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IMPROVEMENT OF FLOW AND BED TOPOGRAPHY IN A CURVED CHANNEL BY REDUCING THE SLOPE OF THE OUTER BANK

Shoji FUKUOKA[1], Tatsuya NISHIMURA[2] and Takeshi SANNOMIYA[3]

[1]Dept of Civil Engineering, Tokyo Institute of Technology, Tokyo 152, JAPAN

[2]CTI Engineering Company Ltd, JAPAN

[3]Sewerage and Sewage Purification Dept, Ministry of Construction, JAPAN

ABSTRACT

The flow in a curved channel causes the bed scour near the outer bank and resulting outer bank erosion. Making the outer bank slope gentler is a useful means of bank-protection. For a curved channel with gentler outer bank slope, the theoretical and experimental studies are conducted, aiming at the improvement of flow and bed topography. The result of calculation agrees well with that of the experimental results with regard to the structure of three dimensional flow and the bed topography, and making the outer bank slope gentler in a curved channel can reduce the bed scour near the outer bank without the rise of water level.

1. INTRODUCTION

The riverside slope of levees and banks is conventionally built at 2H:1V slopes. Making this slope gentler strengthens the levee, stabilizes the river course, makes its easier to control levee vegetation and generally improves the river environment, thereby creating rivers which are safe and gentle to people and living creatures.

However, making the slope of river banks gentler will shift the bank forward: this may reduce the cross sectional area of flow and raise the water level. The effects of a stabilized river course and strengthened bank by making the bank slope gentler have not been quantitatively analyzed, thus gentler slopes are not usually used at present.

The flow in a curved channel is concentrated at the outer bank and scours the river bed near the outer bank. Extra precautions have therefore to be taken to make the outer bank stable. Shifting the bank forward will move the scouring position, which occurs near the channel bank, toward the center of the channel.

In this paper, the three-dimensional flow structure and bed topography when the slope of the outer bank is made gentler is determined from basic equations of motion and continuity equation for sediment. These are simplified by comparing the orders of respective terms of the equations of motion with respect to the uniformly curved channel. The calculated results are also compared with experimental results and the effects of making the bank slope gentler on the improvement of flow and the deformation of the riverbed are reviewed.

2. THEORETICAL ANALYSIS

Basic equations

In order to handle the flow in the curved channel, the orthogonal curvilinear coordinate system represented by s-n-z is used. A hydrostatic pressure distribution in the z-direction is assumed. The following equations in which the three-dimensional equations of motion are simplified, have conventionally been used to analyze the flow in a uniformly curved channel (Engelund, 1974).

$$\frac{1}{r}\frac{\partial}{\partial n}(rv)=0 \quad \cdots\cdots (1)$$

$$\left.\begin{array}{l} -g\dfrac{\partial H}{\partial s}+\dfrac{\partial}{\partial z}\left\{\varepsilon\dfrac{\partial u}{\partial z}\right\}=0 \\ -\dfrac{u^2}{r}=-g\dfrac{\partial H}{\partial n}+\dfrac{\partial}{\partial z}\left\{\varepsilon\dfrac{\partial v}{\partial z}\right\} \end{array}\right\} \quad \cdots\cdots (2)$$

where, s: Longitudinal direction, n: Lateral direction, z: Vertical direction, u, v: Velocity components in s, n direction respectively, H: Water level r: Radius of curvature, ε: Eddy viscosity coefficient.

However, equation (2) is for a uniformly curved channel with a large width and vertical side walls. Thus, equation (2) is not sufficient for a channel with a gentler slope on the outer bank, which is analyzed in the present study.

Therefore, the equations of motion can be simplified as shown in the equations below, by comparing the orders of respective terms of the three-dimensional equations of motion. This is based on the results of experiments for a uniformly curved channel with a gentle outer bank slope as explained in Chapter 3.

$$\underline{\underline{v'\frac{\partial u_0}{\partial n}}}+\underline{\underline{\frac{u_0 v'}{r}}}=-g\frac{\partial H}{\partial s}+\frac{\partial}{\partial z}\left\{\varepsilon\frac{\partial u'}{\partial z}\right\} \quad \cdots\cdots (3)$$

$$-\frac{u_0^2}{r}-2\frac{u_0 u'}{r}-\frac{u'^2}{r}=-g\frac{\partial H}{\partial n}+\frac{\partial}{\partial z}\left\{\varepsilon\frac{\partial v'}{\partial z}\right\} \quad \cdots\cdots (4)$$

where, u_0, v_0 are the depth-average velocity components in the s, n directions, and u', v', are the corresponding fluctuation velocity components changing in the vertical direction.

Equation (3) has terms marked with "═════" which are in addition to equation (2) terms, conventionally used in the flow analysis of a uniformly curved channel. These terms are required for the flow analysis for a gentle outer bank slope.

In the calculations of deformation of riverbed for a uniformly curved channel, the continuity equation for sediment is used.

$$\frac{\partial z_o}{\partial t}+\frac{1}{1-\lambda}\left\{\frac{1}{r}\frac{\partial (r\, q_{Bn})}{\partial n}\right\}=0 \quad \cdots\cdots (5)$$

where, z_o is riverbed height, λ is percentage of void of bed material, q_{Bs}, q_{Bn} show the total sediment load in s, n directions respectively. Mayer-Peter and Muller's equation and Hasegawa's equation (1981) shown below are used here.

$$\left.\begin{aligned} q_{Bs}&=8\sqrt{(\rho_s/\rho-1)gd^3}\,(\tau_*'-\tau_{*c})^{1.5}\\ q_{Bn}&=q_{Bs}\left[\frac{v_b}{u_b}-\sqrt{\frac{\tau_{*c}}{\mu_s\,\mu_k\,\tau_*}}\frac{\partial z_0}{\partial n}\right]\end{aligned}\right\} \quad \cdots\cdots (6)$$

where, τ_*' is dimensionless effective shear stress, τ_{*c} is dimensionless critical shear stress, ρ_s is the density of sand, d is the particle size of bed material, u_b, v_b are velocities near the riverbed in s, n directions, and μ_s, μ_k are the static and dynamic coefficients of friction respectively.

Calculation Method

It is difficult to directly solve the equation of continuity and equations of motion in the uniformly curved channel described by equations (1), (3) and (4). Therefore in this study, to solve the velocity u, v, the velocity distribution in the vertical direction is given by the Fourier series. The coefficients of Fourier series u_o, u_1, u_2, v_o, v_1, v_2 are determined from the equations which are discretized by multiplying the weight of 1, cos $\pi z'$, cos $2\pi z'$ [$z'=(z-z_o)/h$, h is water depth], and by integrating in the water depth direction (Fukuoka et al. 1992).

$$\left.\begin{aligned} u&=u_0+u_1\cos\pi z'+u_2\cos 2\pi z'\\ v&=v_0+v_1\cos\pi z'+v_2\cos 2\pi z'\end{aligned}\right\} \quad \cdots\cdots (7)$$

By substituting equation (7) in equations (3) and (4) and discretizing it by Galerkin's method, the results can be arranged as shown in the following equations.

$$\left.\begin{aligned}
u_0&=\frac{gh}{C_B}\frac{\partial H}{\partial s}\Big/\sqrt{u_b^{\,2}+v_b^{\,2}}-u_1-u_2\\
u_1&=\frac{2h^2}{\varepsilon\pi^2}\left\{g\frac{\partial H}{\partial s}-\underline{\underline{\frac{v_1}{2}\frac{\partial u_0}{\partial n}-\frac{u_o v_1}{2r}}}\right\}\\
u_2&=\frac{h^2}{2\varepsilon\pi^2}\left\{g\frac{\partial H}{\partial s}-\underline{\underline{\frac{v_2}{2}\frac{\partial u_0}{\partial n}-\frac{u_0 v_2}{2r}}}\right\}\\
v_0&=\frac{h}{C_B\sqrt{u_b^{\,2}+v_b^{\,2}}}\\
&\quad\times\left\{\frac{u_0^{\,2}}{r}+\frac{u_1^{\,2}+u_2^{\,2}}{2r}-g\frac{\partial H}{\partial n}\right\}-v_1-v_2\\
v_1&=\frac{2h^2}{\varepsilon\pi^2}\left\{-\frac{u_0^{\,2}-u_0u_1}{r}\right.\\
&\qquad\left.-\frac{u_1^{\,2}+u_2^{\,2}\ u_1u_2}{2r}+g\frac{\partial H}{\partial n}\right\}\\
v_2&=\frac{h^2}{2\varepsilon\pi^2}\left\{-\frac{u_0^{\,2}-u_0u_2}{r}\right.\\
&\qquad\left.-\frac{u_1^{\,2}+u_2^{\,2}}{2r}+\frac{u_1^{\,2}}{4r}+g\frac{\partial H}{\partial n}\right\}
\end{aligned}\right\} \quad \cdots\cdots (8)$$

where, C_B is the coefficient of friction between the riverbed and flow. The terms indicated by "=====" in equation (8) are the terms not considered in equation (2).

By integrating equation (1) in the water depth direction, the equation below is produced, and $v_o=0$ can be obtained:

$$\frac{1}{r}\frac{\partial}{\partial n}(rhv_o)=0 \quad \cdots\cdots (9)$$

From equations (8) and (9), the velocity u_o, u_1, u_2, v_1, v_2 and water level H can be determined. To solve these, it is necessary to give the lateral distribution of u_o, the eddy viscosity coefficient ε and water level variation in the lateral direction, but it is very difficult to find these distributions. To simplify the calculations, initial values are assigned to the velocities u_o, u_1, u_2, v_1, v_2, and the calculations are repeated until the solutions stabilize. While repeating the calculations, the distributions in the lateral direction of $\partial H/\partial n$ and eddy viscosity coefficient ε and u_o can be determined from the velocities calculated by equation (8), and $\partial H/\partial n$ and ε is as indicated below.

The equation below is used for $\partial H/\partial n$, for which $v_o=0$ is used in equation (8).

$$\frac{\partial H}{\partial n}=\frac{1}{g}\left\{C_B\frac{(v_1+v_2)\sqrt{u_b^{\,2}+v_b^{\,2}}}{h}+\frac{u_0^{\,2}}{r}+\frac{u_1^{\,2}+u_2^{\,2}}{2r}\right\} \quad \cdots\cdots (10)$$

The following equation is used for the eddy viscosity coefficient ε.

$$\varepsilon=\frac{\kappa}{6}u_* h \quad \cdots\cdots (11)$$

where, u_* is friction velocity. The shear stress is expressed in terms of velocities at the bed:

$$\left.\begin{aligned} u_*^{\,2}&=\frac{\tau_b}{\rho}=C_B\,(u_b^{\,2}+v_b^{\,2}),\\ u_b&=u_0+u_1+u_2,\quad v_b=v_1-v_2\end{aligned}\right\} \quad \cdots\cdots (12)$$

In order to solve equations (8) and (9), the friction coefficient C_B of the river bed is required. The friction coefficient C_B are divided into two stretches: stretches with gentle slope and stretches other than gentle slope, and based on the results of experiments of depth-average velocity u_o :

$$C_B=\left\{\begin{aligned} &C_{BO},\ \text{stretches other than gentle slope}\\ &C_{BO}\left\{\frac{h}{h_{max}}\right\}^{\frac{r_{max}}{r}},\ \text{stretches with gentle slope}\end{aligned}\right\} \quad \cdots\cdots (13)$$

where, C_{Bo} is the average friction coefficient ($=gn^2/h_o^{1/3}$, n is Manning roughness coefficient; h_o is mean water depth of section), h_{max} is maximum water depth, and r_{max} is the radius of curvature at the place where maximum water depth occurs.

In the calculations of the deformation of the bed, the shear stress at the bed is calculated using the near-bed velocity u_b, v_b. The bed height is then calculated using the sediment continuity equation (5) and the sediment discharge equation (6). The flow calculations are iterated using calculated bed height.

The shear stress for gentle stretches is calculated from the velocity on the slope obtained from the flow calculations. For the bed deformation calculation deposition on slope is considered but not scouring.

3. COMPARISON WITH EXPERIMENTAL RESULTS

In the experiments, a slope of 3H:1V was built at the outer bank side of the channel. An initial bed with a uniformly curved channel as shown in Fig. 1 was used, with central radius of curvature r_c =4.5 m, channel width of 1.0 m and channel length of 24.0 m. The experiment conditions are shown in Table 1. Sand was supplied at the upstream end, water was supplied until the bed reached an equlibrium, and the water level and bed topography were measured after verifying that an equilibrium had been reached. The bed topography was then fixed, water was supplied again, and the distributions of the main current and secondary current were measured.

The change in flow regime and bed shape due to making the slope gentler is shown in Fig.2 and 3.

These figures show that where the sand of the bed is continuously transported by flow, if the slope of the outer bank is made gentler from a vertical wall to 3H:1V, then a deep scour near the outer bank shifts towards the center of the channel. This high-velocity portion which also moves toward the center of the channel from the bank side improves the stability of the bank. The changes in the longitudinal and lateral sectional shapes of the channel also become smaller, the flow is stabilized, the water level does not change and the flow regime can be greatly improved. This occurs because the flow is continuously pushed to the central part after outer bank slope is made gentler, since the bed shape and velocity distribution are improved.

Next, the water level variation and velocity distribution in the lateral direction, and the lateral bed shape obtained by experiment are compared with the results of calculations. The calculation method used is in Chapter 2. In this case, the roughness coefficient was n=0.02, and the static and dynamic coefficients of

Table 1. Experimental conditions of uniformly curved channel

Dischage (l/sec)	18.0
Gradient of initial bed	1/500
Mean water depth (m)	0.047
Bed material (mm)	0.8
Time of supplying water	8 hour
Sand supply (l/min)	0.1

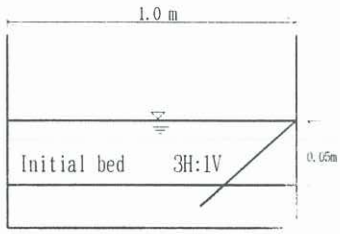

Fig. 1 Cross-section of experiment channel

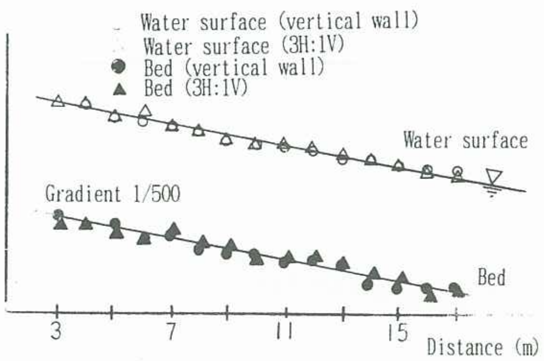

Fig. 2 Mean water level and mean bed height

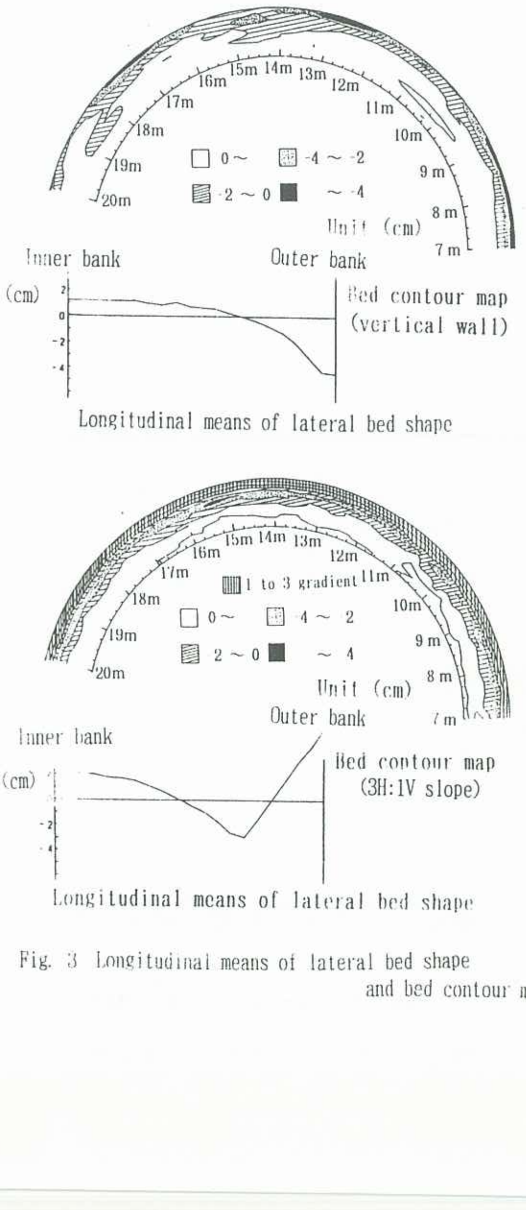

Fig. 3 Longitudinal means of lateral bed shape and bed contour map

friction were μ_s =0.8 and μ_k =0.5, respectively.

Figure 4 compares the calculated values and measured values of water level variation averaged from 10 metres to 18 metres where the cross- section of the bed became almost uniform longitudinally. This graph shows that the calculated results of water level reflects well measured results.

Figures 5 and 6 compare the calculated results and measured results which average the distribution in sections of velocity u and v from 13 metres to 15 metres.

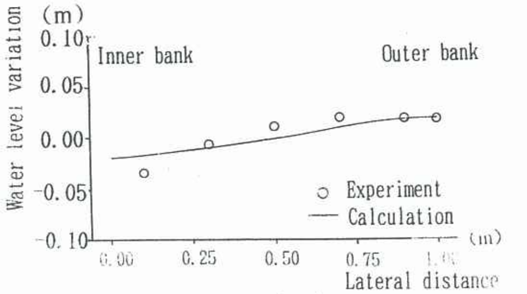

Fig. 4 Distribution of water level variation within section

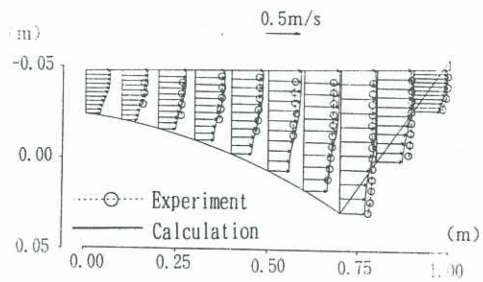

Fig. 5 Distribution of longitudinal velocity within section

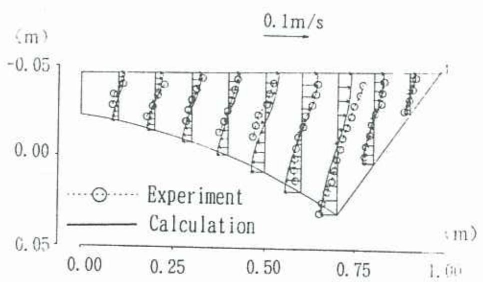

Fig. 6 Distribution of lateral velocity within section

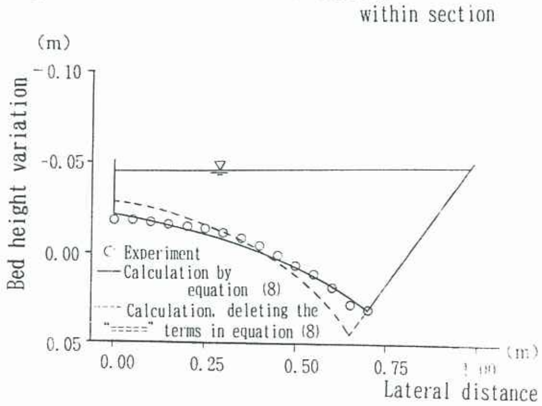

Fig. 7 Comparison of bed shape

These graphs show that the calculations agree quite well with the experiments. Figure 7 compares the calculated results by equation(8) and measured results which average the equilibrium bed shapes in sections from 10m to 18m. This figure also shows the calculated results using equation (2). Figure shows that the calculated bed shape by equation (8) agrees well with experimental bed shape of flow with gentle side slope. The scouring position shifts toward the central part of the channel as a result of making the bank slope gentler. Moreover, in the channel where the bed shape changes due to making the bank slope gentler, the conventional equation of motion (2) is insufficient. The convection terms due to lateral velocity indicated by "=====" in equation(3) become necessary.

4. CONCLUSIONS

The following conclusions can be made from this study

(1) If the sand on a bed moves contionuously even though the slope of the outer bank is made gentler to 3H:1V there is hardly rise in water level as when the side bank is vertical, and the flow regime is improved considerably.

(2) Caluculations and experimental results in an uniformly curved channel were compared. The calculation results almost agree with the water level variation, main current and secondary current distribution of the experimental results, and the effects of improving the flow by making the bank slope gentler can be evaluated from the present theoretical model.

(3) Calculation and experimental results for bed variation were compared, and both agree quite well. The theoretical model can thus simulate bed topography well when the bank slope is made gentler.

(4) For a channel with a gentler slope, the equation of motion conventionally employed is insufficient, and the convection terms, indicated by "=======" of equation(3) are now required.

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