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A TURBULENT LOW REYNOLDS NUMBER k-ε MODEL FOR RIBLET FLOWS

L. DJENIDI and R.A. ANTONIA

Department of Mechanical Engineering
University of Newcastle, NSW 2308, AUSTRALIA

ABSTRACT

An existing low Reynolds number $k-\epsilon$ model is adapted to the calculation of a turbulent boundary layer over a riblet surface. Using orthogonal curvilinear co-ordinates, the equations are solved by a control finite volume procedure coupled with an x-marching method and an iteratively modified strongly implicit algorithm.

The calculated Reynolds shear stress and the average turbulent kinetic energy are consistent with available measurements. However, the return to two-dimensionality as the distance from the wall increases is reproduced inaccurately and the calculated total drag reduction is significantly underestimated, thus underlying the need to improve the near-wall performance of the present model.

INTRODUCTION

A significant amount of experimental work has been done for a turbulent boundary layer over riblets (small longitudinal wall grooves), the general concensus being that a drag reduction of about 8% is possible. In the context of industrial applications, Coustols and Schmitt (1990) have reported that wind tunnel experiments on a model of the Airbus A320 indicate important total drag reductions under cruise conditions. These authors also allude to a favourable performance of the riblets obtained from flight tests of the A320.

Details of the mechanism by which the drag reduction is achieved are not yet understood, mainly due to the lack of reliable experiments within the wall region above the riblet grooves. Experimental attempts (Vukoslavcevic et al., 1988; Benhalilou et al., 1991) to study the flow within and above the grooves have yielded useful results but have not allowed the physics behind the drag reduction to be unravelled. It should be noted that measurements within the grooves are extremely difficult to carry out.

Several numerical studies of riblet flows have been reported. Khan (1986), Djenidi et al. (1991) and Benhalilou et al. (1991) used a mixing length eddy viscosity model. The discrepancies between measurement and calculation indicate that such an approach is inadequate for riblet flows, and a more refined model closure is required.

Launder and Li (1991) used a $k-\epsilon$ model (Launder and Sharma, 1974) to calculate a turbulent channel flow over a riblet walls. The model failed to reproduce the measured drag reduction. Benhalilou's (1992) algebraic stress model also exhibited a number of shortcomings, especially with regard to the calculation of the drag reduction, k and $\overline{uv}$. The present study is an attempt to adapt an already existing $k-\epsilon$ model (Chien, 1982) to the calculation of a turbulent boundary layer over a riblet wall.

COMPUTATIONAL GRID

The V-groove shape considered here is shown in Figure 1 (the flow direction is perpendicular to the $y-z$ plane; ony half a riblet is shown) where θ is the half-angle of the groove. The smooth wall is simulated by $\theta = 90°$. The curvilinear mesh used to compute the flow was derived using a Schwarz-Christoffel transormation. A series of numerical experiments established that the calculation was not particularly sensitive to the grid spacing. It should be noted that despite the rather coarse spacing in the groove, numerical inaccuracies in this region have a negligible influence on the total skin friction. Less than 1% of this friction is associated with this region for a 81×19 grid with $s = 0.7$ mm and $h = 0.35$ mm (s is the spanwise distance between adjacent peaks, and h is the height of the groove).

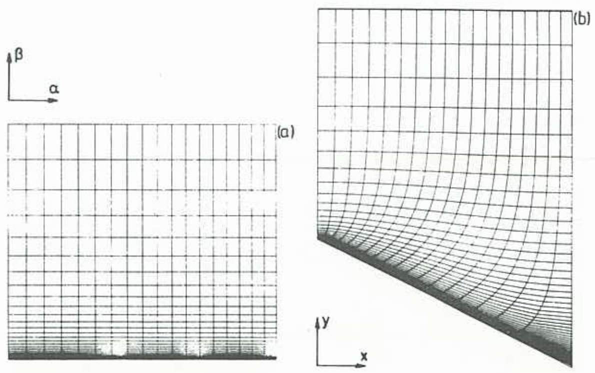

Figure 1 (a) Computation and (b) physical mesh for half a V-groove.

GOVERNING EQUATION OF THE MEAN MOTION

Starting from the ensemble averaged Navier-Stokes equations, the stationary boundary layer assumptions were applied with $\partial P/\partial x = 0$. The resulting equations in an orthogonal curvilinear system (see Djenidi et al., 1991) are written

$$\frac{\partial h^2 U}{\partial x} + \frac{\partial hV}{\partial \beta} + \frac{\partial hW}{\partial \alpha} = 0 \quad (1)$$

$$\begin{aligned}\frac{\partial h^2 UU}{\partial x} + \frac{\partial hUV}{\partial \beta} + \frac{\partial hUW}{\partial \alpha} &= \frac{\partial}{\partial \beta}\left(\nu \frac{\partial U}{\partial \beta} - h\overline{uv}\right) + \\ &\quad \frac{\partial}{\partial \alpha}\left(\nu \frac{\partial U}{\partial \alpha} - h\overline{uw}\right) \quad (2)\end{aligned}$$

where U, V, W, u, v, w are the components of the mean velocity and their fluctuations in the x (streamwise), β and α directions respectively, the local metric h is a function of β, α, y and z (y and z being the normal and spanwise directions in a cartesian system) and ν is the kinematic viscosity of the fluid. The equations have five unknowns, U, V, W, $\overline{uv}$ and $\overline{uw}$. The closure of $\overline{uv}$ and $\overline{uw}$ will be considered in the next section. Equations (1) and (2) are solved for U and V. To obtain W, it is assume that the mean streamwise vorticity is zero (everywhere in the flow), viz.

$$\Omega_x = \frac{1}{h^2}\left(\frac{\partial hW}{\partial \beta} - \frac{\partial hV}{\partial \alpha}\right) \quad . \quad (3)$$

The following evidence can be given in support of (3). The riblets are largely immersed within the inner region ($h^+ \lesssim 15$) and the Reynolds stresses within the grooves are unlikely to be of sufficient strength to generate secondary flows, e.g. counter-rotating vortices. Also the detailed experimental data that are available (Hooshmand et al., 1983; Vukoslavcevic et al., 1987; Benhalilou et al., 1991; Benhalilou, 1992) do not seem to support such motions. In spite of the strong inhomogeneity in the wall region, the mean flow recovers its two-dimensionality ($W = 0$, $\partial/\partial z = 0$) for $y^+ \geq 10$ above the riblets.

TURBULENCE MODELING

An eddy viscosity, ν_t, is used to model the Reynolds stress, viz.

$$\overline{uv} = -\nu_t \frac{\partial U}{h\partial \beta}\ , \quad \overline{uw} = -\nu_t \frac{\partial U}{h\partial \alpha}$$

with

$$\nu_t = c_\mu f_u \frac{k^2}{\epsilon} \quad . \quad (4)$$

The turbulent kinetic energy k and the "isotropic" dissipation, are evaluated by a low Reynolds number $k - \epsilon$ model (Chien, 1982). In the present curvilinear system, the equations are

$$\begin{aligned}&\frac{\partial}{\partial x}(h^2 Uk) + \frac{\partial}{\partial \beta}(hVk) + \frac{\partial}{\partial \alpha}(hWk) = \\ &\quad h^2 P_k - h^2(\tilde{\epsilon} + D) + \frac{\partial}{\partial \beta}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial \beta}\right] + \\ &\quad \frac{\partial}{\partial \alpha}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial \alpha}\right] \quad (5)\end{aligned}$$

$$\begin{aligned}&\frac{\partial}{\partial x}(h^2 U\tilde{\epsilon}) + \frac{\partial}{\partial \beta}(hV\tilde{\epsilon}) + \frac{\partial}{\partial \alpha}(hW\tilde{\epsilon}) = \\ &\quad C_{\epsilon_1} f_1 \frac{\tilde{\epsilon}}{k} h^2 P_k - C_{\epsilon_2} h^2 f_2 \frac{\tilde{\epsilon}^2}{k} + \frac{\partial}{\partial \beta}\left[\left(\nu + \frac{\nu_1}{\sigma_\epsilon}\right)\frac{\partial \tilde{\epsilon}}{\partial \beta}\right] + \\ &\quad \frac{\partial}{\partial \alpha}\left[\left(\nu + \frac{\nu_1}{\sigma_\epsilon}\right)\frac{\partial \tilde{\epsilon}}{\partial \alpha}\right] + E \quad (6)\end{aligned}$$

with $P_k = -h\overline{uv}\,\partial U/\partial \beta - h\overline{uw}\,\partial U/\partial \alpha$, $\tilde{\epsilon} = \epsilon - D$. The definitions of D and E and the magnitudes of the constants C_μ, C_{ϵ_1}, C_{ϵ_2}, σ_k and σ_ϵ are given in Table Ia. The damping functions f_1, f_2 and f_μ are shown in Table Ib (for further details see Chien, 1982). Chien's model was chosen partly because of the ease with which it can be implemented, partly because of its ability to reproduce the most basic features of a flat plate boundary layer and finally because of its accuracy in calculating the skin friction (Patel et al., 1985). This latter feature is important in the context of estimating the drag reduction due to riblets. Rodi and Mansour (1990) evaluated a number of $k - \epsilon$ models using channel flow and boundary layer DNS data. They found that while the Chien model was not as accurate close to the wall as the model they developed, it was more accurate than the Launder-Sharma (1974) and Lam-Bremhorst (1981) models.

D	E	C_μ	C_{ϵ_1}	C_{ϵ_2}	σ_k	σ_ϵ
$\frac{2\nu k}{y^2}$	$-\frac{2\nu\epsilon}{y^2}\exp(-0.5y^+)$	0.09	1.35	1.8	1.0	1.3

Table Ia

f_1	f_2	f_μ
1.0	$1 - 0.22\exp\left[-\left(\frac{Re_t}{\sigma^2}\right)\right]$	$1 - \exp(-0.0115y^+)$

Table Ib $Re_t = k^2/\nu\tilde{\epsilon}$; $y^+ = U_\tau y/\nu$

BOUNDARY CONDITIONS AND NUMERICAL METHOD

The boundary conditions were :
at the wall $\quad U = V = W = k = \tilde{\epsilon} = 0$
at the upper limit $\quad \partial/\partial y = 0$
on the lateral boundaries ($z = 0$, $z = \frac{1}{2}$) $\quad \partial/\partial z = 0$
Initial mean velocity, turbulent kinetic energy and dissipation distribution were applied at the start of the computational domain, which coincides with the beginning of the riblet surface. They were obtained from curve fits to available data (experimental or numerical) for a two-dimensional turbulent boundary layer. A control finite volume procedure (Patankar, 1980), is used for the discretisation of the equations (1–6), based on an orthogonal mesh and a staggered grid where V and W nodes are shifted from U, k and $\tilde{\epsilon}$ nodes. The solution procedure is an x-marching method, where the finite volume scheme was used at each step in the x direction (de St. Victor, 1986). The discretised system was solved with the Modified Strongly Implicit (MSI) algorithm of Schneider and Zedan (1981).

RESULTS

Preliminary calculations for a flat surface (by making θ equal to 90°, cf. Figure 1) showed that no significant errors are introduced by transforming the equations of motion and closure model equations from a cartesian co-ordinate system to a curvilinear one (Djenidi and Antonia, 1992). Furthermore, it was also observed that the model, originally developed for large R_θ seemed to perform satisfactorily at relatively smal values of R_θ. This is rather attractive in the present context since, in order to obtain detailed measurements in riblet flows, experiments have to be carried out with relatively large grooves and hence at low Reynolds numbers.

Figure 2 shows the calculation of $\overline{u^+v^+}$ and k^+ over a smooth flat plate and above riblet peaks and valleys for $R_\theta = 1000$. Results obtained for $R_\theta = 7700$ (not shown here) are similar to those of Figure 2. Further, both quantities exhibit identical behaviours within and just above ($y^+ \leq 15$) the groove.

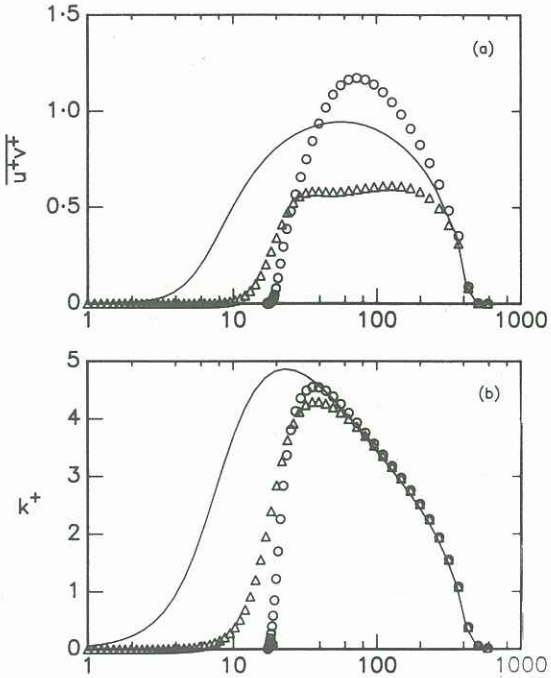

Figure 2 Distributions of Reynolds shear stress (a) and kinetic energy (b). $R_\theta = 1000$. ——, smooth flat plate; o, riblet peak; Δ, riblet trough.

Above the riblets ($y^+ \gtrsim 20$), $\overline{u^+v^+}$ is significantly different from k^+. As y^+ increases above the riblet surface, the present calculations of $\overline{u^+v^+}$ are unlikely to be correct. For example, by comparison to the unpublished data of Benhalilou (1992) [these are not shown here], it would appear that the calculation overestimates $\overline{u^+v^+}$ above a peak and underestimates it above a valley. This is probably caused by the manner in which the near-wall effect is taken into account by the model. Specifically, the present relation for f_μ does not appear to reproduce the apparent return to a two-dimensional state as y^+ increases. The use of U_τ in $y^+ = yU_\tau/\nu$ may not be appropriate since it varies significantly along the groove contour. Since $\overline{u^+v^+}$, within the inner region of the flow, depends strongly on f_μ, cf. Eq. (4), its spanwise variation may be incorrectly reproduced as the wall distance increases.

The Reynolds shear stress and turbulent kinetic energy distributions exhibit a common feature : they are all strongly damped within the groove, emphasising the importance of viscous effects on the near-wall turbulence. Moreover, this effect is restricted mainly to the inner flow region, highlighting a fundamental difference between riblet and corner flows (Djenidi and Antonia, 1992).

The calculated fritional drag ratio, $D = F_R/F_0$, where F_R is the riblet frictional force and F_0 is the frictional force on a smooth flat plate, is shown in Figure 3 for several values of h^+ (with $s^+ = 2h^+$). The calculated drag reduction strongly underestimates the measurements of Walsh (1982). Note also that, when $h^+ \to 0$, D exceeds 1 whereas

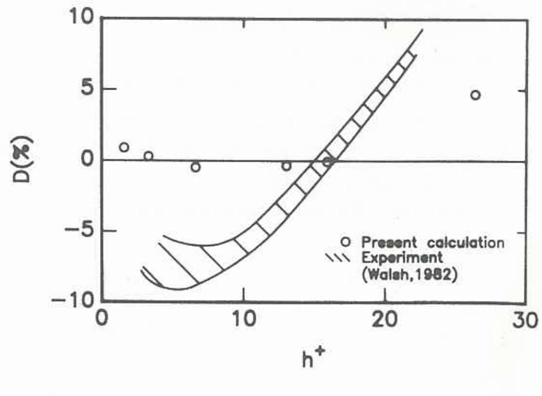

Figure 3 Dependence on h^+ of the ratio of the total frictional drag over riblet surface to the total frictional drag over a smooth flat plate.

it should certainly approach 1 (e.g. $h^+ = 0$ is equivalent to a smooth wall). Launder and Li (1991) obtained a similar result for the calculated drag in a turbulent channel flow over riblets using another version of a low Reynolds number $k - \epsilon$ model (Launder and Sharma, 1974). Low Reynolds number $k - \epsilon$ models in their present two-dimensional forms clearly fail to mimic the spanwise inhomogeneity of the turbulent flow field over riblets.

CONCLUSIONS AND FINAL DISCUSSION

A low Reynolds number $k - \epsilon$ model has been used to calculate a turbulent boundary layer over riblets. This model does not reproduce the disappearance of the spanwise variation in the turbulence quantities as the distance from the wall increases. In particular, the return towards two-dimensionality appears to be too slow which affects the simulation of the log region. While the introduction of the damping function is necessary, the choice of f_μ has a significant effect on the calculation of k and ϵ (Mansour et al., 1989) and the modelling of k and ϵ may need to be reassessed. This may in turn require the various empirical constants (C_μ, C_{ϵ_1}, C_{ϵ_2}, σ_k, σ_a) as well as the functions f_1 and f_2 to be modified. The damping effect on $\overline{uv}$ may be better simulated by using $\overline{v^2}/k$ instead of f_μ, the rationale being that the rapid reduction in $\overline{uv}$ as the wall is approached is mainly due to the damping of $\overline{v^2}$ rather than to the viscous effect (Launder and Tselipidakis, 1990). This approach avoids the use of U_τ, despite the necessity of introducing an equation for $\overline{v^2}$. Using this approach, good agreement with DNS smooth wall channel flow data was obtained by Durbin (1990) and Kawamura (1991).

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