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ON THE FLOW GENERATED BY THE UNSTEADY MOTION OF A HORIZONTAL CYLINDER THROUGH A STABLY-STRATIFIED FLUID

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ABSTRACT

Laboratory experiments are described in which a horizontal cylinder of diameter d has been translated with time-dependent velocity $u(t) = u_0[1 - \cos(\omega t)]$ and frequency ω through a stably-stratified fluid having buoyancy frequency N and mean kinematic viscosity ν. Parametric studies have been undertaken to determine and classify the flow types associated with ranges of values of the quantities ω/N, Fr $(= u_0/Nd)$ and Re $(= u_0d/\nu)$, and measurements have been made of wake overshooting behaviour. The results illustrate the dependence of the forms and degree of overshooting upon the derived parameters $u_0/\omega d$ (the Keulegan Carpenter number) and the acceleration ratios $u_0\omega/N^2d$ and $(u_0)^2/\omega\nu$, and measurements confirm that the widths of the turbulent wakes scale with $(Fr)^{1/2}$ as for the steady flow counterpart cases.

INTRODUCTION

Consideration in this paper is given to the flow generated by the periodic motion of a horizontal cylinder through an initially-quiescent stably stratified fluid in which there is a linear density gradient. Laboratory experiments have been conducted to determine the dependence of the characteristic features of the flow upon (i) the Froude and Reynolds numbers of the flow and (ii) the normalised frequency and amplitude of the periodic forcing. In the past, most studies of time-dependent flows past cylindrical obstacles have been concerned with homogeneous fluids. These studies have often been driven by considerations of wave- and current-induced loadings on flexible offshore structures, and, for this reason, attention has been confined primarily to vertical cylinders (see, for example, Keulegan and Carpenter (1958), Griffin and Ramberg (1976), Bearman and Graham (1980), Sumer *et al.* (1989, 1991)). Counterpart studies of stratified fluids have been restricted so far to experiments by Stevenson and Thomas (1969) and Stevenson (1973) on the phase configurations of internal waves generated by the travelling oscillatory motion of a horizontal cylinder in a stratified fluid.

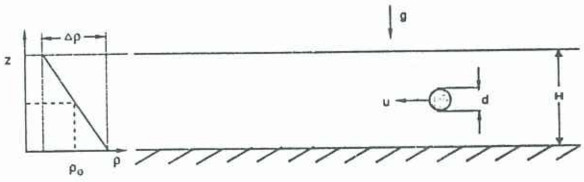

Fig 1. Schematic view of experimental arrangement

In the present study, the results of Boyer *et al* (1989) on steady forcing have been extended to incorporate periodicity, and particular attention has been paid to the resulting effects upon wake flows. Internal wave motions have not been of primary interest.

NOTATION

Fig 1 shows the problem under consideration. A horizontal circular cylinder (diameter d) is moved horizontally with frequency ω and time-dependent velocity

$$u(t) = u_0 - u_1 \cos \omega t, \qquad (1)$$

through a stratified fluid of mean kinematic viscosity ν and depth H, in which there is a linear density gradient

$$\rho(z) = \rho_o + (\Delta\rho/2)(1 - 2z/H), \qquad (2)$$

In (1) and (2), ρ_o is the density of the fluid at mid depth and $\Delta\rho$ is the density difference between the bottom ($z = 0$) and the top ($z = H$) of the fluid, and u_0 and u_1 are the steady and oscillatory components of the motion.

The resulting flow can then be described in terms of:

$Re = u_0 d/\nu$, the Reynolds number,
$Fr = u_0/Nd$, the internal Froude number,
$\Phi = u_1/u_0$, the velocity ratio, and
$\Theta = \omega/N$, the frequency ratio,

and the geometrical ratios d/H, d/W and d/L. Here, W and L are the width and length respectively of the (rectangular) fluid container, and N is the buoyancy frequency (defined in the usual way as $N^2 = g\Delta\rho/\rho_0 H$). The Keulegan-Carpenter number $K = u_1/\omega d$, the ratio of the amplitude of the oscillation to the cylinder diameter, may be derived easily from the above set. The range of values investigated is indicated in Table 1

Table 1.

$d = 1.6$ cm; $H = 18.0$ cm
$\Phi = u_1/u_0 = 1.0$; $0.85 \leq N \leq 1.15\ s^{-1}$
$0.5 \leq u_0 \leq 3.0$ cm s^{-1}
$76 \leq Re \leq 648$
$0.20 \leq Fr \leq 3.70$
$0.29 \leq \Theta \leq 6.91$
$0.11 \leq K \leq 5.03$

EXPERIMENTAL ARRANGEMENT

The experiments were conducted in a long channel of length 15m, using the towing arrangement described by Boyer *et al* (1989). Brine and fresh water solutions were used to stratify the channel, and flow visualisation was accomplished through the use of a shadowgraph. A video recorder and still camera mounted on the towing carriage recorded the shadowgraph images for later quantitative analysis. The form of the motion of the cylinder was preset by entering the required values of u_0, u_1, and ω in the control software for the towing carriage.

RESULTS

As anticipated, effects of periodicity in the cylider motion are most noticeable for high values of the forcing

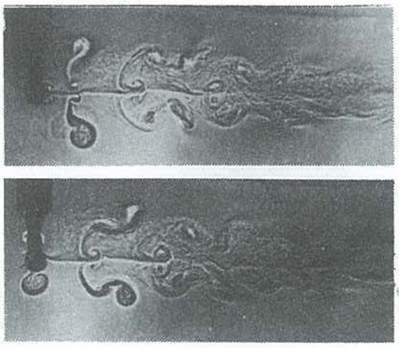

Fig 2. Flow pattern for Re = 397, Fr = 1.59, $\Phi = 1$, $\Theta = 2.45$ and ωt = (i) $\pi(2n + 1)$, (ii) $2n\pi$

amplitude and frequency. Fig 2 shows an example of a flow sequence for a case characterised by relatively high values of Θ and Re, where the turbulent wake is disrupted and modified by the small scale structures associated with the periodic forcing. Examination of the flow visualisation data revealed that the flows could be conveniently classified into 13 flow types in Re:Fr:Θ parameter space (Davies *et al.* 1992).

Overshooting

Upstream overshooting of the cylinder by downstream fluid is a common occurrence for certain parameter ranges, as the cylinder decelerates to rest in part of its cycle. For cases in which viscous effects are negligible, the overshooting phenomenon may be analysed in terms of the relative importance of (i) the inertial drag force per unit mass u_0^2/d exerted on the wake fluid (ii) the buoyancy acceleration N^2d, and (iii) the body acceleration $u_0\omega$. (For cases in which viscous effects cannot be neglected, the viscous acceleration $\nu^{1/2}\omega^{3/2}$ must also be considered). The likelihood of overshooting occurring can then be quantified in terms of the dimensionless parameters $K_0 = u_0/d\omega$, $R = N^2d/u_0\omega$ and $B = u_0^2/\nu\omega$.

Unsteady effects

Because of the variations in the values of the instantaneous Reynolds and Froude numbers of the flows during a given forcing cycle, the structures of the flows vary accordingly during the cycle. An intriguing question is then the degree to which the flow which is observed at any instant is representative of the known *steady* flow (Boyer *et al.* 1989) at the appropriate instantaneous Reynolds and Froude numbers.

Conditions for the occurrence of phenomena associated specifically with periodicity of forcing can be estimated for the extreme cases of buoyancy-dominated (Fr<<1) and inertia-dominated (Fr>>1) flows as

$\Theta > (\Theta)_{crit}$, and
$K << K_{crit}$

respectively. Since $K = (Fr)/\Theta$, the value of the frequency ratio Θ at which effects of unsteadiness will be observed for Fr >> 1 will be dependent upon the Froude number of the flow. In particular, as the value of Fr increases, unsteady forcing effects will be expected only if Θ assumes progressively greater values to ensure that the ratio Fr/Θ remains less than a critical value K_{crit}.

Wake dimensions

Measurements were made of the maximum dimensionless width $\gamma^* = (\gamma/d)$ of the wake in a given forcing cycle, for all cases in which the flow in the wake was turbulent. Following Boyer *et al* (1989), the γ^* data were then plotted against $(Fr)^{1/2}$, where, it is remembered, Fr is defined in terms of the steady velocity u_0. Fig 3 shows that the data are represented satisfactorily by the $(Fr)^{1/2}$ scaling,

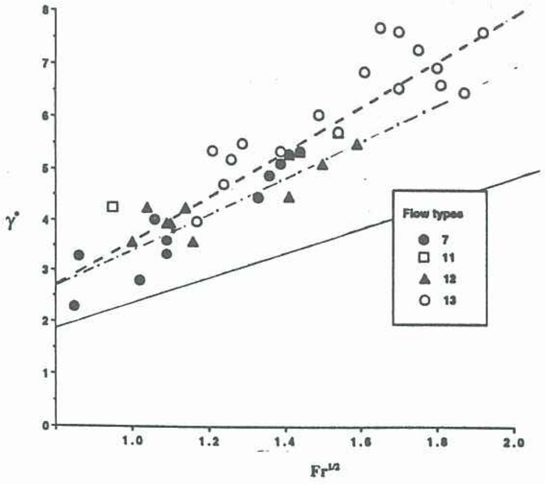

Fig 3. Plot of γ^* versus $(Fr)^{1/2}$ for turbulent wakes.

though the dimensionless maximum wake widths are significantly greater than those obtained by Boyer *et al* (1989) for otherwise-identical steady forcing conditions. The discrepancy is reduced significantly if the Froude number is redefined in terms of the maximum velocity ($2u_0$) generated within the forcing cycle (see Fig 3). Attempts to scale the instantaneous maximum wake width at any stage in the forcing cycle with the square root of the relevant instantaneous Froude number are also promising, but the scatter in the data preclude the drawing of firm conclusions on this issue.

CONCLUDING REMARKS

The study has revealed a number of flow phenomena which are associated specifically with the periodicity of the flow, and simple scaling considerations support the data in this respect. Attempts to classify the flow visualisation data in Re:Fr:Θ parameter space have been made, and this classification has been shown to accord well with the scaling arguments advanced for periodicity effects. The phenomenon of overshooting has been described, and data have been presented to show that the degree of overshooting is controlled essentially by the relative values of the parameters K, B and R. Experimental data on wake widths for turbulent flows accord well with scalings based upon $(Fr_{max})^{1/2}$ and $(Fr_{inst})^{1/2}$, where the relevant Froude numbers are scaled with the maximum and instantaneous values respectively of the cylinder velocity.

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