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TURBULENT BOUNDARY LAYERS AT LOW REYNOLDS NUMBERS

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ABSTRACT

Low-Reynolds-number turbulent boundary layer measurements reported by Erm & Joubert (1991) are discussed and comparisons are made between some of these data and Spalart's (1988) numerical predictions. The experimental data used in the comparisons corresponded to a nominal reference velocity of 10.0 m/s and were for three different tripping devices, viz. a wire, distributed grit and cylindrical pins. In the comparisons, the current experimental data corresponded to approximate values of R_θ of 713 and 1544 and Spalart's numerical predictions were for approximate values of R_θ of 670 and 1410. Mean-flow and broadband-turbulence data were compared. The experimental data and numerical predictions generally showed good agreement.

1. INTRODUCTION

A turbulent boundary layer is considered to be a low-Reynolds-number flow when the Reynolds number based on momentum thickness, R_θ, is less than about 6000. These flows are important in many fluid-flow problems, such as flow through turbomachinery, numerical modelling and model testing in wind tunnels. The flows have been the subject of increasing attention in recent times and have been studied using a number of different approaches, viz. measurements have been taken using pressure probes and/or hot-wire probes, flow visualisation has been employed to investigate the structure of the flow, and flow prediction, using different types of models, has been used. The first and third approaches are considered in this paper.

Comprehensive surveys of literature on low-Reynolds-number turbulent boundary layer measurements have been given by Erm & Joubert (1991) (see also Erm, 1988) and full details of these surveys need not be reproduced here.

Considering high-Reynolds-number flows, an extensive study of mean velocity profile measurements was carried out by Coles (1956) and he proposed that the velocity profile outside the viscous sublayer could be accurately described by

$$\frac{U}{U_\tau} = \frac{1}{\kappa}\ln\left(\frac{yU_\tau}{\nu}\right) + C + \frac{\Pi}{\kappa}w\left[\frac{y}{\delta}\right] \qquad (1)$$

U is the longitudinal mean velocity at a distance y from the surface and U_τ is the friction velocity, given by $U_\tau = (\tau_w/\rho)^{0.5}$ where τ_w is the wall shear stress and ρ the fluid density. ν denotes the kinematic viscosity and κ and C are the logarithmic-law constants which have values of 0.40 and 5.1 respectively. Π is a profile parameter which has a value of approximately 0.55 for zero-pressure-gradient flows. The function $w[y/\delta]$, where δ is the boundary-layer thickness, is termed the "law of the wake".

Turbulent boundary layers in a zero pressure gradient are known to be affected by low Reynolds numbers and over the years investigations have been undertaken to study how the flow changes. Some of these will now be briefly discussed.

Coles (1962) analysed virtually all of the published data on low-Reynolds-number flows on smooth flat surfaces in nominally zero pressure gradients. He identified a normal state for such flows and expressed this state in terms of a relationship between $\Delta U/U_\tau$ and R_θ (see Figure 1), where $\Delta U/U_\tau$ is the maximum deviation of a profile from the logarithmic law. Coles indicated that Equation (1), which was developed for high Reynolds numbers, is valid provided $\Delta U/U_\tau$, or equivalently $2\Pi/\kappa$, varies with R_θ in the specified way. The asymptotic value of $\Delta U/U_\tau$ given by Coles is about 2.7, which corresponds to a value of Π of about 0.55.

Murlis (1975) and Murlis, Tsai & Bradshaw (1982) presented mean-flow and broadband-turbulence quantities for values of R_θ ranging from 791 to 4750. A wire was used for the tripping device and the nominal velocity was 50 ft/s (15.2 m/s). Turbulence quantities for u and v, but not w, were given, where u, v and w denote the fluctuating components of velocity about the mean in the longitudinal or x, normal or y and transverse or z direction respectively. Purtell (1978) and Purtell, Klebanoff & Buckley (1981) presented profiles for mean velocities and turbulence intensities for values of R_θ varying from 465 to 5200. The velocities ranged from 2.3 m/s to 11.6 m/s and two sandpaper tripping devices were used. Smits, Matheson & Joubert (1983) presented mean-flow quantities, but not turbulence quantities, for values of R_θ less than 3000 and the layers were generally tripped using cylindrical pins.

More recently, Erm & Joubert (1991) presented a comprehensive range of mean-flow and broadband-turbulence quantities, as well as spectra, for low-Reynolds-number flows. Although it was known that the flows are affected by the actual low value of R_θ, prior to this research it was not known how they were affected by the type of tripping device used as well as variations in free-stream velocity for a given device. Consequently, the experimental program was devised to investigate systematically the effects of each of these three factors independently. An empirical technique was devised to determine the heights of tripping devices to match a velocity so that the resultant flows were correctly stimulated, i.e. they followed Coles' (1962) curve (see Figure 1). Three different types of device were chosen and these were a wire, distributed grit and cylindrical pins. The nominal free-stream velocities used were 8.0, 10.0 and 14.0 m/s, corresponding to understimulated, correctly stimulated and overstimulated flow respectively. Most measurements were taken for values of R_θ varying between about 715 and about 2810.

Considering prediction of low-Reynolds-number flows, Spalart (1988) used numerical simulation to predict the behaviour of a low-Reynolds-number turbulent boundary layer and presented both mean-flow and turbulence simulations for approximate values of R_θ of 225, 300, 670 and 1410. The three-dimensional time-dependent Navier-Stokes equations were solved using a spectral method with up to about 10^7 grid points and the computations were performed on the NASA Ames Cray computer. With the development of more-

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powerful computers, numerical methods for predicting flow behaviour are playing an increasingly important role in fluid-flow studies. However, for numerical prediction methods to be credible, there has to be some verification of the method by comparing predictions with actual experimental data.

Mean-flow data associated with the initial stages of the current investigation, which are not included in this paper, have been given by Erm, Smits & Joubert (1985). Some of these data were used by Spalart (1988) when he checked his numerical predictions. The low R_θ data of Erm & Joubert (1991) are of considerable interest since they enable further comparisons to be made with Spalart's predictions. This will be done in this paper. When making the comparisons, mean-flow and broadband-turbulence data for the three tripping devices will be used, but the data will only be for correctly-stimulated flows.

In order that the current paper is self contained, some of the work covered in earlier publications will first be revised before comparisons are made with Spalart's predictions.

2. EXPERIMENTAL APPARATUS AND METHODS

The wind tunnel used was an open-return suction type of conventional design. The working section had cross-sectional dimensions of 613 mm by 309 mm at the inlet and was 2.5 m long. It had three fixed walls and an adjustable straight wall which was used to set a nominal zero pressure gradient. The smooth flat vertical surface upon which measurements were taken formed one of the walls of the working section and was opposite the adjustable wall. The free-stream turbulence intensity in the working section was about 0.32% for a free-stream velocity of about 9.5 m/s.

The tripping devices were glued onto accurately-machined metal inserts that could be bolted into a recess in the smooth wall so that the outer surface of an insert was flush with the smooth wall to high accuracy. The centrelines of the wire and pins and the upstream extremity of the grit were located 80 mm downstream of the contraction outlet. This was the origin for all x distances.

To obtain consistent sets of measurements throughout the course of the investigation, reference conditions were set so that they corresponded to a given reference Reynolds number per meter, but to simplify presentation in this paper, reference conditions will simply be referred to in terms of the corresponding nominal reference velocity.

Details of the instrumentation used to take the measurements, as well as the measurement techniques, are given by Erm & Joubert (1991).

3. ESTABLISHMENT OF ACCEPTABLE FLOWS

An empirical technique has been devised to establish correctly stimulated low-Reynolds-number turbulent boundary layers in a zero pressure gradient that show good agreement with Coles' (1962) curve of $\Delta U/U_\tau$ *vs* R_θ (see Figure 1). The heights of the three tripping devices determined by the technique for correct stimulation at a nominal reference velocity of 10.0 m/s, i.e. the chosen design velocity, are given in Table 1, where details of the devices are summarized.

Table 1. Details of Tripping Devices

Wire:	Diameter = 1.2 mm
Grit:	Height approx. 1.6 mm (distance from smooth surface to outermost peaks) Streamwise extent = 50 mm
Pins:	Height = 2.0 mm, diameter = 3.0 mm, spacing = 9.0 mm Pins are of circular cylindrical form

The details of the empirical technique will not be given here, but the resultant $\Delta U/U_\tau$ *vs* R_θ curves for the three devices for nominal reference velocities of 8.0, 10.0 and 14.0 m/s are shown in Figure 1. In all cases the under- and overstimulated data differ noticeably from the design data, whereas for the design flows, all three devices have approximately the same curve and these show good agreement with the curve of Coles. In addition, the balances of momentum for these nine flows were found to be acceptable.

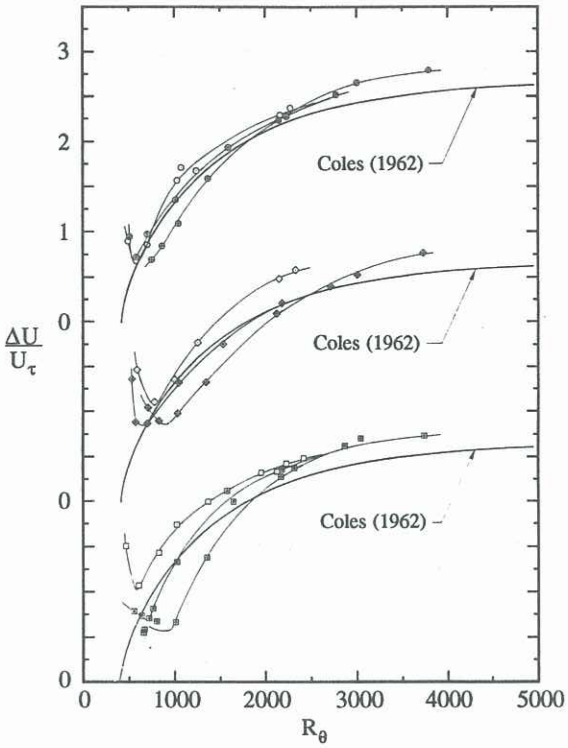

Fig. 1 Variation of $\Delta U/U_\tau$ with R_θ. Note shift in ordinate.
Velocities given below are nominal values.
Wire: ⊙, 8.0 m/s; ◉, 10.0; ⊕, 14.0.
Grit: ⟐, 8.0 m/s; ◈, 10.0; ⟐, 14.0.
Pins: ⊡, 8.0 m/s; ▣, 10.0; ⊞, 14.0.

4. EXPERIMENTAL RESULTS

To enable the above nine flows to be systematically compared with each other, it was essential that measurements were taken in each flow for values of R_θ close to those at which the comparisons were to be made. Measurements corresponding to five different ranges of values of R_θ were taken and details of these are given by Erm & Joubert (1991). The mid point values of R_θ for the five groups are 713, 1020, 1544, 2175 and 2810. By appropriately selecting measurements, it was possible to compare the different flows in such a manner that the effects of R_θ, tripping device and different amounts of stimulation, each considered independently, could be determined.

The mean-flow and broadband-turbulence data showed variations with R_θ, as expected. Profiles were found to be affected very little by the type of device used for $R_\theta \approx 1020$ and above, indicating an absence of dependence on flow history for this range of R_θ. Profiles were also compared at both $R_\theta \approx 1020$ and $R_\theta \approx 2175$ to see if they were dependent on how R_θ was formed (i.e. the combination of velocity and momentum thickness used to determine R_θ). There were noticeable differences for $R_\theta \approx 1020$, but these differences were only convincing for the pins, and there was a general overall improvement in agreement for $R_\theta \approx 2175$.

To give some credibility to the measurements, it was necessary to perform a number of spot checks to verify their accuracy. This involved comparing results, obtained in different ways, to see if they were consistent. The checks were made using the measurements taken with the Pitot probe and the single- and crossed-wire probes for the 1.2 mm wire for the design flow for the most downstream location where the value of R_θ was 2788.

Figure 2 shows mean velocities determined using the single hot-wire probe compared with those using the Pitot probe. The agreement between the results for the two instruments is very good. Close to the wall there is a discrepancy of about 3% between the velocities measured by the two instruments, but these differences diminish as y increases.

Profiles of $(\overline{u^2})^{0.5}/U_\tau$ *vs* $\log(yU_\tau/\nu)$ for the single-wire probe, the crossed-wire probe in uv mode and the crossed-wire probe in uw mode, are shown superimposed in Figure 3. As previously, the alternative sets of results agree very well, and in this case the agreement between the three sets of results is generally within 1 or 2%.

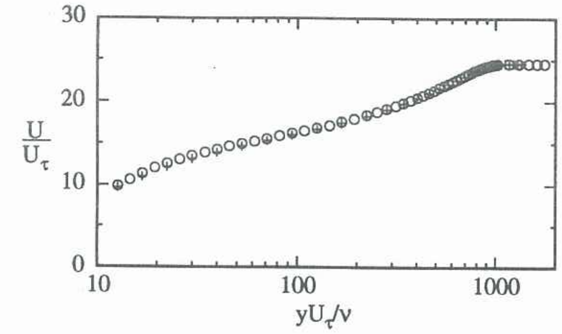

Fig. 2 Mean velocity profiles for $R_\theta = 2788$ for wire tripping device for design flow.
○, Pitot probe; +, single hot-wire

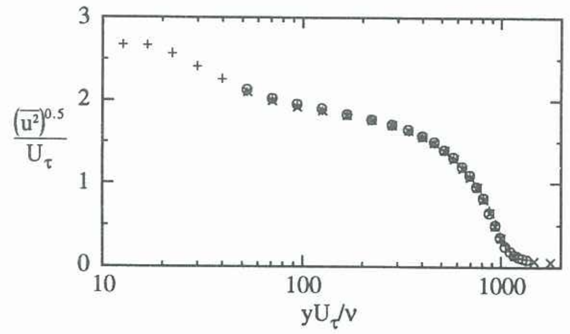

Fig. 3 Turbulence profiles for $R_\theta = 2788$ for wire tripping device for design flow. +, single hot-wire;
○, uv mode crossed hot-wire;
x, uw mode crossed hot-wire

The good agreement in the above checks is encouraging. Furthermore, the current design-flow data agree well with Coles' (1962) curve of $\Delta U/U_\tau$ *vs* R_θ, as shown in Figure 1. The current data are therefore credible and thus subsequent conclusions can be made on a sound basis. For any comparisons between experimental data and Spalart's (1988) numerical predictions to be meaningful, it is essential that credible data be used.

5. EXPERIMENTAL DATA COMPARED WITH NUMERICAL PREDICTIONS

As already indicated, Spalart (1988) used some of the early results of the current investigation, documented by Erm, Smits & Joubert (1985), to check his numerical predictions. Now that later, more comprehensive, experimental data are available, it is possible to make further comparisons with Spalart's predictions.

The experimental data used in the comparisons correspond to a nominal reference velocity of 10.0 m/s and are for the three tripping devices. Current data having approximate values of R_θ of 713 and 1544 are compared with Spalart's predictions for approximate values of R_θ of 670 and 1410 respectively. Both mean-flow and broadband-turbulence data are compared. In the following plots, values of U_τ used to non-dimensionalise the current data were determined using the method of Coles (1962), whereby data were fitted to a log line having constants $\kappa = 0.41$ and $C = 5.0$. These constants comply with those used by Spalart.

Figure 4 shows mean velocity profiles from both investigations compared at the lower and higher values of R_θ. The effect of the device on the profiles is minimal and there is very good agreement between measurements and predictions when semi-logarithmic coordinates are used.

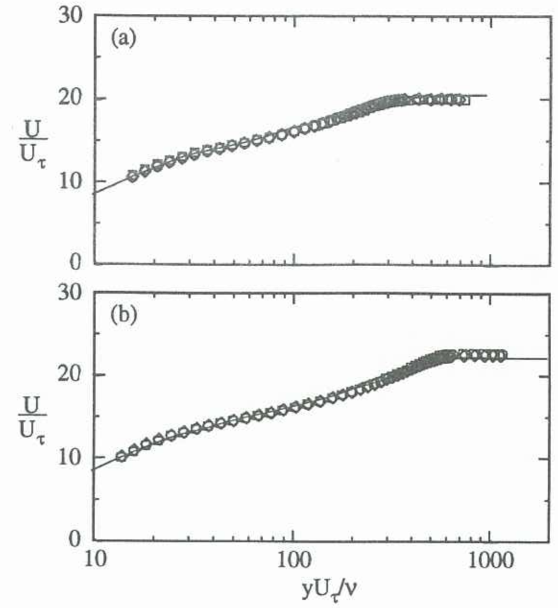

Fig. 4 Mean-flow velocity profiles for design flows for three devices compared with Spalart's numerical predictions
○, wire; ◇, grit; □, pins; ——, Spalart.
(a) $R_\theta \approx 713$ (current data), $R_\theta \approx 670$ (Spalart).
(b) $R_\theta \approx 1544$ (current data), $R_\theta \approx 1410$ (Spalart).

Further comparisons between mean-flow measurements and predictions are shown in Figure 5, where velocity-defect coordinates are now used. U_e is the free-stream velocity at the edge of the boundary layer. For the current data, δ is defined as the location where the velocity is 0.995 of its asymptotic value. Spalart uses a slightly different definition of δ, which is based on the shear stress profile. At $y = \delta$, the value of U/U_e is equal to 0.9974 and 0.9977 for values of R_θ of 670 and 1410 respectively. From Figure 5 it can be seen that the

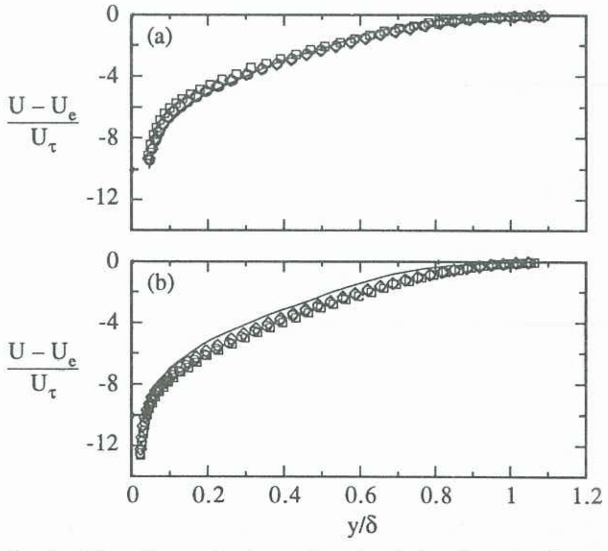

Fig. 5 Mean-flow velocity profiles for design flows for three devices compared with Spalart's numerical predictions
○, wire; ◇, grit; □, pins; ——, Spalart.
(a) $R_\theta \approx 713$ (current data), $R_\theta \approx 670$ (Spalart).
(b) $R_\theta \approx 1544$ (current data), $R_\theta \approx 1410$ (Spalart).

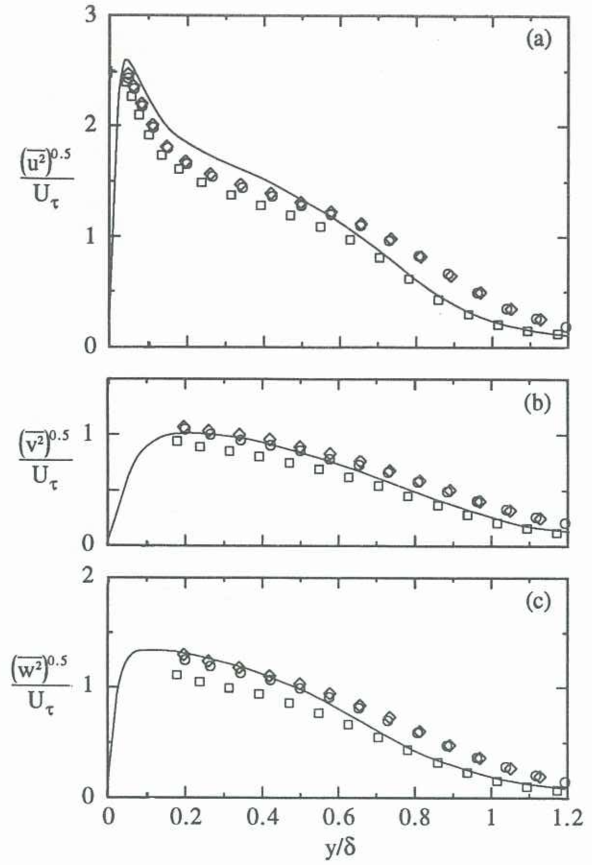

Fig. 6 (a-c) For caption see next column.

effect of device on the profiles is once again minimal. There is very good agreement between measurements and predictions at the lower values of R_θ and reasonable agreement at the higher values of R_θ.

Profiles of root-mean-square turbulence intensities, non dimensionalised by U_τ, for the u, v and w components of the turbulence, are compared in Figure 6. For all three components, there is a significant variation in the experimental profiles at the lower values of R_θ, resulting from using different devices, and thus it is not possible to make precise comparisons with Spalart's predictions. However, it is a matter of interest that the profiles of Spalart are generally located within the range of variation of the experimental profiles. At the higher values of R_θ, the type of device has a negligible effect on the profiles and there is good agreement between measurements and predictions, except perhaps for the u component of the turbulence.

In the above comparisons at the higher values of R_θ, some differences may be attributable to the fact that moderately different values of R_θ were used in the two cases (1544 and 1410), but the R_θ effects have been shown to be only minor.

6. CONCLUDING REMARKS

Comparisons have been made between low-Reynolds-number turbulent boundary layer measurements of Erm & Joubert (1991), having approximate values of R_θ of 713 and 1544, and Spalart's (1988) numerical predictions, for approximate values of R_θ of 670 and 1410. It is encouraging that, for the comparisons made, profiles from the two investigations generally showed good agreement. At the lower values of R_θ, the experimental profiles showed some significant variations, due to using different tripping devices, but the predictions of Spalart were generally within the range of variation of these profiles.

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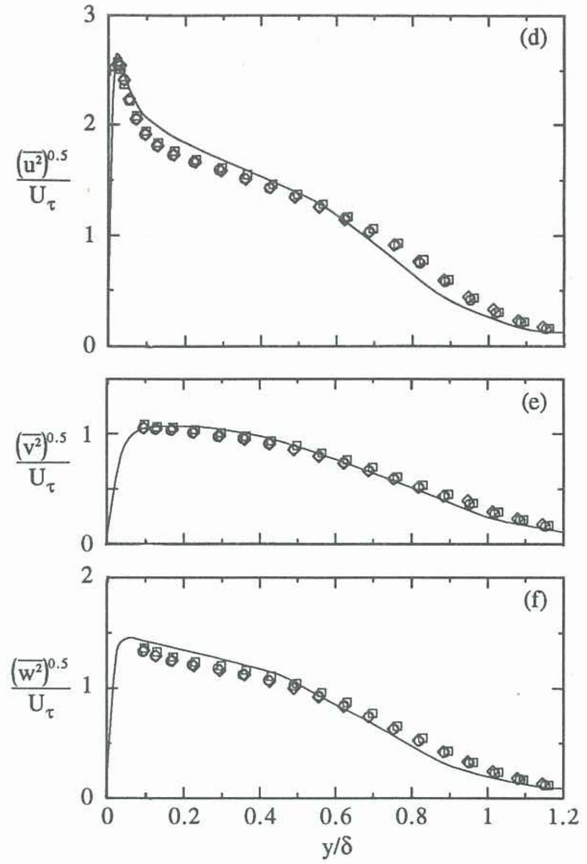

Fig. 6 Turbulence profiles for design flows for three devices compared with Spalart's numerical predictions ○, wire; ◇, grit; □, pins; ——, Spalart. (a-c) $R_\theta \approx 713$ (current data), $R_\theta \approx 670$ (Spalart); (d-f) $R_\theta \approx 1544$ (current data), $R_\theta \approx 1410$ (Spalart)

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