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ON EFFICIENT SHOCK-FOCUSING CONFIGURATIONS

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ABSTRACT

The production of converging cylindrical or conical shocks is discussed. Based upon topological arguments, it was earlier conjectured (Dumitrescu 1983) that it is impossible to convert an initially plane shock into an uniform cylindrical implosion, by shaping a channel, in a plane (2D) configuration; later, Saillard & al. (1985) produced a counterexample. The latters' contruction is hereby given a formal proof of validity; however, for practical purposes, such a design is shown to be inefficient.The commonly-used focusing device (Perry & Kantrowitz 1951) is, on the other hand, fraught with instability problems; a modification of this geometry is therefore proposed, making use of some features of the spiral contraction device, which should improve the stability of imploding shocks.

INTRODUCTION

Shock-wave focusing finds manifold applications, ranging from the colapse of cavitation bubbles to thermonuclear fusion devices.The commonly-used configuration is Perry & Kantrowitz's "teardrop" (1951); Australian researchers have extensively studied the logarithmic spiral (Milton & Archer 1969). At an earlier meeting in Australia, (Dumitrescu 1983), certain developments of the subject have been discussed; in particular, the following problem was posed: is it possible to turn an initially plane shock into an uniform cylindrical implosion, by suitably shaping the channel, in a plane (2D) configuration (by contrast to that of Kantrowitz, which is three-dimensional)?

On topological grounds, it was conjectured that the answer is no; however, Saillard & al. (1985), later come up with a counterexample (but not a proof). Further thoughts on the matter have convinced us that, indeed, our conjecture was unjustified; and we shall hereby give a formal proof to Saillard's construction. However, it will be shown that, for all intents and purposes, such a device is unpractical, and that Kantrowitz's solution remains preferable. On the other hand, this latter is not without certain drawbacks, regarding the stability of the imploding shock. A modification of this geometry will therefore be proposed which, we believe, should generate "cleaner" implosions.

PRODUCTION OF CONVERGING SHOCKS BY WALL-SHAPING

It is well-known that sending a plane shock into a wedge-shaped (or conical) cavity will not generate an uniform cylindrical (or conical) implosion (Setchell & al. 1972): successive diffractions at the cavity walls will constantly disturb the shock, although its strength will continue to increase while it travels into the cavity. The problem is to properly shape the contracting walls. Following, in a slightly modified way, Saillard's argument, let us suppose that the solution has been found, i.e. that, at a certain instant, one has succeeded in producing a cylindrical uniform shock CD, of strength M_1 (Figure 1); this will further propagate, if upstream waves do not come to disturb it, into the wedge-shaped cavity of half-angle 2θ.

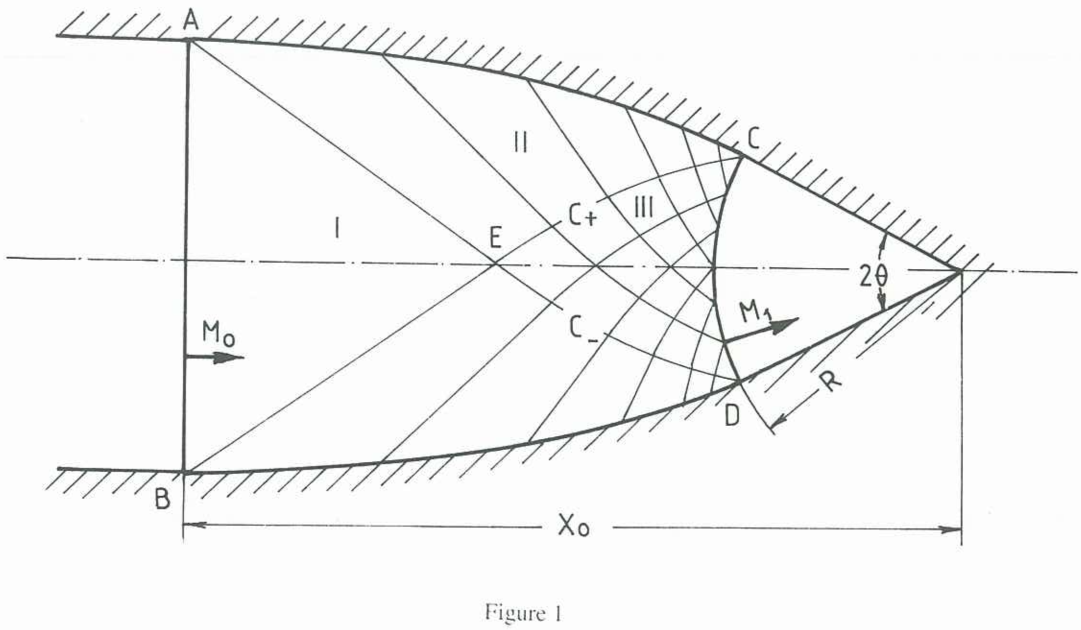

Figure 1

Using Witham's (1951) ray-theory, we are in the presence of a problem in the theory of characteristics for a hyperbolic system of eqs. We see that the shock may be distorted only by perturbations situated within the curvilinear triangle bounded by the two C+ and C_ characteristics issuing from points C, D (the domain of influence of point E). This region (III) must therefore, be one of an uniform cylindrical implosion; the gasdynamic parameters can be determined, since we know the boundary conditions along the circular arc CD, which does not coincide with a characteristic. Such a problem is well-posed (Courant & Friedrichs 1948); moreover, since, looking backwards (upstream), the motion corresponds to an expansion, the charateristics of each family fan out in region III, and will not cross. Actually, the characteristics are identical, being simply shifted along CD; in the strong-shock approximation, they are logarithmic spirals. Thus, the gasdynamic parameters along the characteristic CE are known; in addition, the C_ characteristic AE has to be straight, in order for the state in region I to be uniform (the initial shock AB is supposed straight). Then, for the computation of region II, one recovers again a well-posed problem, in Courant's sense: we are given the boundary-values along two intersecting characteristic lines, C_ (AE) and C+ (EC). In fact, region II is a simple wave, being adjacent to an uniform state (I); the characteristics inside it must all be straight and fanning out (and, therefore, cannot cross), as the gas parameters decrease monotonously along the arc CE.

This completes the proof (and constitutes a rebuttal of our former conjecture); however, the design parametetrs (M_1 and θ) cannot be chosen arbitrarily, as there is a condition to be fulfilled: namely, the characteristics issuing from C and D must actually cross (point E has to exist). This is equivalent to stating that the initial shock Mach number M_0 (equal to M_E), as deduced by the above-described construction, has to be greater than unity; a relation between M_1 and θ results which, in the strong-shock approximation, from Saillard's formulae writes:

$$M_1 > \exp\left[(\nu+1)\theta/\sqrt{n}\right]$$

where: $n = 1 + 2/\gamma + \sqrt{2\gamma/(\gamma-1)}$

$\nu = 0$ for a 2D configuration, $\nu = 1$ for a conical contraction.

SOME PRACTICAL IMPLICATIONS

Two cases of interest might be considered: (i) the penetration of a shock inside a shallow cavity (say, $\theta = 15$ degs); and (ii) the production of fully-developed implosions ($2\theta = 180$ degs, Figure 2). Again in the strong shock approximation, Saillard & al. give formulae for the length and width of the contraction zone; and it appears that a proper shaping of the shock requires a considerable length. In the first case, one gets $x_0/R \approx 4$; but, in such a shallow channel, insisting upon a rigorous contouring is irrelevant, as the shock will anyway be distorted by the wall boundary-layers: a simple rounding would do the job.

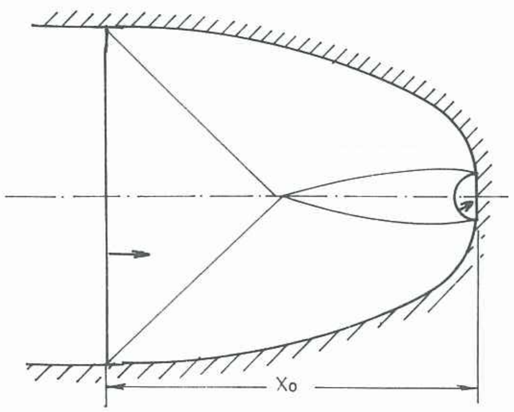

Figure 2.

On the other hand, if one wishes to produce a purely cylindrical shock (case ii), e.g. for investigating its behaviour, the contraction length becomes prohibitive ($x_0/R \approx 120$), and the resulting bubble is much too small; actually, it was impossible to draw Figure 2 to a correct scale. Small contour imperfections, as well as intrinsic instabilities, will destroy the symmetry long before the shock is fully shaped.

We infer that, for all intents an purposes, shock shaping by wall-contouring is impractical, and Kantrowitz's solution is to be prefered. We shall take up some of the latters' own problems; but, first, we shall recall certain points concerning another type of device:

THE SPIRAL CONTRACTION

This is a very efficient means for intensifying a shock; but, it should be stressed that what such a device achieves, is the production of a stronger, but still plane shock, albeit deflected by a certain angle, with minimum distorsion (Dumitrescu 1983).

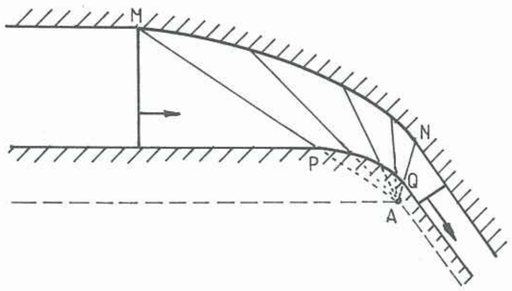

Figure 3.

In the characteristic diagram (Figure 3), all rays emanating from the wall coalesce to point A; however, this is not a cylindrical convergence point. One slight modification of the device, proposed by us earlier, avoids the sudden turn at the corner of the lower flat wall, by replacing this latter with a second contour (PQ), actually homothetic to the upper wall (MN); a smooth turning and strenthening of the shock is achieved, as experimentally demonstrated in our cited paper. We shall see now how these ideas can be used to improve the performance of Kantrowitz's "teardrop".

EFFICIENT GENERATION OF CONVERGING CYLINDRICAL SHOCKS

Figure 4 is a sketch of Kantrowitz' device, copies of which may be found in many laboratories. In all published reports it is stated that, whatever the care taken to minimize all disturbances, the converging shock always displays some degree of instability, which prevents the achievement of a truly point-wise implosion.

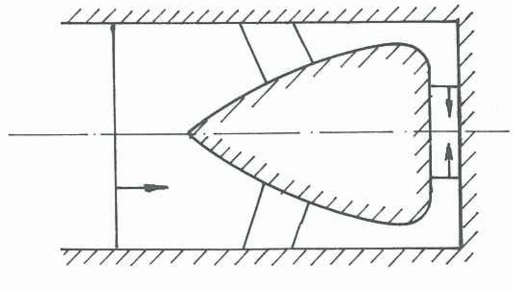

Figure 4.

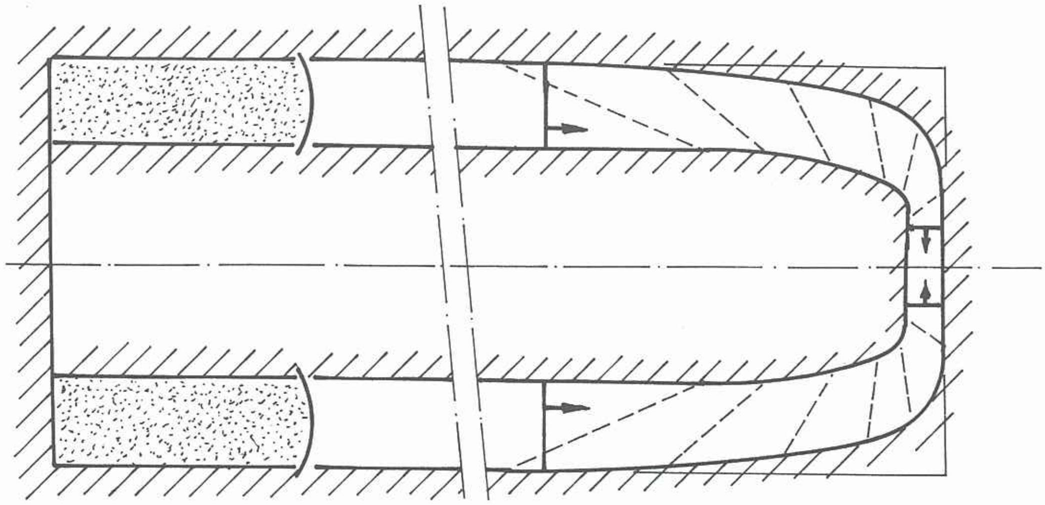

Figure 5.

We believe that the reasons to this are two-fold: (i) one, already discussed in the litterature, is the necessary presence of struts, to support the inner body; these disturb the incoming shock, which cannot fully recover thereafter; (ii) another is the production of disturbances while the shock is made to turn by 90 degs. Many studies of shock propagation in bent ducts (see, eg. Dumitrescu 1966), show that a quite complicated diffraction pattern develops in such cases. These sources of perturbation enhance the intrinsic instability of converging fronts, which, in fact, constitutes the main problem being sought.

Now, a combination of this scheme with an axisymmetric version of the spiral contraction, as sketched in Figure 5, will get rid of disturbances at the corners, as explained above. Whereas the problem of upstream disturbances induced by the struts is automatically solved if one adopts our annular configuration, since the inner body can be supported at the upstream end. An annular diaphragm will have to be employed, made of a material which would shatter upon bursting; scribed metal sheet should be avoided, as a circumferential nonuniformity would then still be imposed.

We believe that the new configuration should display a marked improvement; however, circumstances have prevented us from testing it. As such a modification to an existing device should not be difficult to implement, the author would be pleased to learn that his ideas have been given at least a fair try.

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