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CORRECTIONS FOR FINITE-WATER-DEPTH EFFECTS ON SHIP RESISTANCE

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Abstract

The prediction of ship resistance during steady forward motion has been the subject of research for over a century. Both experimental and theoretical approaches have been proposed for the prediction of full-size drag.

To be presented in this paper is a novel technique of using wave-resistance theory, suitably modified to account for the tank dimensions, to *adjust* towing-tank data. It is shown that the combined method can accurately predict the influence of water depth. Thus, this method can be used to *either* correct experiments in which the tank has the wrong dimensions *or* predict the effect of operating the full-size vessel in varying depths of water.

Introduction

Background

The subject of ship resistance is one which has been studied for over a century now. The work of Michell (1898) was the first which resulted in a usable formula for the *wave* resistance for a ship travelling at a constant speed in deep water. The assumptions in his theory were that the effects of viscosity and surface tension could be ignored. Additionally, the ship was considered to be *thin*.

The wave resistance is defined as the drag associated with generating the wave pattern in the neighborhood of the vessel. In addition to this component of drag, one must add the *viscous* resistance, which can be estimated by one of the flat-plate skin-friction formulas.

More recently, nonlinear wave-resistance theories have been proposed. These theories, which require the development of extensive computer programs have been described, for example, in the series of International Conferences on Numerical Ship Hydrodynamics and Symposiums on Naval Hydrodynamics, both of which are organized by the Office of Naval Research in Washington, DC. These approaches allow one to evaluate the resistance of a thick ship.

Current Work

The work to be described has its origins in a series of collaborative papers by Doctors (1989), Renilson (1989), Doctors, Renilson, Parker, and Hornsby (1991) and Kovacevic (1991). There, both catamarans and a monohull were tested in a towing tank in water of various depths. Attempts to correlate the experimental results for the resistance with the linearized theory were made. It was found that the theory could be used quite accurately to predict the effects of *changes* in the water depth or the spacing between the demihulls of a catamaran.

The intention now is to describe a more detailed series of numerical and experimental investigations in which the concept of using the simple theory to bridge from one water depth to another is examined in detail.

Analytic Work

Linearized Theory

In the current work, the theory of Michell, as extended by Lunde (1951) for a river or canal with a rectangular cross section, has been used. That is, the effects of finite water depth *and* lateral restriction on the width of the waterway are included. The formulation is very similar

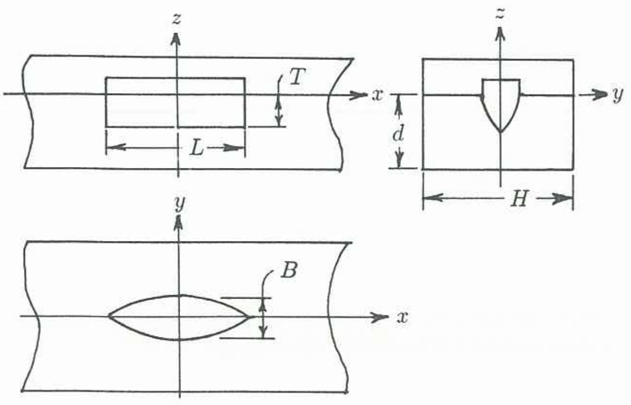

Figure 1: The Towing Tank

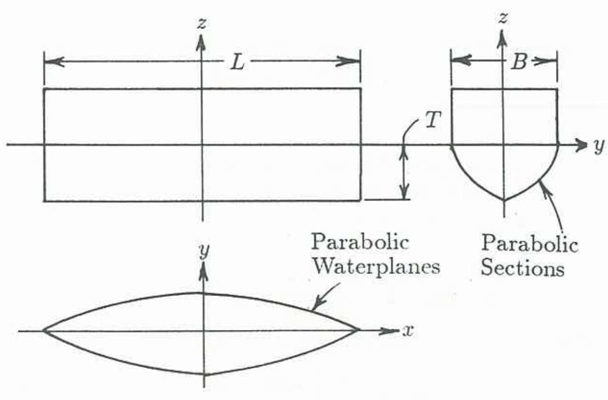

Figure 2: The Wigley Hull

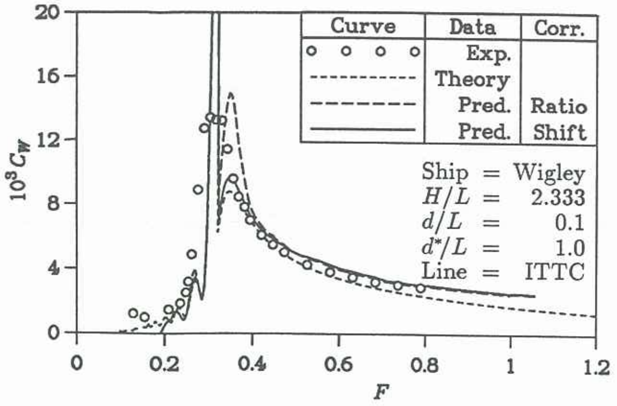

Figure 3: Experiment, Theory and Prediction
(a) $d/L = 0.1$

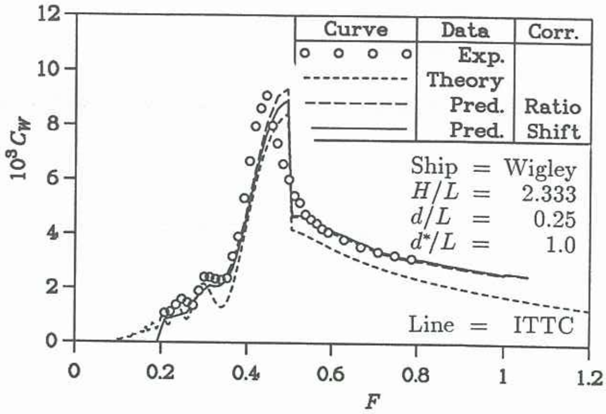

Figure 3: Experiment, Theory and Prediction
(d) $d/L = 0.25$

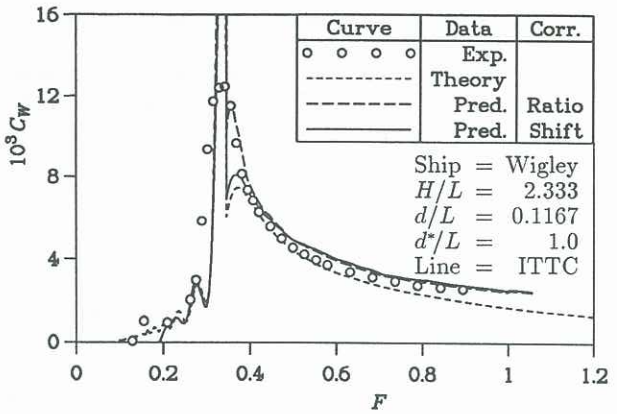

Figure 3: Experiment, Theory and Prediction
(b) $d/L = 0.1167$

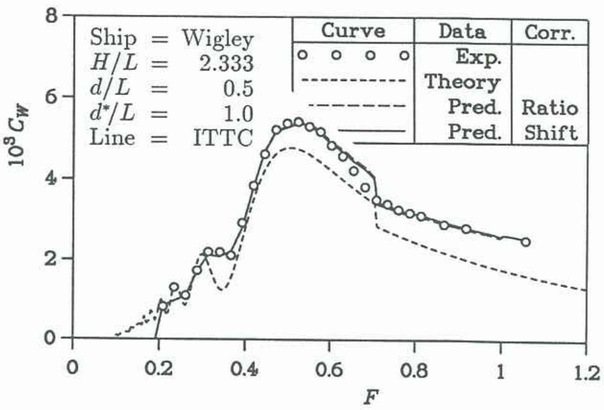

Figure 3: Experiment, Theory and Prediction
(e) $d/L = 0.5$

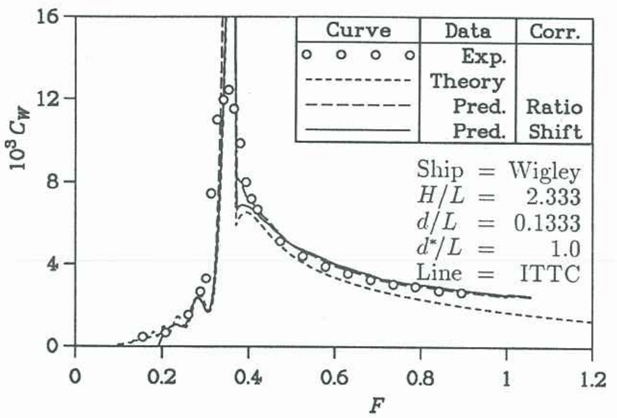

Figure 3: Experiment, Theory and Prediction
(c) $d/L = 0.1333$

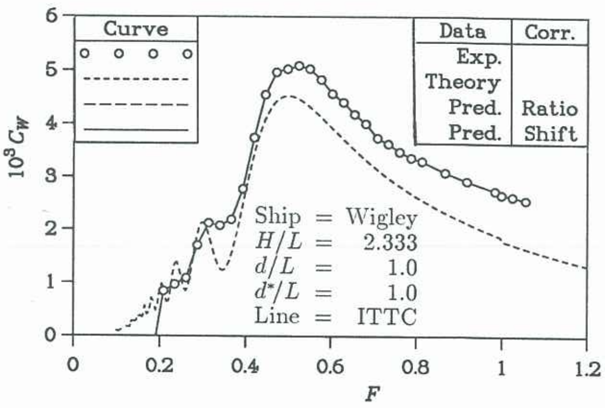

Figure 3: Experiment, Theory and Prediction
(f) $d/L = 1.0$

to that of Doctors (1972) for an air-cushion vehicle.

The experimental setup is shown in Figure 1. The general formula for the wave resistance in a channel of width H and depth d is

$$R_W = \frac{2\rho g}{H} \sum_{i=0}^{\infty} \epsilon_i \frac{w^2 k (U^2 + V^2)}{2k - k_0 \tanh(kd) - k k_0 d \,\mathrm{sech}^2(kd)}, \quad (1)$$

where

$$\epsilon_i = \left\{ \begin{array}{ll} \frac{1}{2} & \text{for } i = 0 \\ 1 & \text{for } i \geq 1 \end{array} \right. \quad (2)$$

and ρ is the water density and g is the acceleration due to gravity.

The longitudinal and transverse wave numbers in Equation (1) are

$$w = \sqrt{k^2 - u^2}, \quad (3)$$
$$u = 2\pi i / H, \quad (4)$$

while the circular wave number k is given by the solution of the implicit dispersion relationship:

$$f = k^2 - k k_0 \tanh(kd) - u^2 \quad (5)$$
$$= 0, \quad (6)$$
$$df/dk = 2k - k_0 \tanh(kd) - k k_0 d \,\mathrm{sech}^2(kd). \quad (7)$$

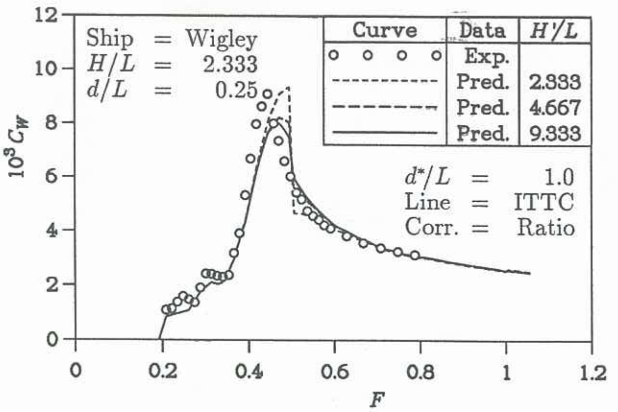

Figure 4: Influence of Artificial Tank Width
(a) $d/L = 0.25$

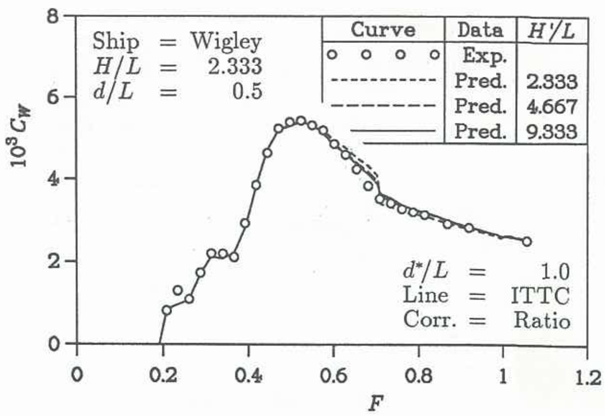

Figure 4: Influence of Artificial Tank Width
(b) $d/L = 0.5$

Finally, the fundamental wave number is

$$k_0 \;=\; g/U^2\,, \tag{8}$$

where U is the speed of the ship.

The index i of the summation in Equation (1) has been dropped from all the symbols except ϵ, for the sake of brevity.

We next consider the two finite-depth wave functions in Equation (1), which are

$$U \;=\; [P^+ + \exp(-2kd)P^-]/[1+\exp(-2kd)]\,, \tag{9}$$
$$V \;=\; [Q^+ + \exp(-2kd)Q^-]/[1+\exp(-2kd)]\,, \tag{10}$$

in which the Michell deep-water wave functions $P^{\pm}$ and $Q^{\pm}$ are defined by

$$P^{\pm} + \mathrm{i}Q^{\pm} \;=\; \int_{S_0} \mathcal{B}(x,z)\exp(\mathrm{i}wx \pm kz)\,\mathrm{d}x\mathrm{d}z\,. \tag{11}$$

Here, x and z are respectively the longitudinal and vertical coordinates and the integration is to be performed over the centreplane area S_0.

The hull defined by Wigley (1934) was used here. The hull has parabolic sections and waterplanes, as shown in Figure 2. The local beam is defined by the formula

$$\mathcal{B} \;=\; B[1-(2x/L)^2][1-(z/T)^2]\,, \tag{12}$$

where L is the length, B is the beam and T is the draft. Because the dependence on x and z can be separated, this hull is called a "simple" ship and the first wave function in Equation (11) can be expressed as

$$P^{\pm} \;=\; P^{(x)}P^{(\pm,z)}\,, \tag{13}$$

while the second wave function $Q^{\pm}$ is zero because of fore-and-aft symmetry. The x-dependent factor in Equation (13) can be obtained by analytic integration of Equation (12) in Equation (11) to give

$$P^{(x)} \;=\; -\frac{4\mathcal{B}}{wA}\,[\cos(A)-\sin(A)/A]\,, \tag{14}$$
$$A \;=\; wL/2\,. \tag{15}$$

Similarly, the result for the z-dependent factor is

$$P^{(\pm,z)} \;=\; \pm\frac{1}{k}[1-2/C^2+2\exp(\mp C)(1/C^2 \pm 1/C)]\,, \tag{16}$$
$$C \;=\; kT\,. \tag{17}$$

Method of Applying Correction

Two approaches for correcting the resistance for the influence of water depth were tried. In the first method, the assumption was made that the influence was to alter the *ratio* of the wave resistance. That is,

$$R_W^{\text{Pred.}}(F,d) \;=\; \frac{R_W^{\text{Theory}}(F,d)}{R_W^{\text{Theory}}(F,d^*)} \times R_W^{\text{Exp.}}(F,d^*)\,. \tag{18}$$

In Equation (18) the experiment is done with a base water depth d^* and the prediction for the resistance at a different depth d is computed at the same Froude number F. The latter is defined in the usual way as

$$F \;=\; U/\sqrt{gL}\,. \tag{19}$$

In order to be able to effect the prediction using Equation (18), one must first subtract the frictional resistance. The frictional drag on the model was computed on the basis of the 1957 International Towing Tank Committee (ITTC) formula, described by Lewis (1988, Section 3.5).

In the second approach, the assumption was made that the influence of depth was to cause a *shift*, or difference, in the wave resistance. That is,

$$R_W^{\text{Pred.}}(F,d) \;=\; R_W^{\text{Theory}}(F,d) - R_W^{\text{Theory}}(F,d^*) + R_W^{\text{Exp.}}(F,d^*)\,. \tag{20}$$

It is interesting to note that using different formulations for the frictional drag will alter the result given by Equation (18). On the other hand, the result of Equation (20) is *unaffected* by the choice of method for the friction calculation.

Results and Discussion

The Wigley model tested had a length of 1.5 m. It had the standard beam-to-length ratio B/L of 0.1 and the standard draft-to-length ratio T/L of 0.0625. A series of six depths was considered. The results for the wave resistance are shown in Figures 3(a) through (f), respectively. The ordinate is the wave-resistance coefficient, defined in the usual way, as

$$C_W \;=\; R_W/\frac{1}{2}\rho U^2 S\,, \tag{21}$$

where S is the wetted-surface area.

The dimensionless base depth d^*/L for use in Equation (18) and Equation (20) was unity. Thus, there is per-

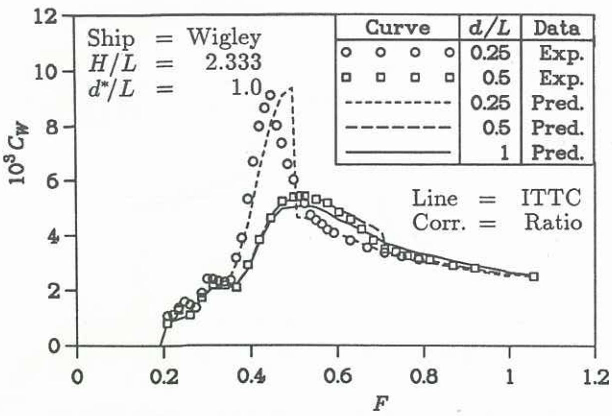

Figure 5: Prediction of the Influence of Depth (a) Ratio Method

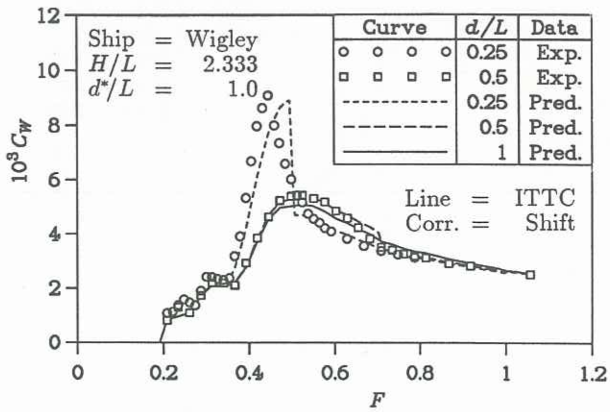

Figure 5: Prediction of the Influence of Depth (b) Shift Method

fect agreement for both types of correction in Figure 3(f), where the depth $d = d^*$.

In examining the six parts of Figure 3, we can see that use of the straight theory substantially underpredicts the wave resistance at high Froude numbers. On the other hand, it predicts unrealistically high values of the resistance when the depth Froude number, given by

$$F_d = U/\sqrt{gd}\,, \tag{22}$$

equals one.

Next, we observe that substantial improvement in agreement with the experiments is obtained by using either of the two corrective approaches described above. Indeed, the agreement is within a few percent at high Froude numbers. At the lower Froude numbers, the interesting jump in the resistance curve at a depth Froude number of unity (which corresponds to a Froude number that depends on the depth) is also predicted well — particularly for the depth-to-length ratios d/L of 0.25 and 0.5 in Figures 3(d) and (e). In very shallow water, such as in Figure 3(a), the prediction of the correction technique has deteriorated.

The corrective method predicts sharp jumps in the wave resistance in Figures 3(d) and (e), which are not observed in practice. An attempt to moderate the predictions was made by using a modified version of Equation (18), in which an *artificially greater* tank width H' was used for the theory. Figures 4(a) and (b) show the effects of using H'/H values of 1, 2 and 4. There is clearly further improvement in the accuracy of the predictions in the neighborhood of the critical speed and it seems that further development of this idea is warranted.

Finally, in the two parts of Figure (5), we see some of the same data presented in a different way. The ratio method in Figure 5(a) and the shift method in Figure 5(b) are both seen to predict the influence of water depth very well.

Conclusions

The research described here shows that there are worthwhile gains to be made in the standard wave-resistance theory, by using simple intuitive corrections. The methods can be used in at least two modes. In the first, they can be used to correct towing-tank data, where either the tank width or tank water depth does not have the desired value. In the second mode, results for other cases for the full-size vessel can be obtained without having to rerun the model tests. In this regard, it should be emphasized that the accuracy is higher for the greater water depths.

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