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TRAPPING OF INTERNAL GRAVITY WAVES IN AN INVERSION

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ABSTRACT: Detailed laboratory experiments were carried out to investigate the trapping of internal gravity waves in an inversion layer of a stratified fluid system. The case in which a strongly linearly stratified inversion layer sandwiched between a turbulent layer and a linearly stratified outer layer was considered. The waves were found to trap within the inversion, which, upon saturation, break to produce isolated patches of turbulence, which coalesce with the mixed layer to increase its depth. The experimental results were compared with previous rapid-distortion-theory based theoretical predictions.

1. INTRODUCTION

There are many flows in which the boundaries of a turbulent region are stratified regions. A geophysical example in this context is the planetary boundary layer of the atmosphere, where the upper stratified layer is separated from the underlying convective boundary layer by a strong temperature inversion. Stratification above the inversion is fairly uniform, and up to a certain height the atmosphere can be treated as a three-layer system with a piecewise continuous buoyancy frequency profile N(z) given by

$$\begin{aligned} N(z) &= 0 && \text{for} \quad o \leq z < h \\ &= N_i && \text{for} \quad h \leq z \leq h + \delta \\ &= N_u && \text{for} \quad h + \delta \leq z \leq h_2, \end{aligned} \tag{1}$$

where z is the vertical coordinate measured from the ground, h is the height of the convective boundary layer within which stratification is neglected due to strong turbulence, N_i is the buoyancy frequency within the inversion of thickness δ and N_u ($< N_i$) is the buoyancy frequency of the outer stratification. In addition, similar stratification profiles can exist beneath the wind-stirred upper ocean mixed layers.

The interaction between turbulence and contiguous stably stratified layers plays a dominant role in determining the transport properties of inversion layers. Internal gravity waves, excited owing to such complex interactions, may saturate and break (Fritts, 1984) thus forming intermittent turbulent patches within which effective transport of buoyancy and other scalars takes place (Fritts et al., 1988). Such localized mixing also contributes to the growth of the mixed layer, which in turn affects the depth of the cloud layers and the dynamics of the clouds. The humidity of the entrained air from the inversion is important in determining the microstructure of the clouds that may be present in the boundary layer (Caughey et al., 1982; Baker et al., 1982; Nicholls and Leighton, 1986).

One of the important consequences of the multi-layered structure in a stratified fluid is the radiation and trapping of internal waves (Carruthers and Hunt, 1986). For example, if it is assumed that h_2 is large and N_u is constant with $N_u < N_i$, which is a good approximation for the atmosphere, the resulting three-layer system entails trapping of waves with frequencies $N_u < N < N_i$, which are excited in the intermediate inversion layer but cannot be propagated into the overlying layer. The reflected waves from the boundaries can interact with the waves that are excited at the lower boundary, resonate and break thus causing turbulent patches in the fluid layer which is saturated with internal waves. The trapping of gravity waves is akin to the *ducting* phenomena in the atmosphere (Fritts, 1989) and the complex phenomena of energy exchange between the resonant modes are sometimes referred to as *kissing* or *embracing* (Jones, 1970). Turbulent patches formed due to the breakdown of gravity waves may ultimately lead to the formation of two-dimensional pancake eddies (Lilly, 1983), which appear in the wave number (k) spectrum as a k^{-3} subrange (Nastrom and Gage, 1985). Although there are no explicit in-site measurements in the atmosphere which report the existence of trapped waves, there are instances where a maximum of vertical velocity has been reported in inversion zones, which can be attributed to the trapped waves (Caughey and Kitchen, 1984; Caughey et al., 1982). Fronts arising from turbulent patches have been discerned (Mahrt and Gamage, 1987) and the presence of two-dimensional motions has been reported (Larsen et al., 1982). Observations made using FM/CW radar observations have revealed wave-like features in the inversion layer (Gossard et al 1982). Large-eddy simulations of Carruthers & Moeng (1978) and the laboratory experiments of Fernando and Long (1985) also indicate the possibility of trapped waves in the inversion layer.

The present work deals with the interaction of turbulence and stratification in a three layer system similar to that described by (1). For experimental convenience, the upper layer is maintained turbulent and the layers below are stably stratified. As a first step, shear-free turbulence induced by mechanical means (an oscillating grid) is used to drive the system. Based on linear internal wave theory, one should expect that waves generated at the inversion have frequencies ω satisfying $\omega < N_i$ and that waves radiated into the outer layer have frequencies $\omega < N_u$. Since $N_i > N_u$, it is further expected that waves in the inversion satisfying $N_u < \omega < N_i$ will be trapped within and, owing to the build up of energy, break, thus dissipating energy. The resulting localized mixing should lead to a reduction of the buoyancy gradient within the interface and to the thickening of the turbulent layer.

2. EXPERIMENT

The experiments were carried out in a Plexiglas tank of cross section 40 x 40 cm and height 60 cm. The tank was filled to a depth of 55 cm, with a three-layer stratified fluid system whose stratification is given by (1). A turbulence-generating mono-planer grid that is capable of making vertical oscillations was positioned horizontally, within the layer where $N(z) = 0$. The grid construction utilized square Plexiglas bars of cross section 1 cm x 1 cm and a mesh size of 4.8 cm; the solidity of the grid was 36%. Oscillating frequencies between 2 to 5 Hz were selected and the stroke was set at 2.6 cm. The three-layer stratification was obtained using a mixture of aqueous salt and alcohol solutions. The alcohol and salt concentrations were chosen in order to match the refractive indices of the three layers, thus obtaining an optically-homogeneous, density-stratified, medium suitable for laser-diagnostic techniques. The salt-alcohol refractive-index matching procedure is described in detail by Hannoun et al. (1988). A minute amount of Rhodamine 6G dye was added to the salt water tank during the stratifying process to establish a passive dye tracer gradient proportional to the salt gradient within the stratified layers. Due to the constraints imposed by the requirement of uniform refractivity index, the strengths of the stratification that could be considered were limited. The density stratification was measured using a calibrated micro-scale conductivity probe and a Laser-induced fluorescence (LIF) technique based on Rhodamine dye distribution was used for (spatial) concentration measurements.

3. RESULTS AND DISCUSSION

Albeit the experiments were started with three layers of given thicknesses, a fourth layer (interfacial layer) was found to form immediately between the inversion layer and the turbulent layer. The bulk of the waves were found to be trapped in this newly formed layer and the energy leakage into the inversion was found to be small. Quantitative flow visualization performed by using the LIF technique clearly showed that wave-breaking occurs only in the interfacial layer (which now acts as an inversion of thickness δ) and that wave activity in the formal inversion of thickness h is small. This observation was a persistent feature throughout the experiments, which often lasted more than five hours. Since the properties of the interfacial layer are governed by the Richardson number $Ri = N_i^2 L_H^2 / u_H^2$, it appears that the formal internal Richardson number $Ri_I = N_i^2 \delta^2 / u_H^2$ plays an insignificant role in interfacial mixing; here L_H and u_H are the length and velocity scales of the (undistorted) turbulence, respectively. The LIF flow visualization also revealed the existence of different mixing mechanisms in different Richardson number (Ri) ranges. At low Ri, say Ri < 15, the interfacial mixing takes place owing to the scouring of the interface by the energetic turbulent eddies in the mixed layer. As Ri increases, wave breaking becomes important and, at Ri > 20, mixing is dominated by the breaking of internal waves. At very high Ri (>60), the interface was found to be rather calm and non entraining; here the interface is dominated by molecular-diffusive processes.

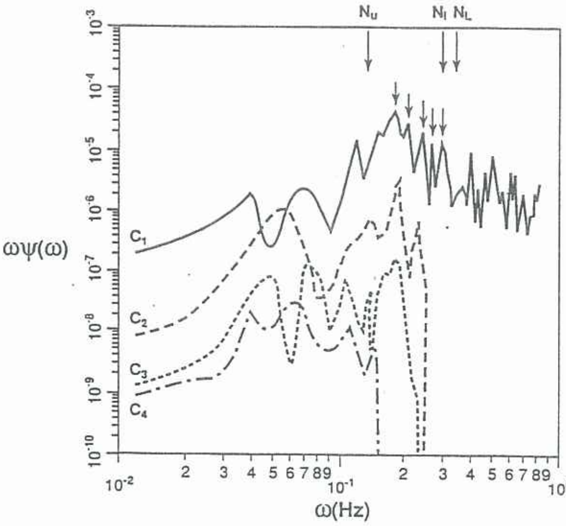

Figure 1: The variance-preserving spectrum of buoyancy fluctuations based on the stationary conductivity-probe measurements. The position of the probes are: Probe C_1 - entrainment interface; Probe C_2 - middle of the inversion layer; Probe C_3 - interface between the inversion layer and the outer stratified layer; and Probe C_4 - middle of the outer layer. The calculated dominant wave frequencies are indicated using vertical arrows; for this case $N_i\delta/u_H = 5.2\pi$ The data were taken 5 minutes after the initiation of grid oscillations; $\varepsilon = 0.47$; Ri = 20. N_L is the buoyancy frequency of the interfacial layer.

The interfacial layer thickness was measured using two techniques. The conductivity measurements showed that the normalized interfacial layer thickness is a slowly varying function of the Richardson number; i.e., $d/L_H \sim Ri^{-1}$. The concentration measurements, on the other hand, showed a much more rapid decrease with Ri; i.e., $d/L_H \sim Ri^{-2}$. The r.m.s. amplitude of the normalized interfacial-wave distortions was found to decrease with Ri as $\left(\overline{\zeta^2}\right)^{1/2}/L_H \sim Ri^{-1}$ and the energy spectrum of the interfacial distortions showed the existence of a region with a -2 slope, in accordance with the predictions of Fernando & Hunt (1992). The r.m.s. fluctuation of normalized interfacial vertical velocity was found to be a decreasing function of Ri as $\left(\overline{w^2}\right)^{1/2}/u_H \sim Ri^{-1}$.

The maxima of the internal wave energy spectra lie between the buoyancy frequencies of the inversion layer N_i and the weakly stratified layer N_u; resonant trapped waves within the interfacial layer could be identified. Rapid-distortion-theory based theoretical analysis of linear gravity waves produced in such a system indicate that the resonant modes should satisfy the criterion (Carruthers and Hunt, 1992)

$$\cot\left(m_1 \delta\right) = -\frac{\gamma}{m_1} + \frac{\left(m_1^2 + \gamma^2\right)}{m_1 (k + \gamma)} \quad , \tag{2}$$

where m_1 is the vertical wave number in the capping inversion,

$$m_1^2 = \left(\frac{N_i^2}{\omega^2} - 1\right)k^2 \quad , \tag{3}$$

and $$\gamma^2 = \left(1 - \frac{N_u^2}{\omega^2}\right)k^2 \quad . \tag{4}$$

The solutions for (2) - (4) indicate that the number of resonant modes excited n satisfies the inequality $(n - 1)\pi < N_i\delta/u_H < n\pi$, where k is the horizontal wave number of turbulent eddies and ω is the frequency. The wave number and frequency of each mode can be evaluated by knowing the ratio $\varepsilon = N_u/N_i$.

The experiments were carried out for different values of N_u/N_i, L_H and u_H and the evolution of the wave field was monitored using the conductivity probes placed in the stratified layers. As shown on Figure 1, distinct resonance modes were identified within the interfacial layer, and the number of modes present and their frequencies were found to be in fair agreement with the theoretical predictions (2)-(4).

Complete description of the experiment and the experimental results is given in Perera, Fernando & Boyer (1992).

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