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TRANSITION MIXING FOR PLUMES IN A CROSS-FLOW WITH SHEAR

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ABSTRACT

Plumes discharged into an ambient shear flow undergo a transition from self-induced mixing to an ultimate state where mixing is dominated by the shear-flow turbulence. Both plume mixing and ambient shear-flow mixing have been separately well characterized by many previous studies and can be thought of as the asymptotic mixing regimes. However, it is the transitional regime that is often of particular engineering interest and that is not well understood.

In this work, we present some results of an experimental analysis of plume mixing in a turbulent shear flow. Our purpose is to investigate the transition from plume to shear-flow mixing. The experimental technique employs buoyant jets that are optically homogeneous with the ambient shear flow. This enables the combined use of laser-Doppler velocimetry and laser-induced fluorescence to measure the velocity and concentration profiles, respectively.

INTRODUCTION

Many of the models in use by regulatory agencies to describe the dilution of wastes and toxic materials released to the environment depend upon one or the other of two basic approaches:

1. Buoyant jet and plume models that define the dilution of emitted materials in a zone near the point of discharge
2. Dispersion models that use knowledge of ambient turbulence to define the dilution in regions removed from the point of discharge

It is often argued that models which ignore ambient turbulence in consideration of buoyant jet mixing are conservative since ambient turbulence should only enhance dilution. Similarly, when dispersion models are considered, the effect of initial buoyant jet mixing can be viewed simply as a shift in origin of the discharged tracer. However, in many situations it is the transition region of the flow field for which reasonably accurate estimates of the dilution are required.

There have been extensive research studies on turbulent buoyant jets and plumes (*e.g.* see List (1982ab), Papanicolaou and List (1987,1988), and Papantoniou and List (1989) for comprehensive reviews). Much of this prior work has resulted in the development of computer models of jet and plume dilution that are widely used in industry and by regulatory agencies (Muellenhof *et al.*, 1985; Hanna *et al.*, 1982; Schatzmann, 1979).

Dispersion in turbulent flows has received a large amount of attention since the first, and probably the most widely referenced, paper by Taylor (1921). Subsequent work by hundreds of others has resulted in three basic approaches:

(a) Simulation models based on statistical methods (Lamb, 1978; Sawford, 1985ab; Legg and Raupach, 1982)
(b) Diffusion equation models (Raupach and Legg, 1983; Nokes *et al.*, 1984)
(c) Langevin equation models (Raupach, 1983; Pearson *et al.*, 1983)

These models have been combined in many variations of "puff" and "Gaussian plume" models. In addition, there are numerous field studies (*e.g.* Sawford *et al.*, 1985; Gudiksen *et al.*, 1984), and laboratory work (*e.g.* Deardorff and Willis, 1984, Nokes and Wood, 1988). It is interesting that despite all of this work on both jets and plumes and on ambient turbulent mixing, there is apparently little work that relates the two mixing processes to each other.

In this paper, we report on the results of a systematic evaluation of the effects of ambient turbulent kinetic energy on the mixing processes within buoyant turbulent jets.

ANALYSIS

It can be easily shown that a buoyant source with volume flow, $Q=UA$, and specific buoyancy flux, $B=\Delta\rho Q/\rho$, released in a steady uniform ambient flow will attain a dilution

$$S \sim \frac{B^{2/3}x^{4/3}}{UQ}, \qquad x >> \frac{B}{U^3} \tag{1}$$

where x is the horizontal distance from the release point (Fischer *et al.*, 1979). This implies that the dilution at a fixed distance from the source decreases with increasing mean flow velocity. However, the minimum dilution of a continuous source that results from mixing induced by turbulent diffusion is estimated to be (Fischer *et al.*, 1979)

$$S \sim \frac{4\pi D x}{Q} \tag{2}$$

where D is a turbulent diffusion coefficient. The use of a constant diffusion coefficient can be justified using Taylor's (1921) argument, provided that a fluid particle has had the opportunity to sample from all possible scales of the turbulence. In general, D is a function of the flow depth, h, and shear velocity, u_*, which are measures of the turbulence scales. In this case,

$$S \sim \frac{u_* h x}{Q} \tag{3}$$

In other words, the rate of dilution is defined by the shear velocity and the size of the largest eddies. Since u_* increases with U, the dilution will increase with mean cross-flow velocity at a fixed distance downstream from the source point . (It is noted in passing that for normal channel flow

$$\left(\frac{f}{8}\right)^{\frac{1}{2}} = \frac{u_*}{U} \tag{4}$$

where f is the Darcy-Weisbach function factor and for a channel

$$f = 4\left[\frac{2ghsU}{U^3}\right] \tag{5}$$

where s is the channel slope. In this case, f is four times the ratio of the turbulent kinetic energy dissipation rate, $2ghsU$, to the mean flow kinetic energy flux. Thus for constant f, u_* increases with U, the mean flow velocity).

As stated above, Eq (1) indicates that at constant x the dilution must ***decrease*** with increasing mean cross-flow velocity. It is useful to imagine that in this regime, mixing is affected by plume-generated vortical interactions with the cross-flow. These interactions require time to mix the source fluid with the cross-flow fluid. Therefore, when the mean cross-flow velocity increases, source fluid reaches the point of observation, x, faster. The effective amount of time for mixing diminishes and the dilution goes down. In essence, the source fluid is swept past the point of observation before it has time to dilute.

Equations (3) and (4), on the other hand, indicate that at constant x the dilution must ***increase*** with increasing mean cross-flow velocity. In the diffusion regime, a larger turbulent diffusion coefficient enhances mixing and the diffusion coefficient increases with the mean cross-flow velocity. Essentially, in this regime, a higher mean cross-flow velocity implies a more turbulent, or "churning" flow, and dilution improves.

Since both these regimes exist asymptotically, there must be a point at which a transition occurs from plume turbulence-induced mixing to mean-flow turbulence-induced (or shear-flow) mixing. This simplistic argument is the basis of the analysis.

THEORY

It is necessary to determine the point at which the turbulent kinetic energy level of a plume becomes equivalent to the turbulent kinetic energy of the cross-flow. This is accomplished below.

The recent work by Papanicolaou and List (1988) shows that the rms velocity in a plume decays with elevation, y, according to

$$\sqrt{u'^2} = 0.57\left(\frac{B}{y}\right)^{1/3} \tag{6}$$

For channel flow, Nezu and Rodi (1986) provided detailed measurements of the distribution of $\sqrt{u'^2}$ and $\sqrt{v'^2}$, and thus, an estimate of the turbulent kinetic energy distribution. They found that away from the bottom, the rms velocity scales with the shear velocity, u_*, and the channel depth, h. (Near the bottom, the appropriate scaling parameters are $y^+ = yu_*/\nu$ and $u^+ = U/u_*$, as determined by Clauser(1956).) Furthermore, Nezu and Rodi (1986) showed that in a channel, $\sqrt{u'^2}/u_*$ is close to unity. For the experiments reported here, we found that

$$\frac{\sqrt{u'^2}}{u_*} = 0.75$$

It is worth noting that the value of this ratio is critical since it appears in the equations to the third power. Thus, the ambient turbulence and plume turbulence will be of the same order when the plume has risen to a height

$$y \cong 0.45\left(\frac{B}{u_*^3}\right) \tag{7}$$

Since the plume trajectory in a cross flow is controlled by the length scale B/U^3 (Fischer *et al.*, 1979), it follows that cross-over from plume mixing to ambient turbulent mixing will occur at an approximate horizontal distance, x, from the source release point defined by

$$x = \left(\frac{B}{u_*^3}\right) f\left(\frac{U}{u_*}\right) \tag{8}$$

where $f(U/u_*)$ is some function. Alternatively, Briggs (1975) argued that the dissipation is the controlling factor and scaled the dissipation by u_* and y, the distance from the wall, so that the basic result of these two approaches is the same. Clearly, if the buoyant jet has significant initial momentum flux, this will introduce a further parameter, the source Richardson number, $R_O = QB^{1/2}/M^{5/4}$, where M is the specific momentum flux of the buoyant jet (Fischer *et al.*, 1979).

In what follows, we apply the above arguments to the results of the experimental program.

RESEARCH APPROACH

The research is primarily experimental. A series of turbulent buoyant jets were discharged into a turbulent cross flow within a laboratory water channel. The concentration of a tracer material was measured along a line within the discharged fluid using the laser-induced fluorescence method developed by Koochesfahani (1984), and modified and refined by Papantoniou and List (1989). Fluid velocities were measured using laser-Doppler velocimetry developed by Gartrell (1979) and modified by Skjelbraia (1987). These optical techniques were made possible by the use of fluids differing in density but rendered optically homogeneous by the methods developed by McDougall (1979) and modified by Hannoun (1985).

DISCUSSION OF EXPERIMENTAL RESULTS

To establish the region of transition from plume mixing to shear-flow mixing, it is useful to nondimensionalize the data in the following manner. Equation (1) leads to the following result in the plume-mixing regime:

$$S_p = \frac{SUQ}{B^{2/3}x^{4/3}} = c_1 \tag{9}$$

In the shear-flow mixing regime, the dilution is governed by Eq (3) which leads to:

$$S_d = \frac{SQ}{u_* hx} \tag{10}$$

Alternatively, this result can be produced from Eq (9) as follows:

$$S_d = S_p \cdot \frac{B^{2/3} x^{1/3}}{u_* U h}$$
$$= S_p \cdot x^+$$

Hence,

$$S_p = S_d \cdot (x^+)^{-1} \qquad (11)$$

Thus, if we plot S_P vs. x^+, we would expect S_P to be equal to a constant, c_1, in the plume-mixing regime since it is independent of x^+. However, as the region dominated by shear-flow is approached, where S_d tends to a constant, we would expect to see an inverse relationship between S_P and x^+.

Figure 1 clearly shows both of these regimes. The wide range of values for the constant c_1 might be explained in two ways. First, the accuracy of the absolute dilution is limited by the calibration procedure. Second, the value of c_1 is sensitive to initial momentum flux effects as suggested by Wright (1977), who also observed a range of values for c_1. In fact, it seems likely that the variance in c_1 can be fully explained by initial jet mixing prior to the onset of the plume-mixing regime. The transition from plume-mixing to shear-flow mixing, however, is still easily seen since the relative dilutions within any one *set* of experiments (indicated by like symbols in Fig. 1) using the same calibration is accurate.

Figure 1 shows that the transition from plume mixing to shear-flow mixing occurs at a constant value of x^+. This leads to the following equation for the distance, x, at which transition occurs

$$x = \frac{\left(x^+ u_* U h\right)^3}{B^2} \qquad (12)$$

where x^+ is obtained from Fig. 1 and ranges from 0.06 to 0.1.

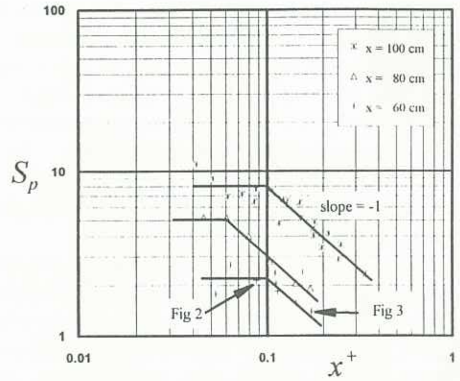

Figure 1. Transition from plume-mixing regime to shear-flow mixing regime

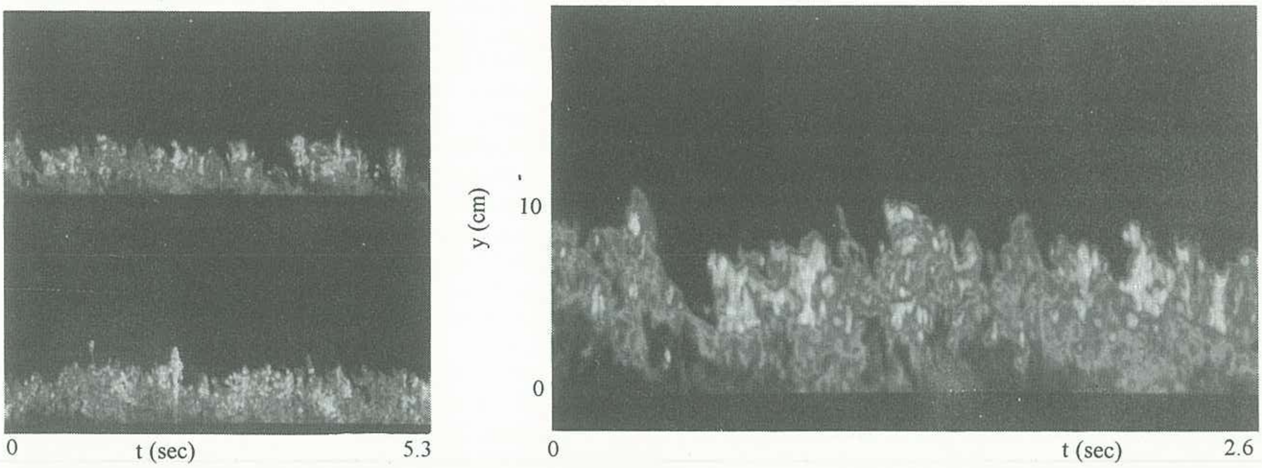

Figure 2. Color-enhanced reproduction of flow in plume-mixing regime (red indicates a value of C/Co = 0.1)

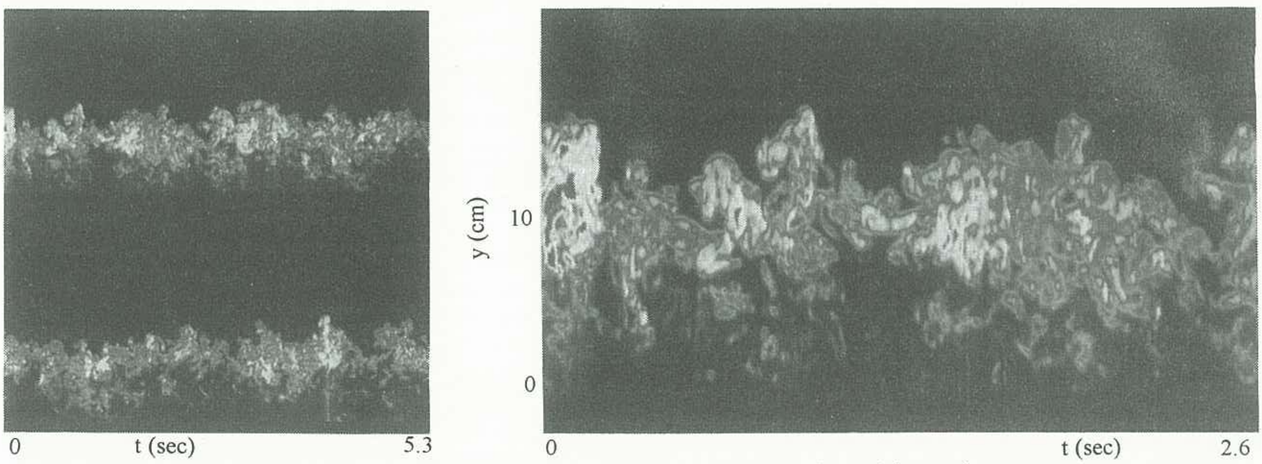

Figure 3. Color-enhanced reproduction of flow in shear-flow mixing regime (red indicates a value of C/Co = 0.1)

Figures 2 and 3 are color-enhanced reproductions, obtained by laser-induced fluorescence, of the flow in the two regimes. To produce these images, a laser line is imaged onto a photo-diode array to create a two-dimensional (time and space) representation of the flow along a vertical line through the centerline of the buoyant jet.

The "puff-like" nature of the flow in Fig. 3 closely resembles the flow behavior typically attributed to plumes. However, in actuality, Fig. 3 shows the flow in the shear-flow regime and Fig. 2 shows the plume regime. This is indicated in Fig. 1.

The source flow conditions were identical for the flows depicted in Figs. 2 and 3. Additionally, the observation location was the same, only the cross-flow velocity was different. Clearly, the plume shown in Fig. 2 met a higher cross-flow velocity than did the plume shown in Fig.3. As discussed above, a higher cross-flow velocity means the tracer fluid is swept past the point of observation faster. In this case, the effect is that the turbulent kinetic energy of the plume still dominates the mixing. In other words, the plume turbulent kinetic energy has not had time to decay.

Interestingly, regions of peak concentration far exceeding mean values persist well into the shear-flow mixing regime as shown in the color-enhancements. This calls into question the validity of using dispersion-type models based on Gaussian concentration profiles to predict peak concentration values even when the flow is assumed to be far into the shear-flow mixing regime.

This result indicates that both plume-mixing and dispersion-type models may provide a false sense of security with regard to the minimum instantaneous dilutions observed in actual flow situations.

REFERENCES

BRIGGS, G A (1975) Plume rise predictions. In Lectures on Air Pollution and Environmental Impact Analyses, D A Haugen, ed, Am Met Soc, Boston, pp 59-111.

CLAUSER, F H (1956) The turbulent boundary layer. Adv in Appl Mech, IV: 2-51.

DEARDORFF, J W and WILLIS, G E (1984) Ground level concentration fluctuations from a buoyant and non-buoyant source within a laboratory convectively mixed layer. Atmos Environ, 18(7):1297-1309.

FISCHER, H B, LIST, E J, KOH, R C Y, IMBERGER, J, and BROOKS, N H. (1979) Mixing in Inland and Coastal Waters, Academic Press, 483p.

GARTRELL, G (1979) Studies on the mixing in a density-stratified shear flow. PhD Thesis, California Institute of Technology.

GUDIKSEN, P H et al., (1984) Field studies of transport and dispersion of atmospheric tracers in nocturnal drainage flows. Atmos Environ, 18(4):713-731.

HANNA, S R, BRIGGS, G A, and HOSKER, R P (1982) Handbook on Atmospheric Diffusion, U S Dept of Energy Report DOE/TIC - 11223, 102p.

HANNOUN, I A (1985) Matching the refractive index in density-stratified flows. Tech Memo. 85-1, W.M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology.

KOOCHESFAHANI, M M (1984) Experiments on turbulent mixing and chemical reaction in a liquid mixing layer. PhD Thesis, California Institute of Technology, Pasadena, CA.

LAMB, R G (1978) A numerical simulation of dispersion from an elevated point source in the convective boundary layer. Atmos Environ, 12:1297-1304.

LEGG, B J and RAUPACH, M R (1982) Markov-chain simulation of particle dispersion in inhomogeneous flows: The mean drift velocity induced by a gradient in Eulerian velocity variance. Boundary-layer Meteorology 24:3-13.

LIST, E J (1982a) Turbulent jets and plumes. Ann Rev Fluid Mech 14:189-212.

LIST, E J (1982b) Mechanics of turbulent jets and plumes pp 1-68 in Turbulent Buoyant Jets and Plumes, W Rodi ed, Pergammon.

McDOUGALL, T J On the elimination of the refractive index variations in turbulent density-stratified liquid flows. J Fluid Mech, 93:83-96.

MUELLENHOF, W P *et al.* (1985) Initial mixing characteristics of municipal ocean discharges. Vol. 1, Procedures and Applications. EPA/600/3-85/073a, November.

NEZU, I and RODI, W (1986) Open-channel flow measurements with a laser Doppler anemometer. J Hyd Eng, ASCE, 112:335-355.

NOKES, R I, MCNULTY, A J and WOOD, I R (1984) Turbulent dispersion from a steady two-dimensional horizontal source. J Fluid Mech, 149:147-159.

NOKES, R I, and WOOD, I R (1988) Vertical and lateral turbulent dispersion some experimental results. J Fluid Mech, 187:373-394.

PAPANICOLAOU, P A and LIST, E J (1988) Investigations of round turbulent buoyant jets. J Fluid Mech, 195:341-391.

PAPANICOLAOU, P A and LIST, E J (1987) Statistical and spectral properties of tracer concentration in round buoyant jets. Intl J Heat and Mass Trans, 30:2059-2071.

PAPANTONIOU, D and LIST, E J (1989) Large-scale structure in the far field of buoyant jets. J Fluid Mech, 209:151-190.

PEARSON, H J, PUTTOCK, J S, and HUNT, J C R (1983) A statistical model of fluid element motions and vertical diffusion in a homogeneous stratified turbulent flow. J Fluid Mech, 129:219-249.

RAUPACH, M R and LEGG, B J (1983) Turbulent dispersion from an elevated line sources: Measurements of wind concentration moments and budgets. J Fluid Mech, 136:111-137.

RAUPACH , M R (1983) Near-field dispersion from instantaneous sources in the surface layer. Boundary-layer Meteorology, 27:105-113.

SAWFORD, B L (1985a) Lagrangian statistical simulation of concentration mean and fluctuation fields. J of Climate and Appl Met, 24(11):1152-1166.

SAWFORD, B L (1985b) Concentration statistics for surface plumes in the atmospheric boundary layer. Proc 7th Symp on Turb and Diff AMX, Boulder, CO, Nov 12-15.

SAWFORD, B L, Frost, C C and Allan, T C (1985) Atmospheric boundary layer measurements of concentration statistics from isolated and multiple sources. Boundary-layer Meteorology, 31:249-268.

SCHATZMANN, M. (1979) An integral model of plume rise. Atmos Environ, 13:721-731.

SKJELBRAIA, J E (1987) Observations of breaking waves on sloping bottoms by use of laser-Doppler velocimetry. PhD Thesis, California Institute of Technology.

TAYLOR, G I (1921) Diffusion by continuous movements. Proc Lond Math Soc, A20:196-211.

WRIGHT, S J (1977) Effects of ambient crossflows and density stratification on the characteristic behavior of round, turbulent buoyant jets. PhD Thesis, California Institute of Technology.