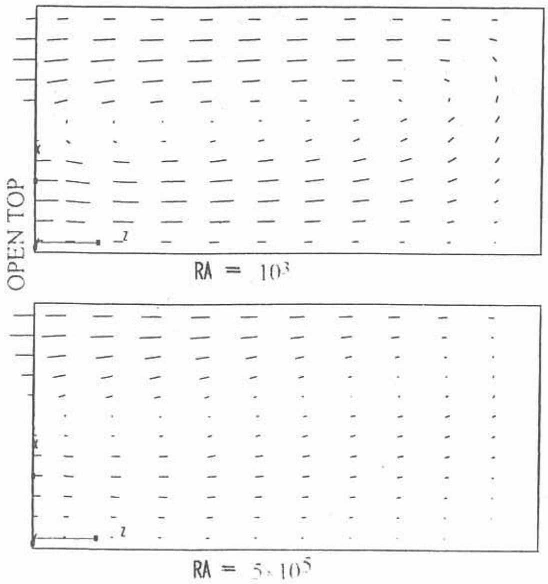

Figure 4: Vector plot on a vertical plane (L/R = 4)

journey. There is a small area near the bottom where velocity is very low. The vector plot (Fig.4) shows more clearly the velocity distribution at the vertical plane. There is a well defined low velocity spot near the bottom.

The above results show that flow-through surfaces can be simulated numerically using the Hirasaki and Hellums(1967) formulation without necessarily imposing a fixed velocity profile. Real evacuated-tube solar absorbers operate at Rayleigh numbers and aspect ratios much higher than the range presented here. This however is a start; this area has not been investigated widely either numerically or experimentally. Tests at higher Ra are being planned for the future with hopes of an experimental comparison.

REFERENCES

ABIB, A.H. and JALURIA, Y. (1988) Numerical simulation of the buoyancy-induced flow in a partially open enclosure. Num. Heat Tr., 14,235-254.

HIRASAKI, GJ and HELLUMS, JD (1967) A general formulation of the boundary conditions on the vector potential in three-dimensional hydrodynamics. Quart.Appl.Math.,26,331-342.

HASEGAWA, S, NISHIKAWA, K and YAMAGATA, K (1963) Heat transfer in an open thermosyphon. Bull. of JSME, 6,no.22,230-250.

LEONG, SS and DE VAHL DAVIS, G (1979) Natural convection in a horizontal cylinder. In Numerical Methods in Thermal Problems (ed.RW Lewis & K Morgan), 287-296, Pineridge.

LESLIE, FM (1960) Free convection in the open tilted thermosyphon. J.Fluid Mech.,7,15-127.

MALLINSON, GD and DE VAHL DAVIS, G (1981) Three-dimensional flow in a closed thermosyphon. J.Fluid Mech.,109,259-275.

MARTIN, BW (1954) Heat transfer by free convection in an open thermosyphon tube. Brit.J.Appl.Phys.,5,91-95.

WONG, AK and REIZES, J (1986) The vector potential in the numerical solution of 3-dimensional fluid dynamics problems in multiply-connected regions. J.Comp.Phys.,62,124-142.

YANG, H and CAMARERO, R (1986) An improved vorticity-vector potential method for 3-dimensional duct flow simulations. Int.J.Num.Meth.Fluids,6,35-45.